Newton's Clock
Chaos in the Solar System

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and his contemporaries, though lacking many of the advanced mathematical tools that mathematicians now take for granted, could solve in a few minutes numerous problems that a majority of present-day mathematicians would have to struggle with far longer to solve.

Despite the mathematical language that Newton employed, the publication of the *Principia*, fueled by Halley’s promotion of an atmosphere of keen anticipation, created a tremendous stir. Whether or not the book was actually read, the philosophy it embodied had an immediate impact, and it colored philosophical discourse ever afterward. It also represented the true beginning of theoretical physics.

The book’s introduction contains the famous three laws of Newton, the seeds of which had appeared in many earlier writings, especially those of Galileo Galilei, René Descartes, and Pierre Gassendi. Newton’s contribution was the expression of these three laws of motion in a terse, quantitative form, gathered together to form the foundation of theoretical mechanics. As part of this formulation, he presented a novel and invaluable definition encapsulating the concept of “quantity of motion.” He stated it (in Latin) as follows: “The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.” In modern terms, this means that every object has an associated quantity, called momentum, obtained by multiplying its mass and its velocity.

The first law, derived from the investigations of Galileo, Descartes, and others, specifies that “every body continues in its state of rest, or of uniform motion in a [straight] line, unless it is compelled to change that state by forces impressed upon it.” Often called the law of inertia, this statement expresses the counterintuitive fact that, contrary to the evidence of everyday experience that all things—from a stone rolling down a hillside to a spinning top—will eventually stop moving, no force at all is required to sustain motion, and that a moving body naturally travels at a constant speed in an unchanging direction. This uniform motion persists in the absence of such forces as air resistance and friction.

Newton and his contemporaries understood the importance of the law of inertia in explaining the motion of a planet around the sun. In the simplest possible case—that of a circular orbit—the planet at each instant moves uniformly along a tangent to its orbit; simultaneously, it is drawn by the ever-present attraction of the sun. These two motions combine in such a way that the planet traces out a circular path. Planets and comets, meeting less resistance in emptier, wider spaces, preserve their motions for much longer than ordinary terrestrial objects.

The second law states that “the change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which the force is impressed.” In other words, Newton related force to a change in momentum (not to acceleration, as most contemporary physics textbooks state). If the mass remains constant, a change in motion is then equivalent to a change in velocity; that is, to the acceleration of a material object. Thus, all forces cause acceleration (a change in the speed or direction of an object), and doubling the force means doubling the acceleration. Interestingly, Newton’s second law also describes a situation in which the mass of an object changes. This type of motion didn’t become important until the advent of rockets, which propel themselves by spewing out hot gases and thus continuously change their mass.
How a sequence of discrete tugs of attraction influences a planet’s motion. Without a tug at B, the planet P would move off in the direction of X. When the gravitational force exerted by the sun acts continuously, the planet takes an elliptical path.

The importance of Newton’s choice of momentum as a key concept in dynamics lies in the notion that momentum is one of two quantities that, taken together, yield everything there is to know about a dynamical system at a given instant. The second quantity is simply position, which determines the force’s strength and direction. Newton’s insight concerning the pairing of momentum and position was placed on firmer ground more than a century and a half later by the mathematicians William Rowan Hamilton and Karl Gustav Jacob Jacobi. It is a duality that lies at the heart of modern dynamical theory.

The third law says that “to every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.” Often misunderstood,