EP711: COMPUTATIONAL ATMOSPHERIC DYNAMICS EP711

Spring 2012, J. B. Snively

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Take-Home Exam

For each problem, provide your solutions, codes, figures, and any discussion in a digital form. Please feel free to provide additional derivations, drawings, or figures in scanned PDF or handwritten (paper) form. Please include citations and a list of references (if needed).

1) <u>Assess the stability of a "typical" atmosphere.</u>

Use the meridional and zonal wind profiles stored in the following txt file:
http://webfac.db.erau.edu/~snivelyj/ep711/wind.txt
To describe the ambient atmospheric background, use the NRLMSISE-00 empirical model. Assume typical solar activity, specify profiles from 0 to 150 km altitude, with 100 m resolution, over Daytona Beach, Florida (29.2°, -81.1°), at local midnight, on January 1, 2012.

Plot the following (over the full altitude range):

a) Meridional and Zonal Wind Speed
b) Brunt-Väisälä Frequency
c) Gradient Richardson Number

Answer the following questions:

a) Discuss briefly the overall stability characteristics of the model atmosphere.
b) What strength shear (dU/dz) would be necessary for dynamic instability at mesopause?
c) What strength shear (dU/dz) would be necessary for dynamic instability at 115 km?

2) <u>Investigate the importance of molecular viscosity.</u>

Using your NRLMSISE-00 data from (1), calculate kinematic viscosity and thermal diffusivity (both having units m^2/s) assuming that the dynamic viscosity is constant over altitude and equal to 1.8×10^{-5} Pa•s, and that the Prandtl number is Pr=0.7.

Plot the following on a semi-log scale:

a) Kinematic viscosity ν
b) Thermal diffusivity α
c) Effective Reynold's number for a gravity wave, as given by [e.g., *Fritts et al.*, 2009]: $\mathrm{Re} = \dfrac{\lambda_z^2}{\nu \tau_b}$ where wavelength λ_z=5 km, τ_b is Brunt-Väisälä period $2\pi/N$, and ν is kinematic viscosity.

Answer the following questions:

a) What is the minimum number vertical grid points per wavelength that would be required to *directly simulate* the turbulent evolution of a λ_z =5 km gravity wave at 75 km altitude in your model profile?
b) Repeat this calculation at 105 km altitude.
c) Comment on the results – Is it realistic to expect to fully describe the dynamics of the atmosphere in a single numerical model? (at least in our lifetimes?)

3) <u>Determine the Stability of the FTCS Method for Advection.</u>

To date, we've used the Lax-Friedrichs, Lax-Wendroff, and finite volume methods for simple hyperbolic problem solutions (i.e., systems made up of advection equations). One notable feature of the Lax-Friedrichs method, which also makes it quite numerically-diffusive, is the "averaged" term on the right hand side:

$$u_j^{n+1} = \frac{1}{2}(u_{j-1}^n + u_{j+1}^n) - \frac{\Delta t}{2\Delta x}(F_{j+1}^n - F_{j-1}^n)$$

A simpler method, where Euler's method is used directly to evolve the time-step, is given by:

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{2\Delta x}(F_{j+1}^n - F_{j-1}^n)$$

Let's assume *constant* coefficient advection at a speed c_o, where $F=c_o u$. *Show via a stability analysis that this "Forward in Time, Centered in Space" method is entirely useless.*

4) <u>Graphically determine whether this flux-limiter is TVD.</u>

The "*Koren*" flux limiter function is given by the following expression:

$$\phi(\theta) = \max[0, \min(2\theta, (1+2\theta)/3, 2)]$$

Graphically determine whether this limiter satisfies TVD criteria.

5) <u>Explain the CFL stability constraint for multi-dimensional methods.</u>

Note on page 72 of Potter the equation 3.105, describing the stability constraint for an N-dimensional explicit advection scheme where the grid spacing is equal in each direction:

$$\Delta t \leq \frac{\Delta x}{|\vec{v}|\sqrt{N}}$$

Here, $|\boldsymbol{v}|$ is the magnitude of the fastest vector velocity on the grid.

Explain and derive this constraint using geometric arguments.

Comment on the implications of this constraint for "dimensionally-split" methods.

6) <u>Confirm that the flux form of Lax-Wendroff does what it claims.</u>

Recall that "flux form" finite difference and finite volume methods for hyperbolic problems adhere to form given by:

$$Q_j^{n+1} = Q_j^n - \frac{\Delta t}{\Delta x}(F_{j+1/2}^n - F_{j-1/2}^n)$$

A flux function *F* for constant-coefficient advection is given on Slide 11 of Lecture 9.

Confirm algebraically that this reduces to the "standard" Lax-Wendroff method.

7) <u>Linearize the Momentum Equation.</u>

On Slide 3 of Lecture 13, a nonlinear conservative-form expression for the Momentum equation is given. On Slide 8 of Lecture 13, a linearized set of equations has appeared.

Starting with Equation 2 on Slide 3, expand, simplify, and linearize to obtain Equations 1 and 2 on Slide 8!