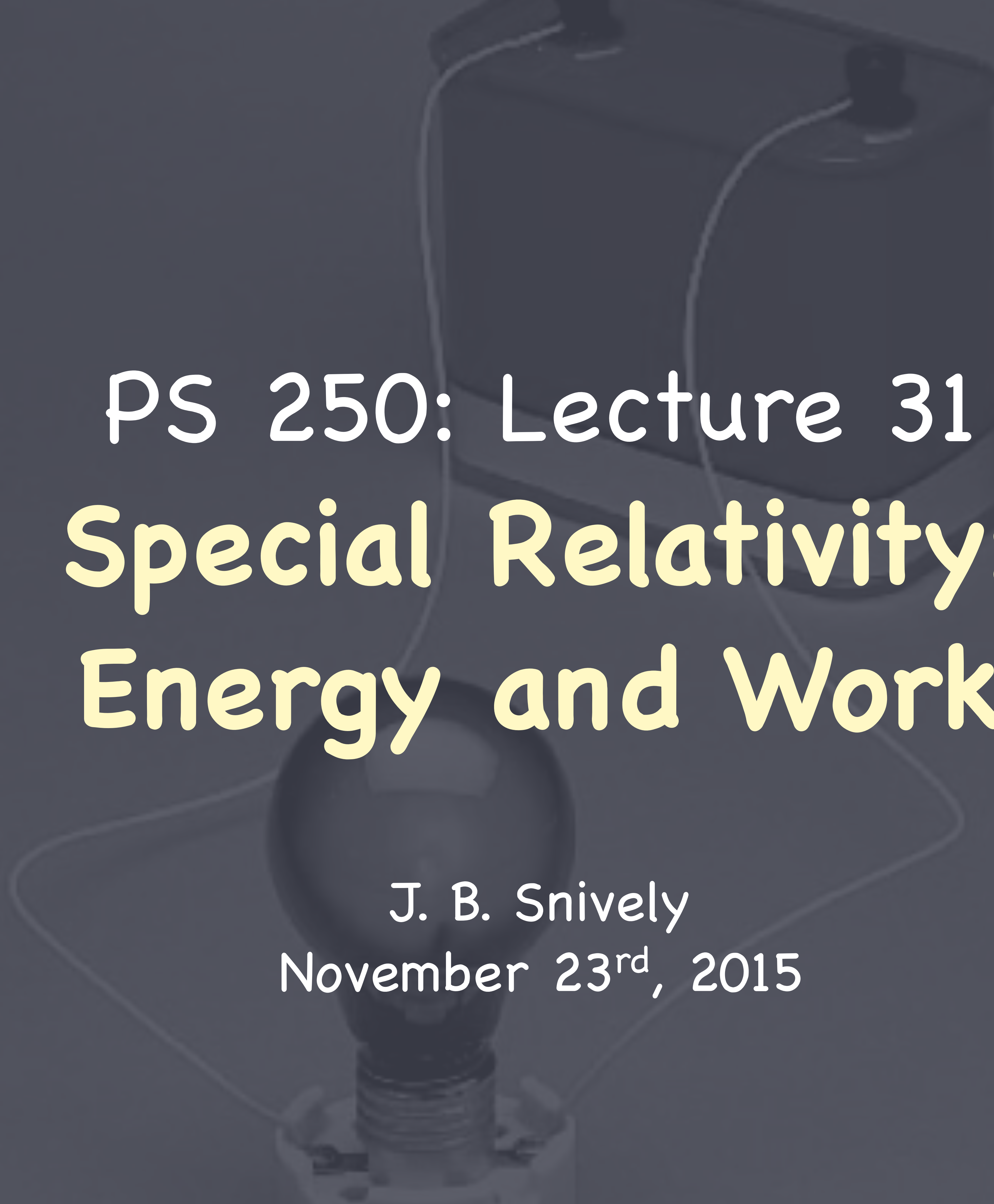

PS 250: Lecture 31

# Special Relativity: Energy and Work

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# Today's Class

- $\boxed{\text{Time Dilation / Length Contraction}}$
- Momentum, Energy and Work
- Summary

# Recall: Einstein's Postulates

## Foundation of Special Relativity

- **Principle of relativity:**

  The laws of physics are the same in every inertial (non-accelerating) reference frame.

- **Speed of Light is a Constant:**

  The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.

# Two Big Conclusions:

**Moving clocks run slow.**
(Time Dilation)

**Moving objects are shortened.**
(Length Contraction)

# Time Dilation

**Elapsed $\Delta t > \Delta t_o$ on the Ground:**

$$\boxed{\Delta t = \frac{\Delta t_o}{\sqrt{1 - u^2/c^2}}}$$

**Or, $\Delta t = \gamma \Delta t_o$, where $\gamma$ is given by:**

$$\boxed{\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}}$$

# Length Contraction

As a result of time dilation, length of the moving object is reduced relative to external observers:

**Observed $l < l_o$ on the Ground:**

$$\boxed{l = l_o\sqrt{1 - u^2/c^2}}$$

**Or, $l = l_o/\gamma$, where** $\gamma$ is given by:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}}$$

# Today's Class

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# Relativistic Momentum

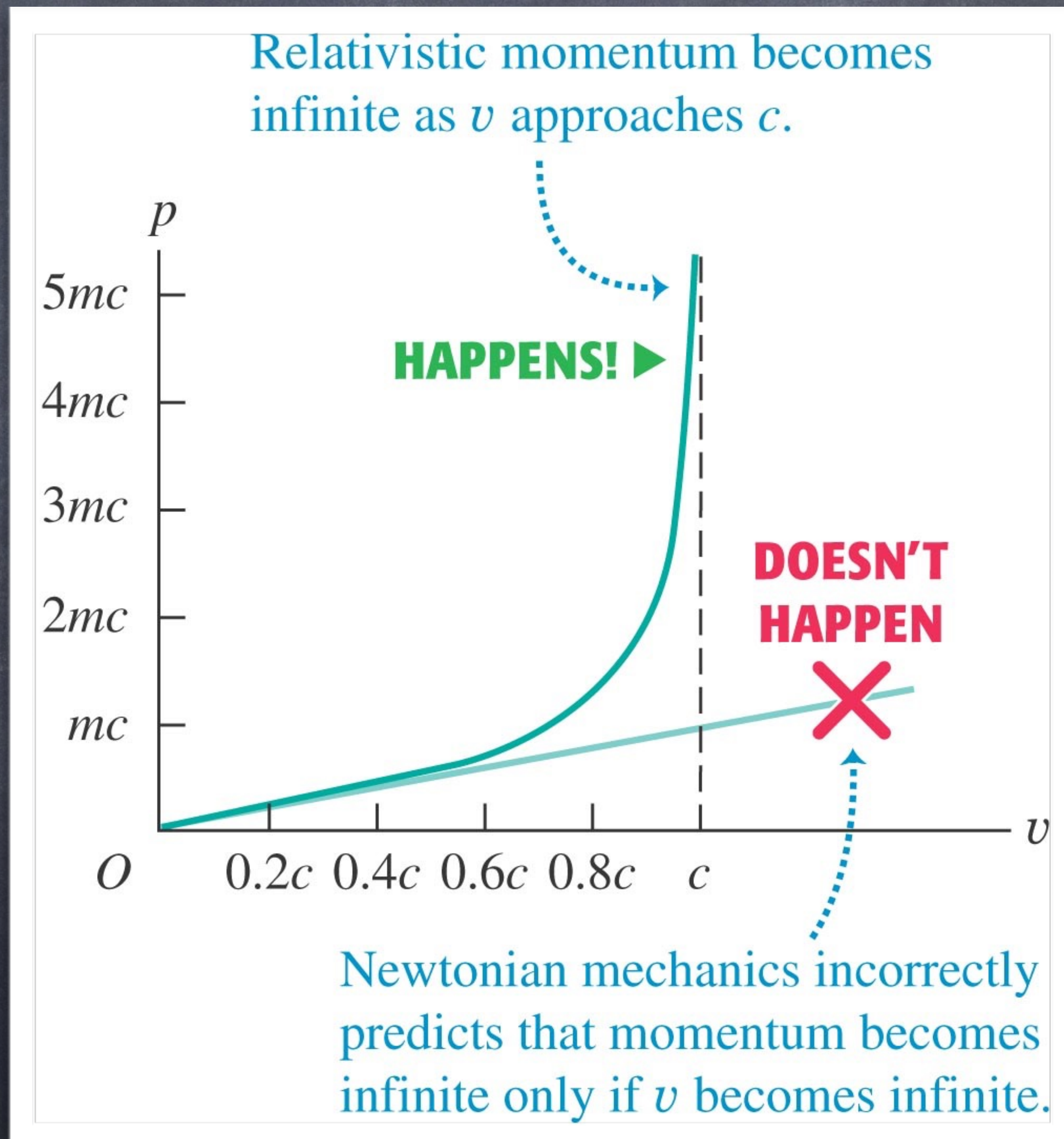
As v approaches c,
momentum approaches ∞!

$$\boxed{\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}}$$

Or, in terms of "gamma":

$$\boxed{\vec{p} = \gamma m\vec{v}}$$

# Relativistic Momentum



# Newton's Laws + Relativity

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{so:} \quad \vec{F} = \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

For force, velocity, and acceleration in x-direction:

$$\boxed{F_x = \frac{ma_x}{(1 - v_x^2/c^2)^{3/2}}}$$

Acceleration, for F || a || v:

$$a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

**Important Conclusion:** As speed approaches c, acceleration approaches zero regardless of F!

# Relativistic Energy

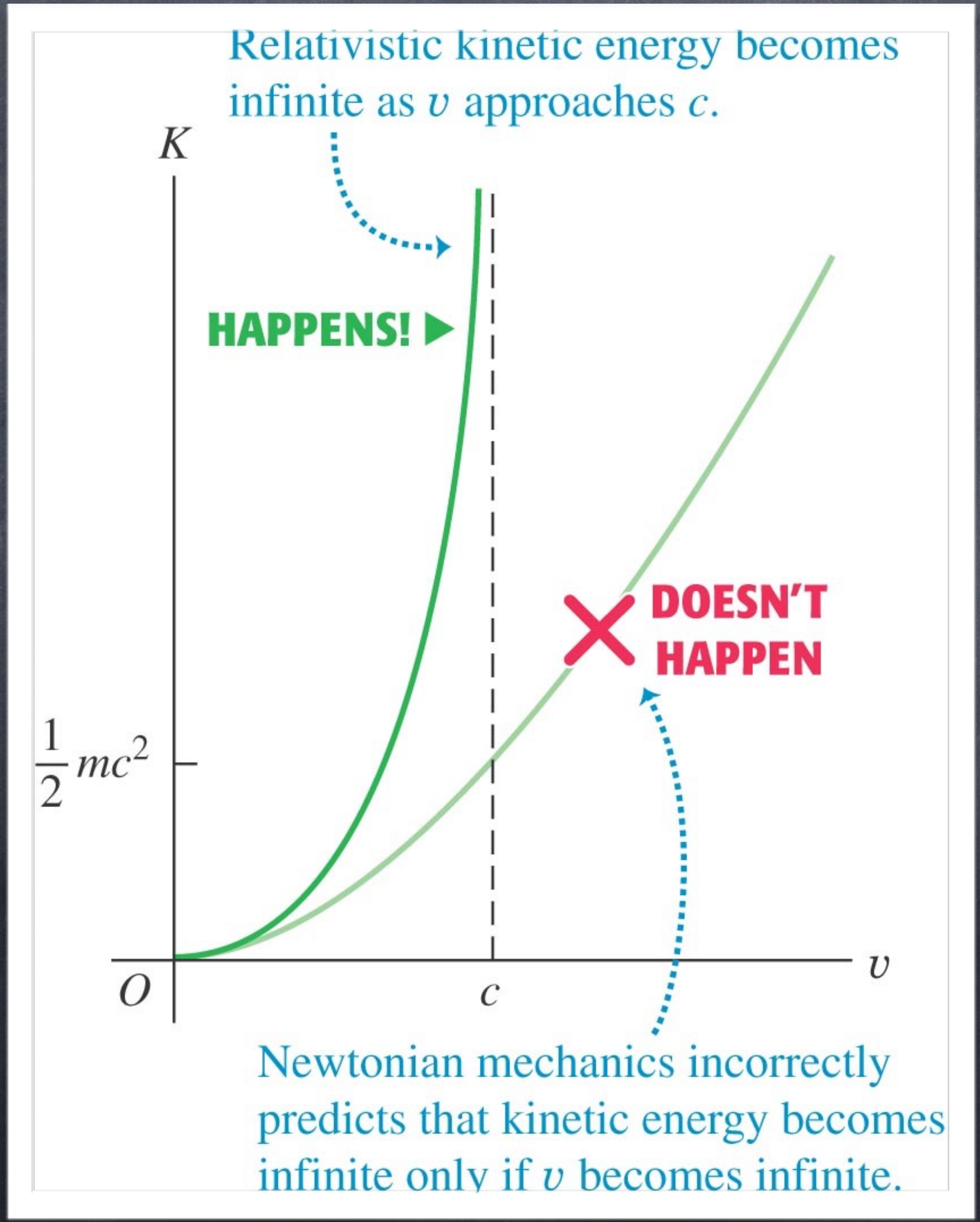
To accelerate a particle from 0 to v,
Work done = Kinetic energy imparted:

$$W = K = \int_0^v \frac{m v_x dv_x}{(1 - v_x^2/c^2)^{3/2}} \boxed{= \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2}$$

Or, in terms of "gamma":

$$K = \gamma mc^2 - mc^2 \boxed{= (\gamma - 1)mc^2}$$

# Relativistic Energy



# Relativistic Energy

Consider Kinetic Energy:

$$K = \gamma mc^2 - mc^2 \boxed{= (\gamma - 1)mc^2}$$

Total Energy        Rest Energy

Therefore, Total Energy **E = Kinetic + Rest**:

$$E = \gamma mc^2 = K + mc^2$$

# “Rest Energy” E=mc$^2$

For a particle at rest, energy exists that is proportional to its mass:

$$\boxed{E = mc^2}$$

Total Energy (Rest+Kinetic) can be expressed in terms of momentum and energy:

$$E^2 = (mc^2)^2 + (pc)^2$$

A moving particle with zero rest mass also has energy and momentum:

$$\boxed{E = pc}$$

# What this means...

- Particle rest mass can be converted into energy ($E=mc^2$). The energy released can be related to the change in mass of the system!
- Likewise, energy can be converted to rest mass (i.e., rest energy of a new particle produced during a collision)!
- Particles without mass (photons, for example) still carry energy and momentum

# Summary / Next Class:

- **Mastering Physics** for Next Monday
- **Mastering Physics** for Next Wednesday