PS 250: Lecture 30

Special Relativity: Time Dilation and Length Contraction

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November 20th, 2015

Today's Class

- Special Relativity
- Time Dilation
- Length Contraction
- Summary

Einstein's Postulates

Foundation of Special Relativity

- **Principle of relativity:**

  The laws of physics are the same in every inertial (non-accelerating) reference frame.

- **Speed of Light is a Constant:**

  The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.

What this implies...

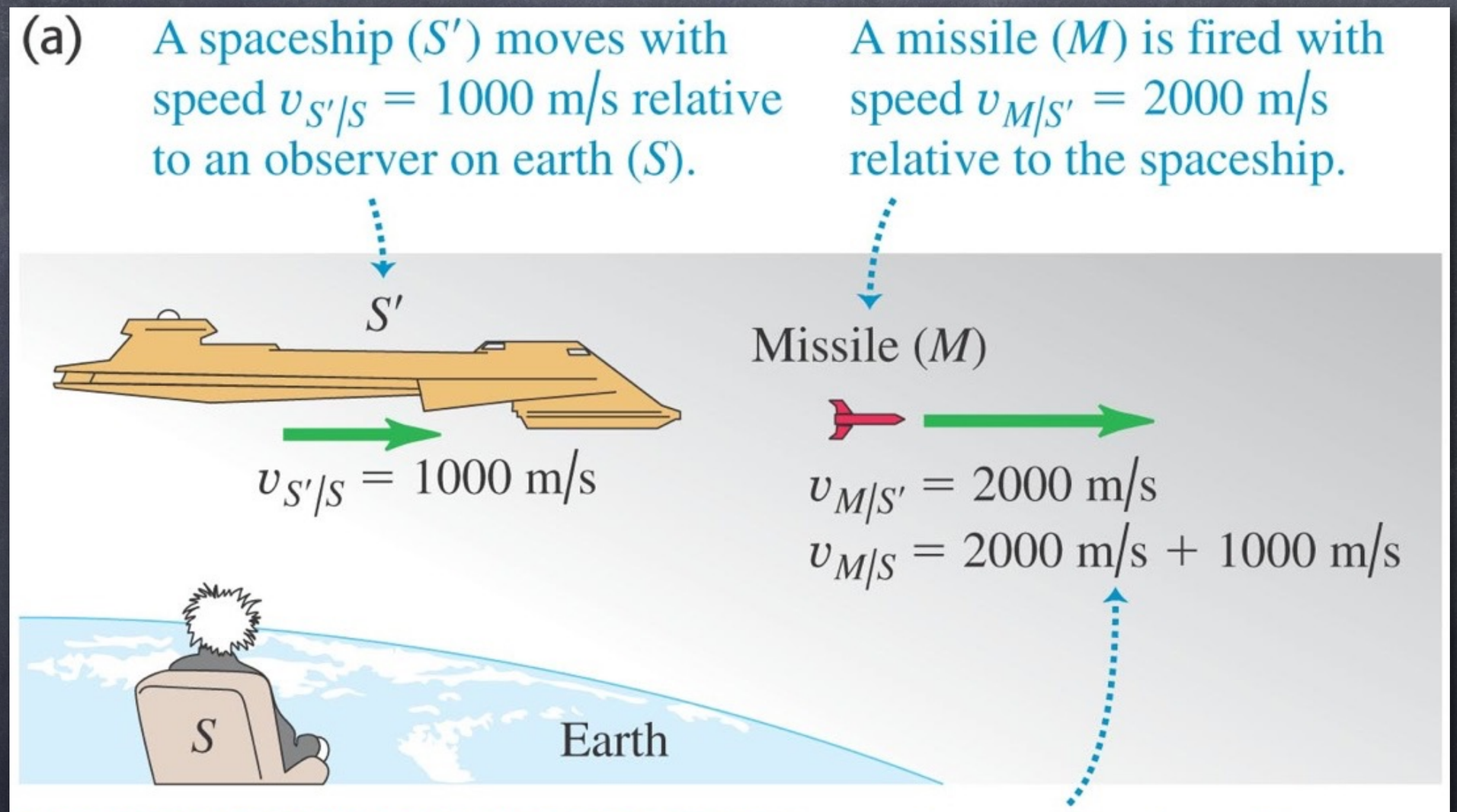
- There is no specific medium that light propagates through - no "ether" - and (in vacuum) it has the same speed in any direction relative to any observer.
- Laws of electro-magnetics apply in any inertial reference frame. (You can move a magnet inside a solenoid to produce an EMF, or a solenoid around a magnet!)
- It is not possible for an observer to travel at the speed of light.

Two Big Conclusions:

Moving clocks run slow.
(Time Dilation)

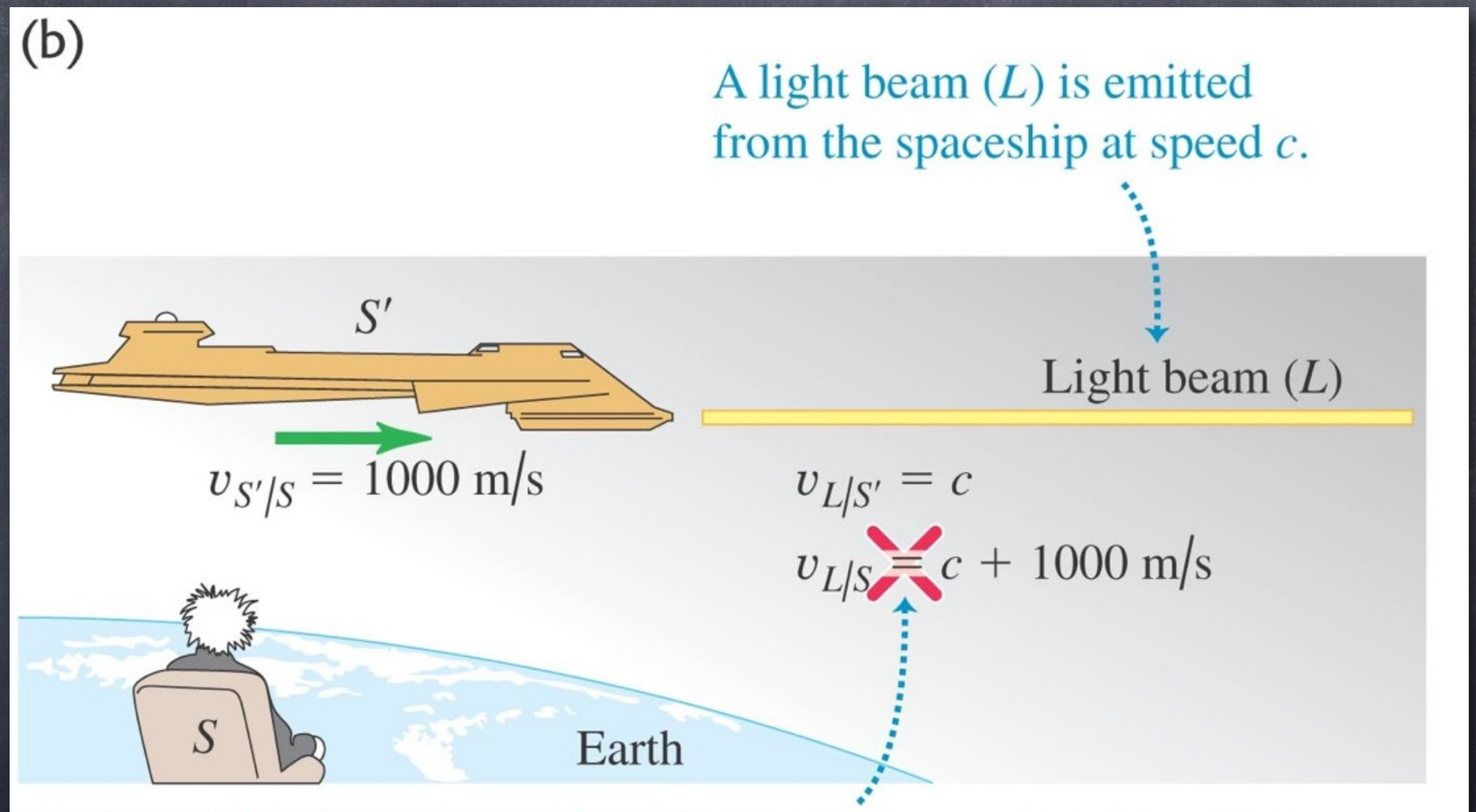
Moving objects are shortened.
(Length Contraction)

Newton vs. Einstein



NEWTONIAN MECHANICS HOLDS: Newtonian mechanics tells us correctly that the missile moves with speed $v_{M/S}$ = 3000 m/s relative to the observer on earth.

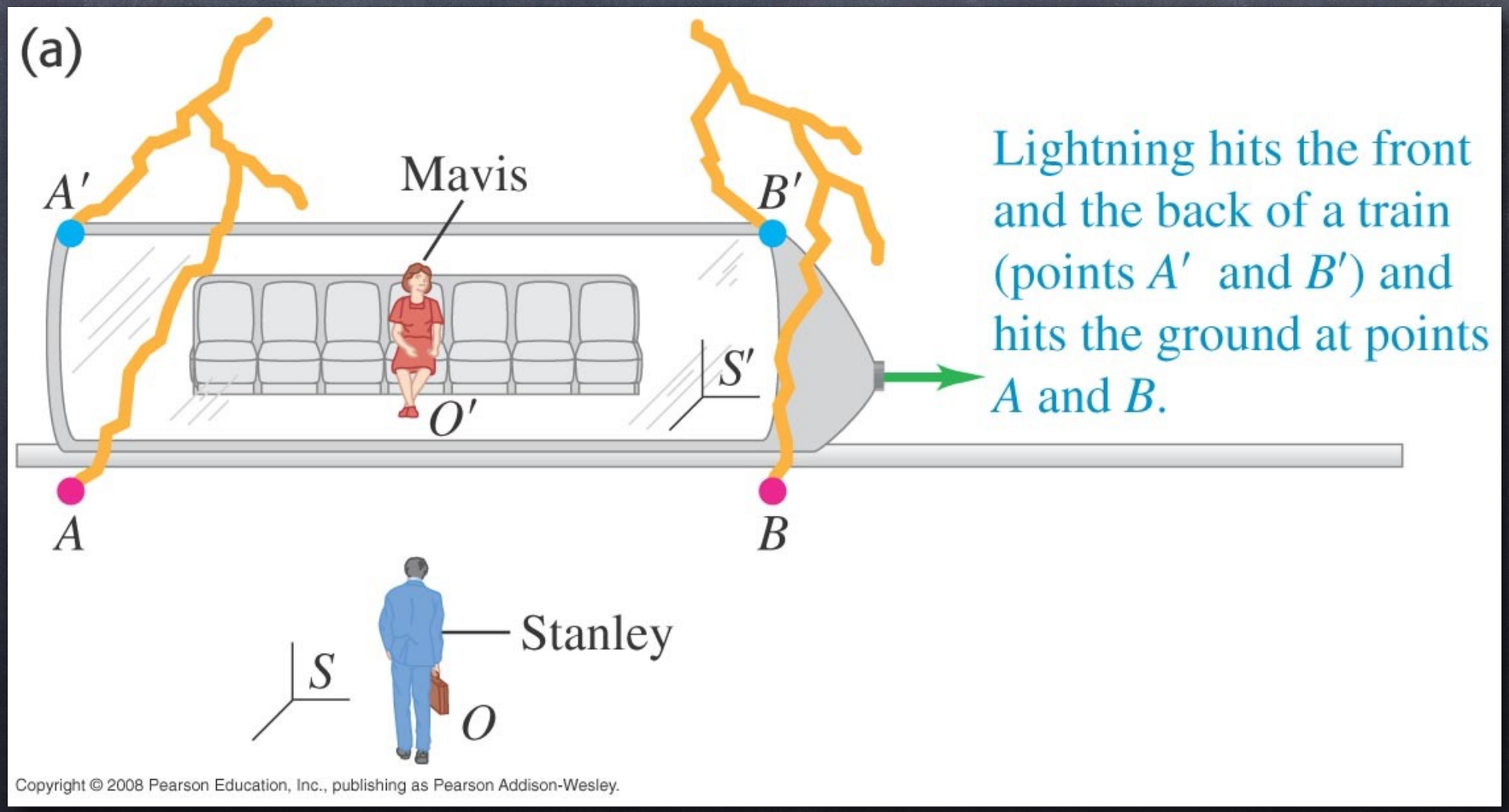
Newton vs. Einstein

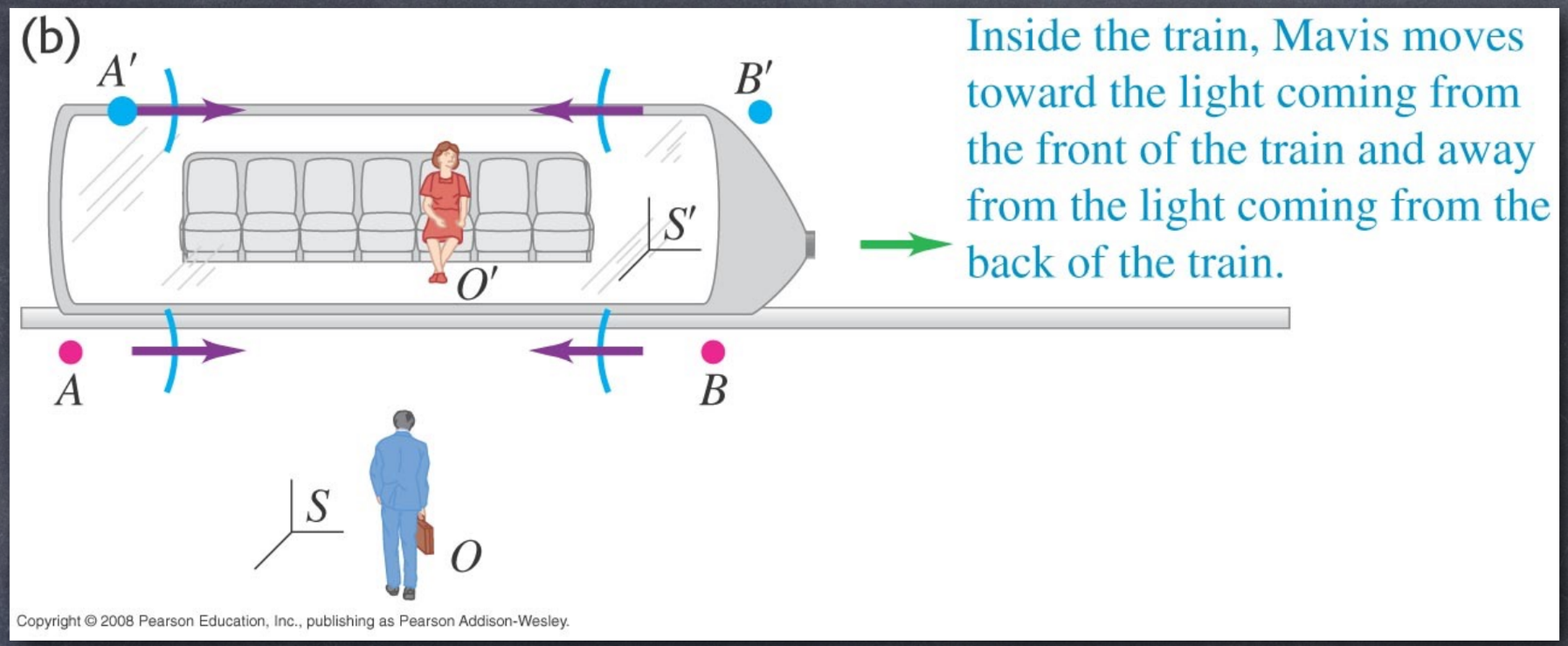


NEWTONIAN MECHANICS FAILS: Newtonian mechanics tells us *incorrectly* that the light moves at a speed greater than c relative to the observer on earth ... which would contradict Einstein's second postulate.

Relativity of Simultaneity

"Two events that are simultaneous in one inertial system are not, in general, simultaneous in another"




(b)
A′
B′
S′
O′
A
B
S
O
Inside the train, Mavis moves toward the light coming from the front of the train and away from the light coming from the back of the train.
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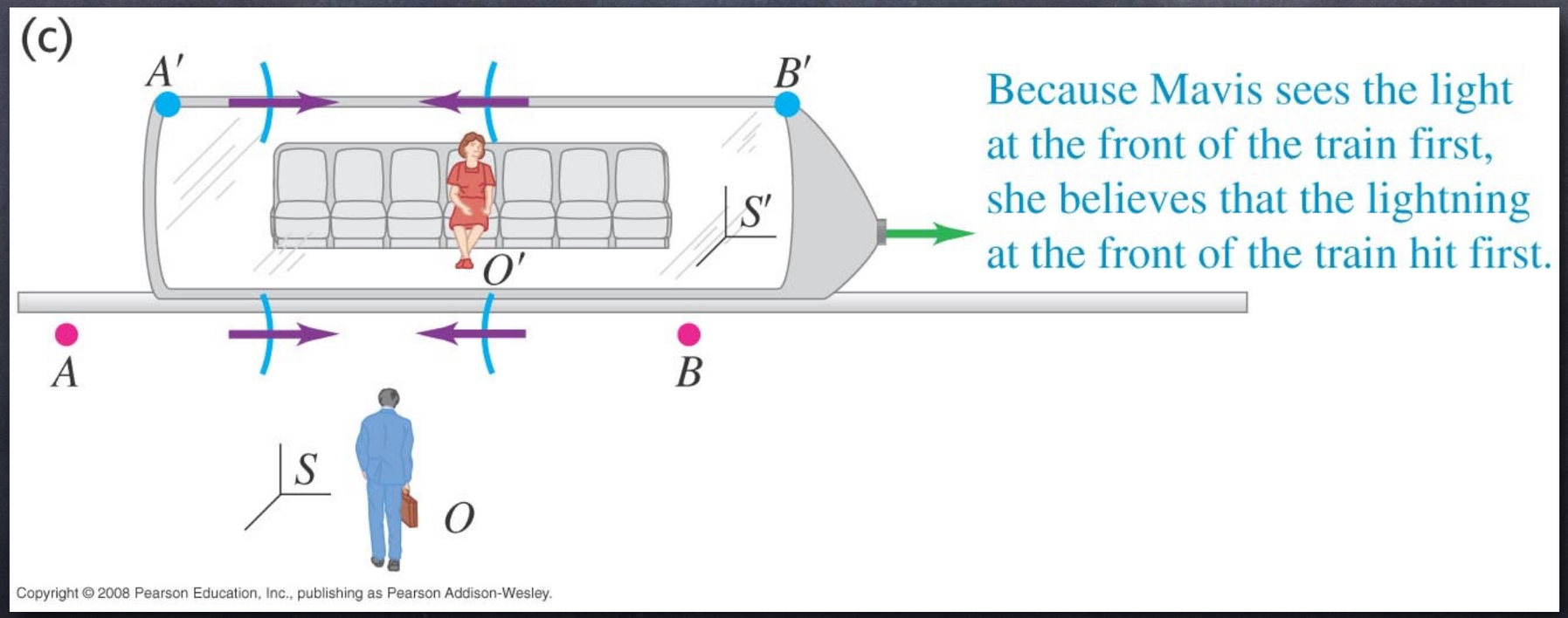
(c)
A′
B′
S′
O′
A
B
S
O
Because Mavis sees the light at the front of the train first, she believes that the lightning at the front of the train hit first.
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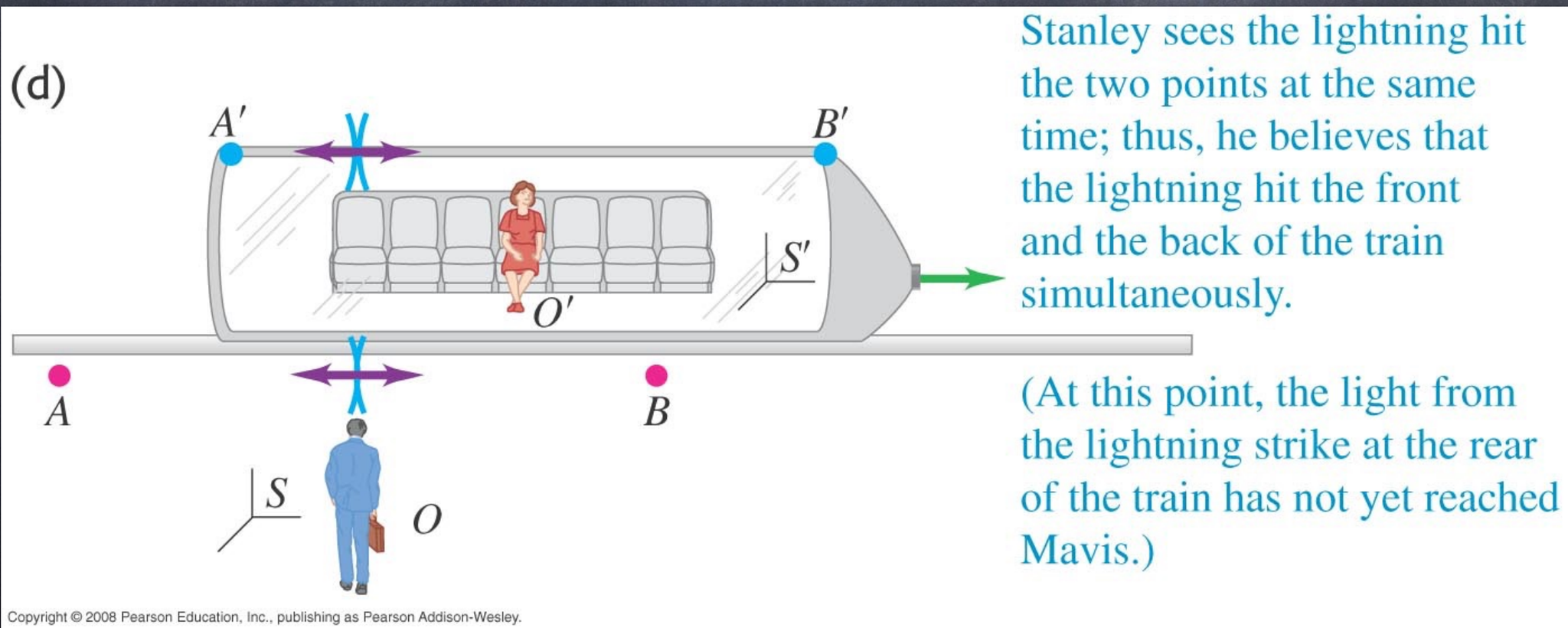
Relativity of Simultaneity

In Mavis' reference frame, the strikes could not have been simultaneous: She is sitting in the middle of the train, and the speed of light is constant!



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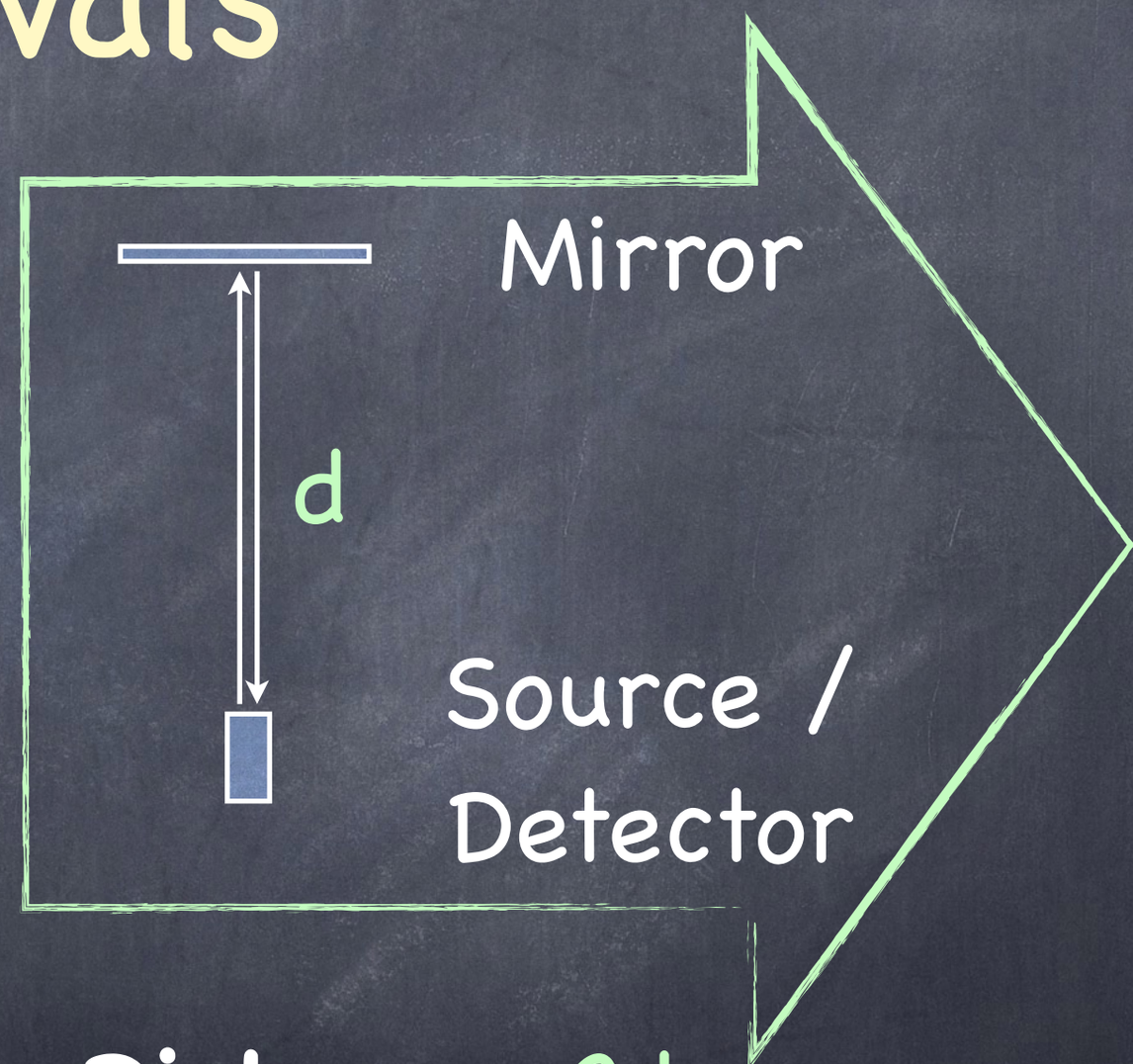
Relativity of Time Intervals

On the Spaceship
(inertial reference frame, moving very quickly relative to the ground):

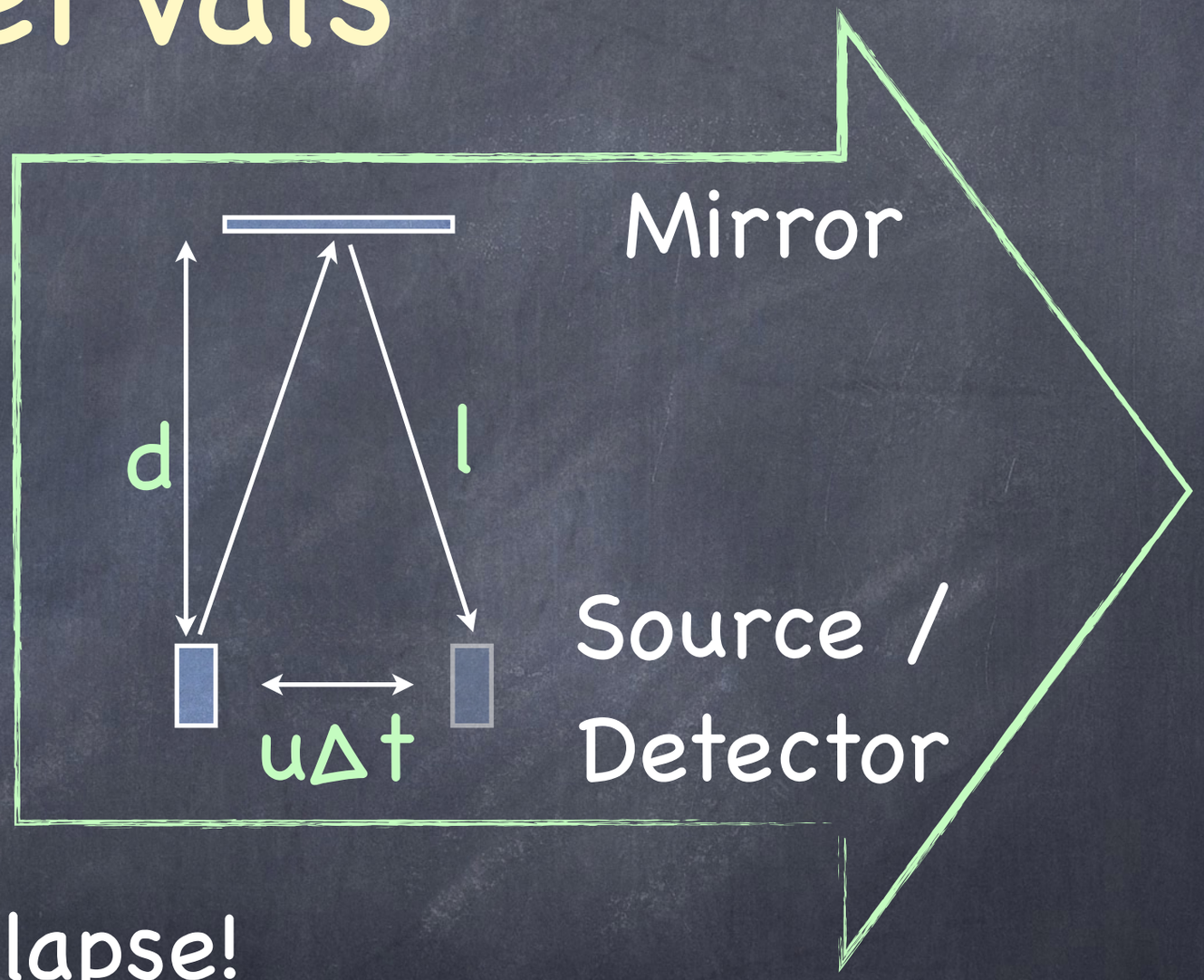


Light Propagation Distance: $2d$

Propagation Time: $\Delta t_0 = 2d / c$

Relativity of Time Intervals

On the Ground (inertial reference frame, fixed relative to the spaceship):



Some $\Delta t > \Delta t_0$ must elapse!

Light Propagation Distance 2l, where l is given by:

$$l = \sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

Time Dilation

Elapsed $\Delta t > \Delta t_o$ on the Ground:

$$\boxed{\Delta t = \frac{\Delta t_o}{\sqrt{1 - u^2/c^2}}}$$

Or, $\Delta t = \gamma \Delta t_o$, where γ is given by:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}}$$

Today's Class

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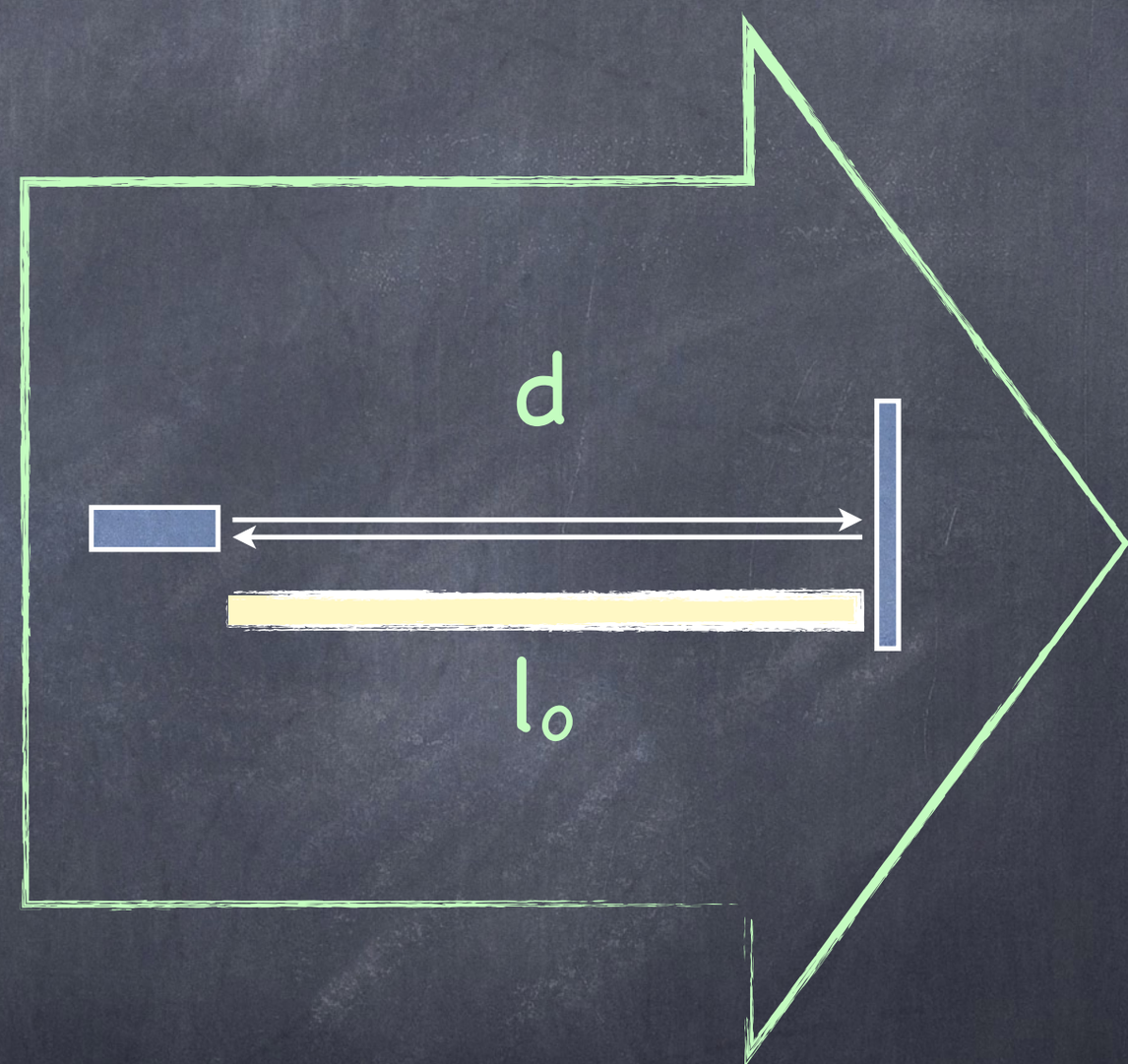
Relativity of Length

On the Spaceship
(inertial reference frame,
moving very quickly
relative to the ground):

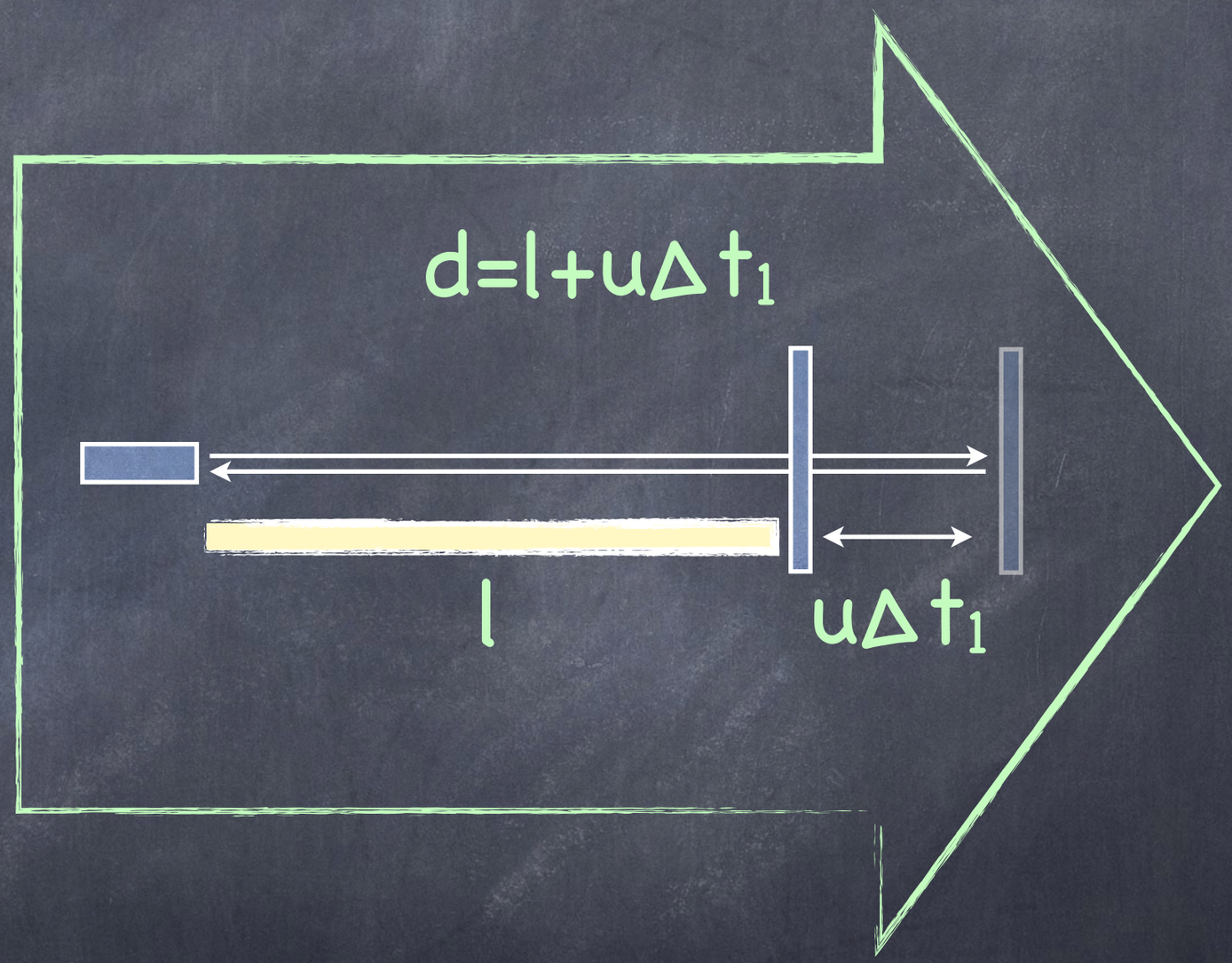


Light Propagation Distance: $2d = 2l_0$

Propagation Time: $\Delta t_0 = 2l_0 / c$

Relativity of Length

On the Ground (inertial reference frame, fixed relative to the spaceship):



Propagation Distance $d_{out} = l+u\Delta t_1$, $d_{back} = l-u\Delta t_2$

Total time: $\Delta t = \Delta t_1 + \Delta t_2 = \dfrac{l}{c-u} + \dfrac{l}{c+u}$

Length Contraction

As a result of time dilation, length is reduced:

Observed $l < l_o$ on the Ground:

$$l = l_o\sqrt{1 - u^2/c^2}$$

Or, $l = l_o/\gamma$, where γ is given by:

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Summary / Next Class:

- **Homework** for Today
- **Mastering Physics** for Next Monday
- **Homework** for Next Wednesday