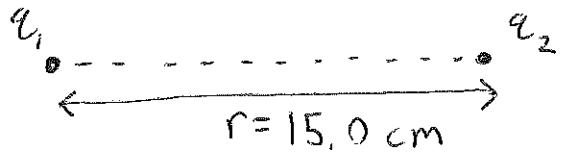
21.9 Two spheres with positive charges




a) In this case, we assume both charges are equal (Note that, because both charges are positive, there is a force trying to push them apart)

-Apply Coulombs law to this problem

$q_1 = q_2 = q$

$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2}$ Now solve for $q$,

$$q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\,m\sqrt{\frac{0.220\,N}{8.988\times10^9\,N\cdot m^2/C^2}} = \boxed{7.42\times10^{-7}\,C}$$

On each sphere

b) Here, one sphere has 4 times the charge of the other

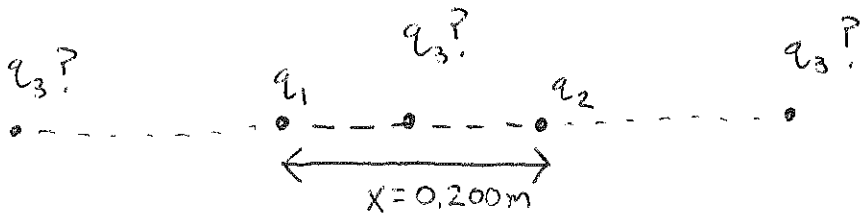
$q_2 = 4q_1$, Apply Coulombs law the same as before

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \rightarrow q_1 = r\sqrt{\frac{F}{4(1/4)\pi\epsilon_0}}$$

$$q_1 = \boxed{3.71\times10^{-7}\,C}$$

remember! $q_2 = 4q_1 = \boxed{1.48\times10^{-6}\,C}$

21.19 Here, we have 3 charges arranged along the x-axis. We know where 2 of the charges are located, and we know the net force on one of the charges. We want to find the location of the third charge (this is a common problem, remember how to do it!)




Where should the third charge be located? It could be to the left, right, or in between $q_1$ and $q_2$. Lets look at the force $q_2$ exerts on $q_1$:

$$F_{2-1} = K\frac{|q_1 q_2|}{r_{2-1}^2} \quad \text{where } K = \frac{1}{4\pi\epsilon_0},$$

$q_1 = +3.00\,\mu C$
$q_2 = -5.00\,\mu C$
$r_{2-1} = 0.200\,m$

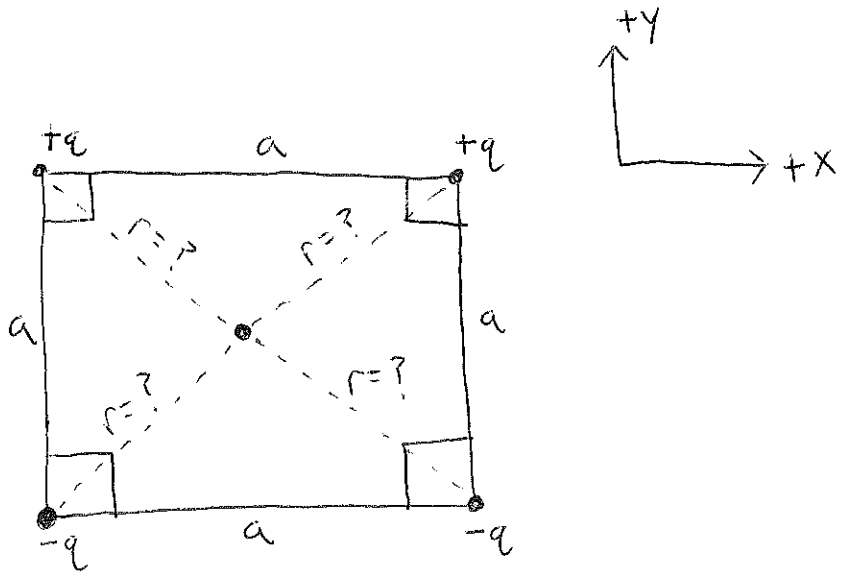
so $F_{2-1} = 3.37\,N$, recall the two charges are opposite (one positive the other negative), so it is an attractive force. Now, lets look at the sum of the forces on $q_1$:

$$\Sigma F_{net} = F_{2-1} + F_{3-1} \rightarrow F_{3-1} = -F_{2-1} + F_{net} = -3.37N - 7.00N = -10.37$$

Since $F_{3-1}$ is negative, it must be to the left of $q_1$,

$$F_{3-1} = K\frac{|q_1 q_3|}{r_{3-1}^2} \rightarrow |r_{3-1}| = \sqrt{\frac{K|q_1 q_3|}{F_{3-1}}} = 0.144m \text{ so } \boxed{r_{3-1} = -0.14}$$

21.30




- Before we do anything, lets use some simple math to find the distance of the point of interest from each charge: $r = \sqrt{(a/2)^2 + (a/2)^2} = a/\sqrt{2}$

- Next, we want to find the magnitude of each E-field

$E_q = \frac{kq}{r^2} = 2\frac{kq}{a^2}$ (This is a very simplified assumption, in the future we will take a more rigorous approach)

- Next, we know there will be x and y components of the E-field. If you look at the sign of the charges, you can see that the $E_x$ component will cancel out and the $E_y$ component will add (If it is not clear, you can do the calculation and see for yourself). So now we have:

$$E_{qy} = -E_q \cos 45^\circ = -\frac{E_q}{\sqrt{2}} = \frac{-2kq}{\sqrt{2}a^2} = -\frac{\sqrt{2}kq}{a^2}\hat{j}$$

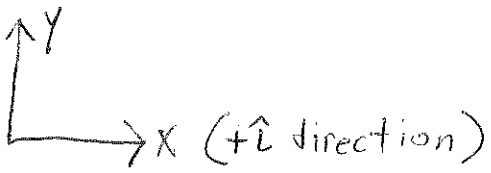
There are 4 components, so the final answer is

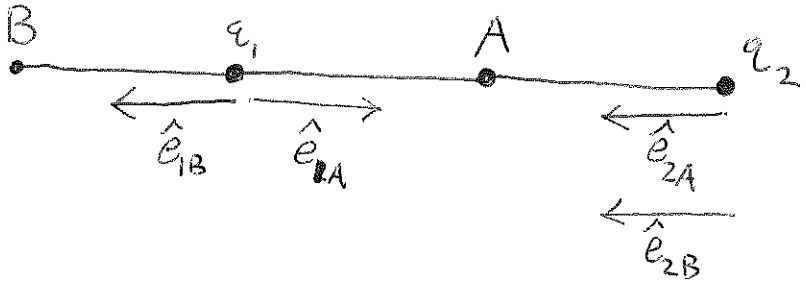
$$\boxed{\sum E = -\frac{4\sqrt{2}kq}{a^2}\hat{j}}$$

where $k = \frac{1}{4\pi\epsilon_0}$ (Coulombs constant)

($\hat{j}$ denotes y-direction, get used to showing this)

21.31




This problem lies only on one axis, which makes identifying the unit vectors much easier. Remember, unit vectors point from the Point Charge TO the point of interest! Lets list our unit vectors and distances below:

$\hat{e}_{1B} = -\hat{\imath}$ $\quad \hat{e}_{1A} = +\hat{\imath}$ $\quad \hat{e}_{2B} = -\hat{\imath}$ $\quad \hat{e}_{2A} = -\hat{\imath}$

$r_{1B} = 0.100\,m$ $\quad r_{1A} = 0.150\,m$ $\quad r_{2B} = 0.350\,m$ $\quad r_{2A} = 0.100\,m$

And our charges: $q_1 = -6.25 \times 10^{-9}\,C$ and $q_2 = -12.5 \times 10^{-9}\,C$

well that was easy! Now lets do a) and b)

a) $\vec{E}_A = \vec{E}_{1A} + \vec{E}_{2A} = K\frac{q_1}{r_{1A}^2}\hat{e}_{1A} + K\frac{q_2}{r_{2A}^2}\hat{e}_{2A}$

$\rightarrow \vec{E}_A = K\frac{(-6.25\times10^{-9}C)}{(0.150m)^2}(+\hat{\imath}) + K\frac{(-12.5\times10^{-9}C)}{(0.100m)^2}(-\hat{\imath})$

$= -(2.50\times10^{3}N/C)\hat{\imath} + (1.124\times10^{4}N/C)\hat{\imath} = \boxed{(8.74\times10^{3}N/C)\hat{\imath}}$

b) $\vec{E}_B = \vec{E}_{1B} + \vec{E}_{2B} = K\frac{(-6.25\times10^{-9}C)}{(0.100m)^2}(-\hat{\imath}) + K\frac{(-12.5\times10^{-9}C)}{(0.350m)^2}(-\hat{\imath})$

$\rightarrow \vec{E}_B = (5.619\times10^{3}N/C)\hat{\imath} + (9.17\times10^{2}N/C)\hat{\imath}$

$\rightarrow \vec{E}_B \boxed{= (6.54\times10^{3}N/C)\hat{\imath}}$

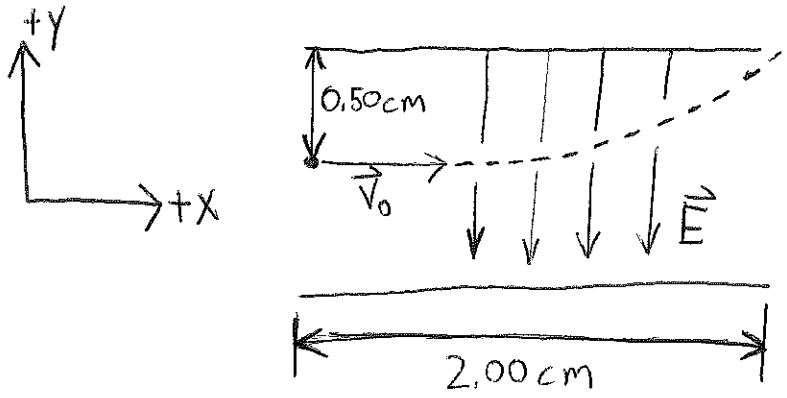
c) If we put a proton at point A and wanted to find the force on it, we simply use

$\vec{F} = q\vec{E}_A$, where $q$ is the charge of a proton!

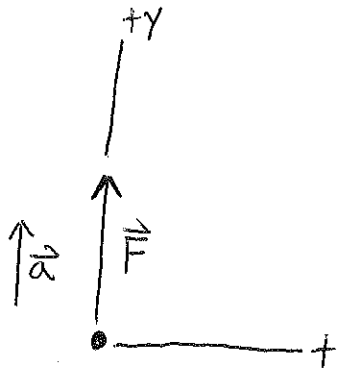
$$\vec{F} = (1.60 \times 10^{-19} C)([8.74 \times 10^{3} N/C]\hat{i}) = \boxed{[1.40 \times 10^{-15} N]\hat{i}}$$

Easy!!!

21.33 Particle Accelerator... sort of!




Remember $\vec{F} = q\vec{E}$ and $q = -e$ so the force points the opposite direction of the E-field




-The particle gets pushed up

- We know the electron just misses the upper plate so lets use Kinematics to find the acceleration, which we can use with $\vec{F} = m\vec{a}$ to find the force and the E-field

a) $x - x_0 = 2.00\text{cm}$ $\quad y - y_0 = 0.500\text{cm}$

$$v_0 = (1.60 \times 10^6 \text{ m/s})\hat{i} = v_{0x}$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 \rightarrow x - x_0 = v_{0x}t \rightarrow t = \frac{x - x_0}{v_{0x}} = 1.25 \times 10^{-8}\text{s}$$

Now $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \rightarrow a_y = \frac{2(y - y_0)}{t^2} = \left[6.40 \times 10^{13} \text{ m/s}^2\right]\hat{j}$

Now that we know acceleration lets use the force equatio

$$\vec{F} = m_e\vec{a} = q\vec{E} \rightarrow \vec{E} = \frac{m_e\vec{a}}{q} = \frac{(9.109 \times 10^{-31}\text{kg})(6.40 \times 10^{13} \text{ m/s}^2)\hat{j}}{-1.602 \times 10^{-19}\text{C}}$$

$$\boxed{\vec{E} = (-364 \text{ N/C})\hat{j}}$$

Note, because of the mass of the electr
the force of gravity has little impac

b) For part b, lets use Newton's second law again but solve for the acceleration using the mass of a proton

$$m_p\vec{a} = q\vec{E} \rightarrow \vec{a} = \frac{q\vec{E}}{m_p} = \frac{(1.602\times10^{-19}C)(-364\ N/C)\hat{j}}{1.673\times10^{-27}kg}$$

$$\rightarrow \vec{a} = [-3.49\times10^{10}\ m/s^2]\hat{j}$$

-This acceleration is orders of magnitude less than that of the electron. So, if the electron just missed the edge of the plate, there is no way the proton got near it!

-The proton has the same initial (and totally horizontal) speed so it takes the same time to clear the plates, lets look at the vertical displacement:

$$y - y_0 = \cancel{v_{0y}^{0}}t + \frac{1}{2}at^2 = \frac{1}{2}(-3.49\times10^{10}\ m/s^2)\hat{j}\,(1.25\times10^{-8}\ s)^2$$

$$= \boxed{[-2.73\times10^{-6}\ m]\hat{j}}$$

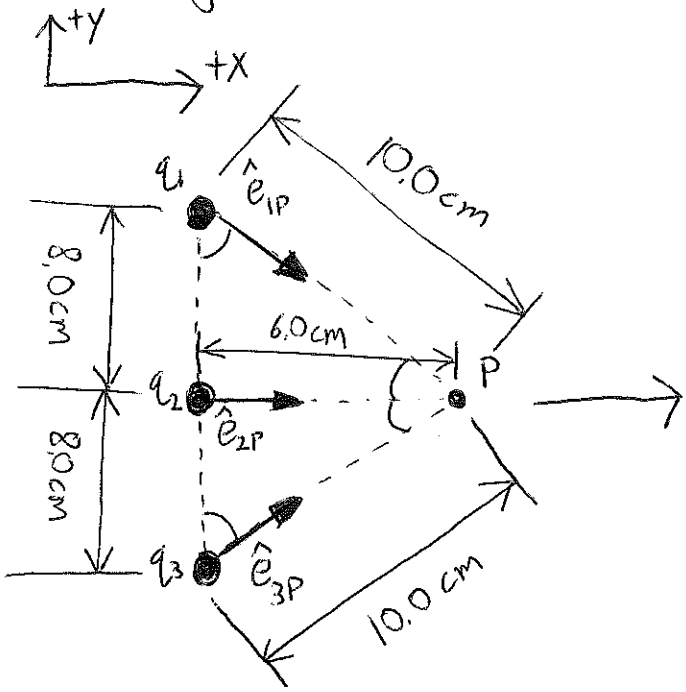
c) The (mass!) of the proton causes it to experience a much slower acceleration due to Newton's second law and the force exerted by the Electric Field. Further, the directions of motion are opposite due to the signs of the charge of protons and electrons

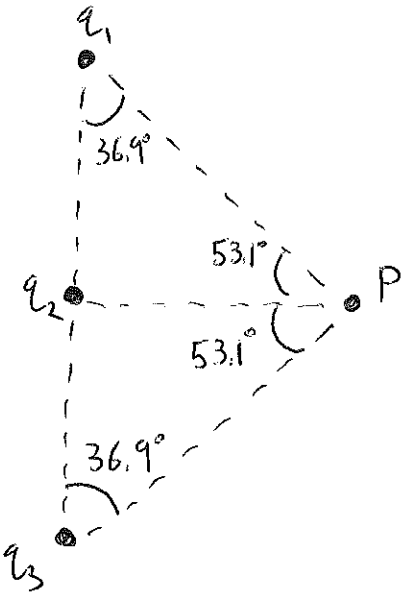
d) $a >> g$, we can ignore gravity because it counts for so little

21.47

Remember that $E = k\frac{q}{r^2}\hat{e}_r$ and $\Sigma E_{net} = E_1 + E_2 + E_3$

- Set the origin point at the point of interest (or not, it doesn't matter)




- Using Geometry I have found the distances and angles need
- You can do this using trig with the angles, but we want to get used to a proper, rigorous approach. Lets use unit vectors! We know that: $\hat{e}_r = \frac{\vec{e}_r}{\|\vec{e}_r\|}$ where $\vec{e}_r$ is the vecto pointing from the charge to the point of interest, and $\|\vec{e}_r\|$ i its magnitude. Now lets find all three unit vectors

$$\hat{e}_{1P} = \frac{[0.06\hat{i} - 0.08\hat{j}]\,m}{0.1\,m} \qquad \hat{e}_{2P} = \frac{[0.06\hat{i}]\,m}{0.06\,m} \qquad \hat{e}_{3P} = \frac{[0.08\hat{j} + 0.06\hat{i}]\,m}{0.1\,m}$$

$$\hat{e}_{1P} = 0.6\hat{i} - 0.8\hat{j} \qquad \hat{e}_{2P} = +\hat{i} \qquad \hat{e}_{3P} = 0.6\hat{i} + 0.8\hat{j} \quad \checkmark$$

- Now lets compute all the Electric Fields

$$\vec{E}_{1P} = K\frac{q_1}{r_{1P}^2}\hat{e}_{1P} = K\frac{(-5.00\times10^{-6}C)}{(0.1\,m)^2}\left[0.6\hat{i} - 0.8\hat{j}\right]$$

$$= (-4.49\times10^{6}\,N/C)\left[0.6\hat{i} - 0.8\hat{j}\right]$$

$$= \left[(-2.69\times10^{6}\,N/C)\hat{i} + (3.59\times10^{6}\,N/C)\hat{j}\right]$$

$$\vec{E}_{2P} = K\frac{q_2}{r_{2P}^2}\hat{e}_{2P} = K\frac{(-2.00\times10^{-6}C)}{(0.06\,m)^2}\left[+\hat{i}\right] = \left[-4.99\times10^{6}\,N/C\right]\hat{i}$$

$$\vec{E}_{3P} = K\frac{q_3}{r_{3P}^2}\hat{e}_{3P} = K\frac{(-5.00\times10^{-6}C)}{(0.1\,m)^2}\left[0.6\hat{i} + 0.8\hat{j}\right]$$

$$= (-4.49\times10^{6}\,N/C)\left[0.6\hat{i} + 0.8\hat{j}\right]$$

$$= (-2.69\times10^{6}\,N/C)\hat{i} + (-3.59\times10^{6}\,N/C)\hat{j}$$

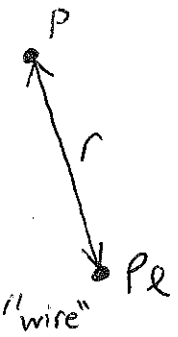
So now we know $\sum\vec{E} = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$

$$\rightarrow \sum\vec{E} = (-2.69\times10^{6} - 4.99\times10^{6} - 2.69\times10^{6})\,N/C\,\hat{i} + (3.59\times10^{6}\,N/C - 3.59\times10^{6}\,N/C)\hat{j}$$

$$\boxed{= \left[1.04\times10^{7}\,N/C\right]\hat{i}}$$

See how much clearer this is when we use a proper method?

21.52

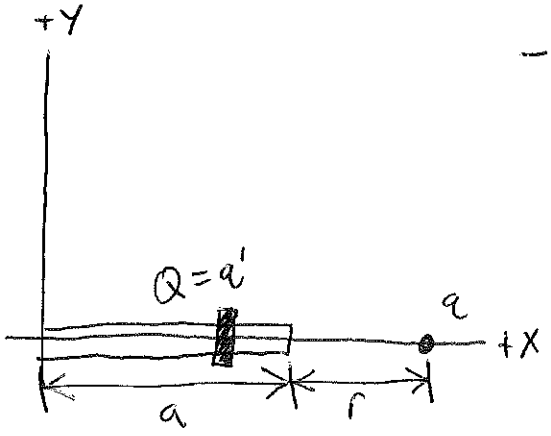



- We know from example 21.10 that $E = \frac{\rho_\ell}{2\pi\epsilon_0 r}$ so $r = \frac{\rho_\ell}{2\pi\epsilon_0 E}$, we know the line charge and E-field magnitude so this problem should be fairly easy,

$$r = \frac{1.5 \times 10^{-10}\ C/m}{2\pi\epsilon_0 (2.50\ N/C)} = \boxed{1.08\ m}$$

- For a more detailed derivation of the expression refer to 21.10 Example

21.89




-Lets do a thorough approach here

(a) For this part we are ignoring the point charge and focusing on the distributed rod

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq'(\bar{r}_p - \bar{r}')}{|\bar{r}_p - \bar{r}'|^3} \quad \text{for } x > a, \quad \bar{r}_p = (a+r)\hat{\imath}, \quad \bar{r}' = x'\hat{\imath}$$

$$(\bar{r}_p - \bar{r}') = (a+r-x')\hat{\imath} \qquad |\bar{r}_p - \bar{r}'| = \sqrt{(a+r-x')^2} = a+r-x'$$

$$\rightarrow d\vec{E} = \frac{dq'}{4\pi\epsilon_0}\frac{(a+r-x')\hat{\imath}}{(a+r-x')^3} = \frac{dq'}{4\pi\epsilon_0}\frac{1}{(a+r-x')^2}\hat{\imath}$$

Note that $dq' = \frac{Q\,dx'}{a}$ so $d\vec{E} = \frac{Q}{4\pi\epsilon_0 a}\frac{dx'}{(a+r-x')^2}\hat{\imath}$

$$\rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(a+r-x')^2}\hat{\imath} = \boxed{\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{r} - \frac{1}{a+r}\right)\hat{\imath}}$$

b) $\vec{F} = q\vec{E}$ and remember $a + r = x$ and $r = x - a$ So

$$\vec{F} = q\frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{r} - \frac{1}{a+r}\right)\hat{\imath} = \frac{Qq}{4\pi\epsilon_0 a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{\imath}$$

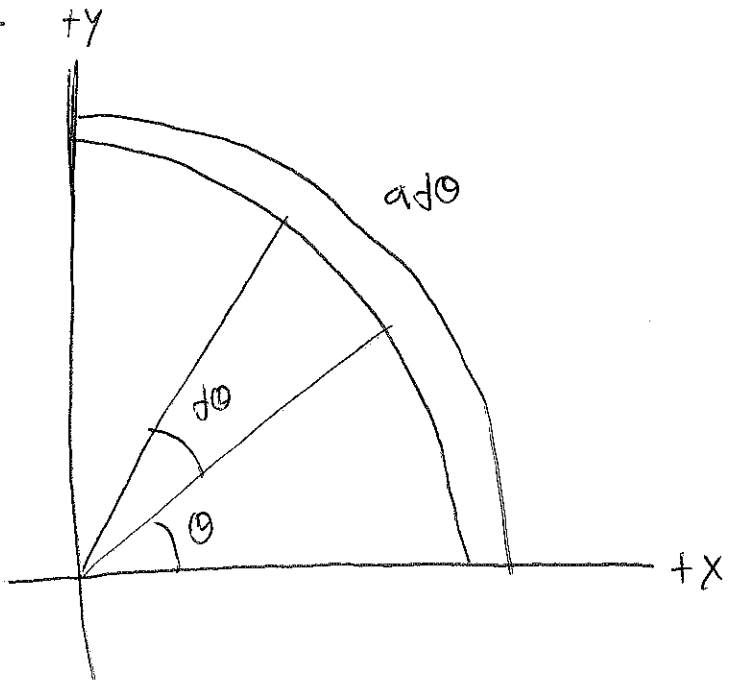
c) What if $r >> a$?

This also means $r = x - a \approx x$ so

$$\frac{1}{x-a} \quad \frac{1}{x} = \frac{x}{x^2 - ax} - \frac{x-a}{x^2-ax} = \frac{a}{x^2 - \cancel{ax}\ 0} = \frac{a}{x^2}$$

so $\vec{F} = \frac{Qq}{4\pi\epsilon_0 \cancel{x}}\left(\frac{a}{x^2}\right) = \boxed{\frac{Qq}{4\pi\epsilon_0 x^2} \text{ or } \frac{Qq}{4\pi\epsilon_0 r^2}}$

21.97




- The hardest part of this problem is finding the dQ element. We can solve it by using the ratio of the arclength ($a\,d\theta$) to the length of the entire segment, which is a quarter of the circumference of a circle, so

$$dQ = \left(\frac{a\,d\theta}{\frac{1}{2}\pi a}\right)Q = \frac{2Q}{\pi}d\theta$$

- Now lets find our vectors,

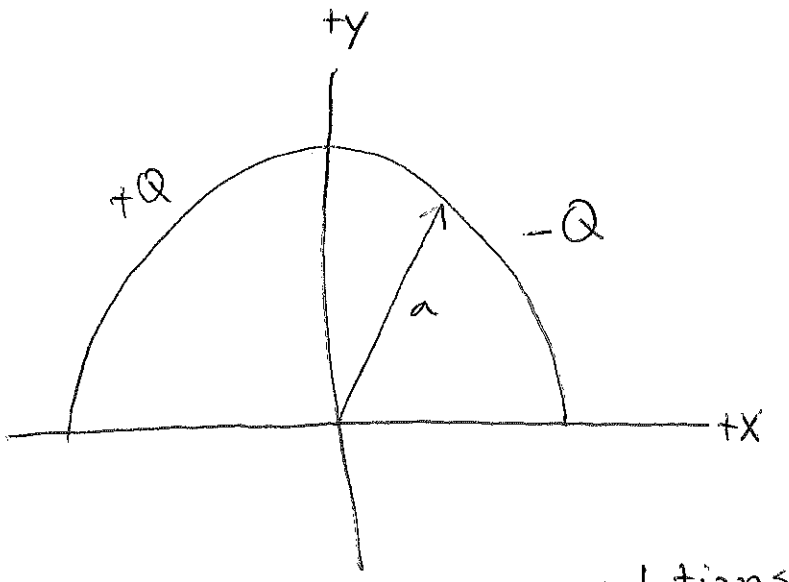
$$\bar{r}_p = 0\,,\quad \bar{r}' = a\cos\theta\hat{\imath} + a\sin\theta\hat{\jmath} = a\left[\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right]$$

$$\left|\bar{r}_p - \bar{r}'\right| = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta} = a$$

$$\text{so}\quad d\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{dQ}{\left|r'_p - r'\right|^3}\left(r'_p - r'\right) = \frac{Q\,d\theta}{2\pi^2\epsilon_0}\left[\frac{-a\left[\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right]}{a^3}\right]$$

$$\bar{E} = \frac{-Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2} d\theta\left[\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right] = \boxed{-\frac{Q}{2\pi^2\epsilon_0 a^2}\left[\hat{\imath} + \hat{\jmath}\right]}$$

21.98




- We might as well use our solutions from the last problem right?

We know the E field in any direction is

$$E = \frac{-Q}{2\pi^2 \epsilon_0 a^2}$$

The only difference is we have a region of negative charge and direction $[\hat{i} + \hat{j}]$ and a region of positive charge and direction $[-\hat{i} + \hat{j}]$. So:

For +Q region: $\boxed{\bar{E}_+ = \frac{Q}{2\pi^2 \epsilon_0 a^2}\left[\hat{i} - \hat{j}\right]}$

For -Q region: $\boxed{\bar{E}_- = \frac{Q}{2\pi^2 \epsilon_0 a^2}\left[\hat{i} + \hat{j}\right]}$

so $\sum \bar{E} = \bar{E}_+ + \bar{E}_- = \boxed{\frac{Q}{\pi^2 \epsilon_0 a^2}\hat{i}}$