Intuitive Introduction To Acoustic-gravity Waves

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Contents

- **Observational Basis**
- Theoretical Basis
- Propagation Characteristics

Atmospheric gravity waves...

- Comprise the range of atmospheric waves governed primarily by *buoyancy*, rather than rotation or compression.
- Exhibit spatial scales ranging from kilometers to hundreds of kilometers, and time scales from a few minutes to a hours.
- Propagate to high (middle and upper atmospheric) altitudes, where they may interact, break, trigger instabilities, and ultimately dissipate.
- Provide coupling by transporting energy and momentum between atmospheric regions.
- Are readily observable via perturbations to clouds, ice layers, and airglow.



[Images from: *Wikipedia Commons*]

This thing.

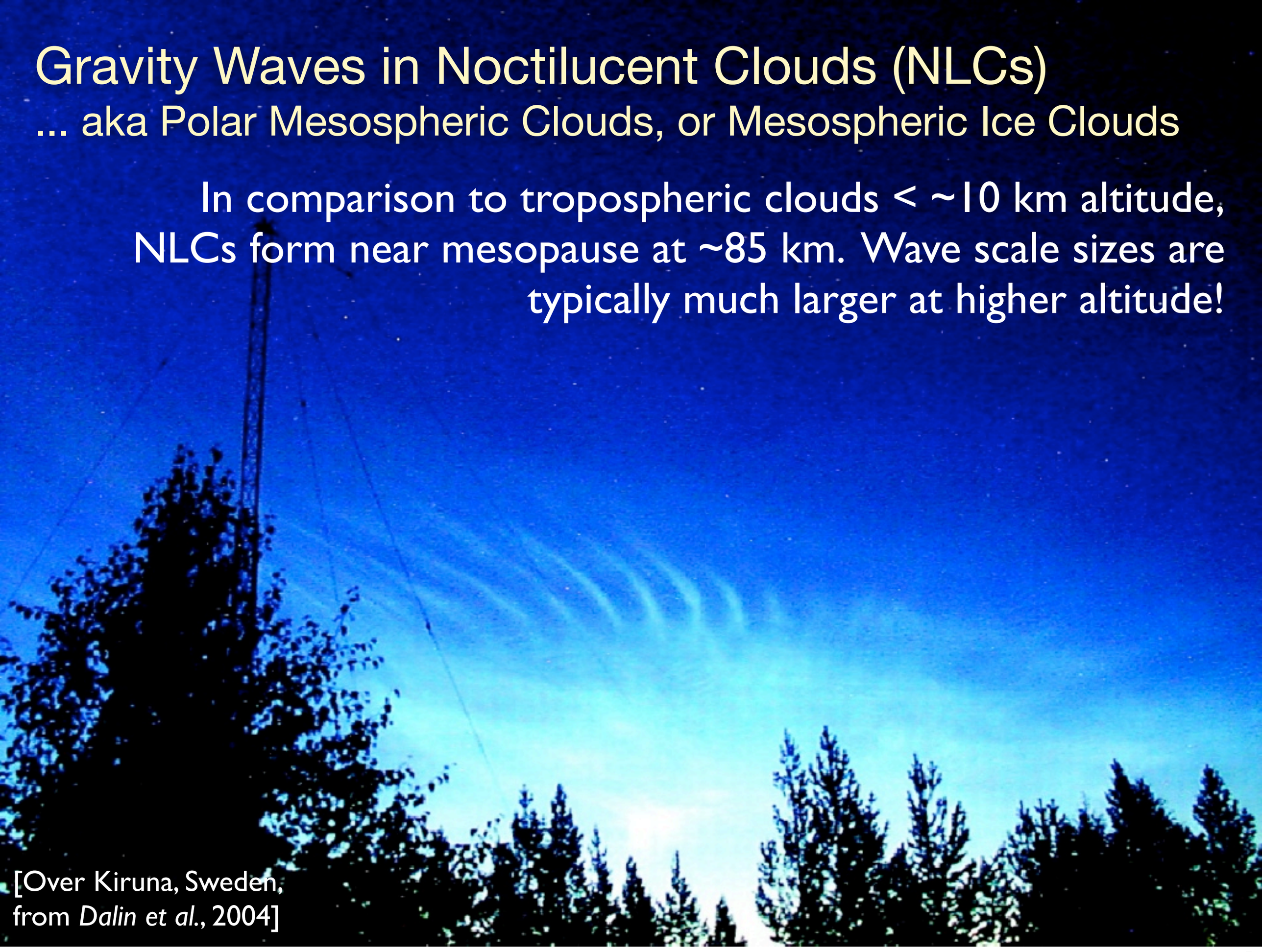
Solitary Wave Roll Cloud?

... Extends as far as visible from ERAU Lehman Building!

Gravity Waves in Noctilucent Clouds (NLCs)

... aka Polar Mesospheric Clouds, or Mesospheric Ice Clouds

In comparison to tropospheric clouds < ~10 km altitude, NLCs form near mesopause at ~85 km. Wave scale sizes are typically much larger at higher altitude!



[Over Kiruna, Sweden, from *Dalin et al.*, 2004]

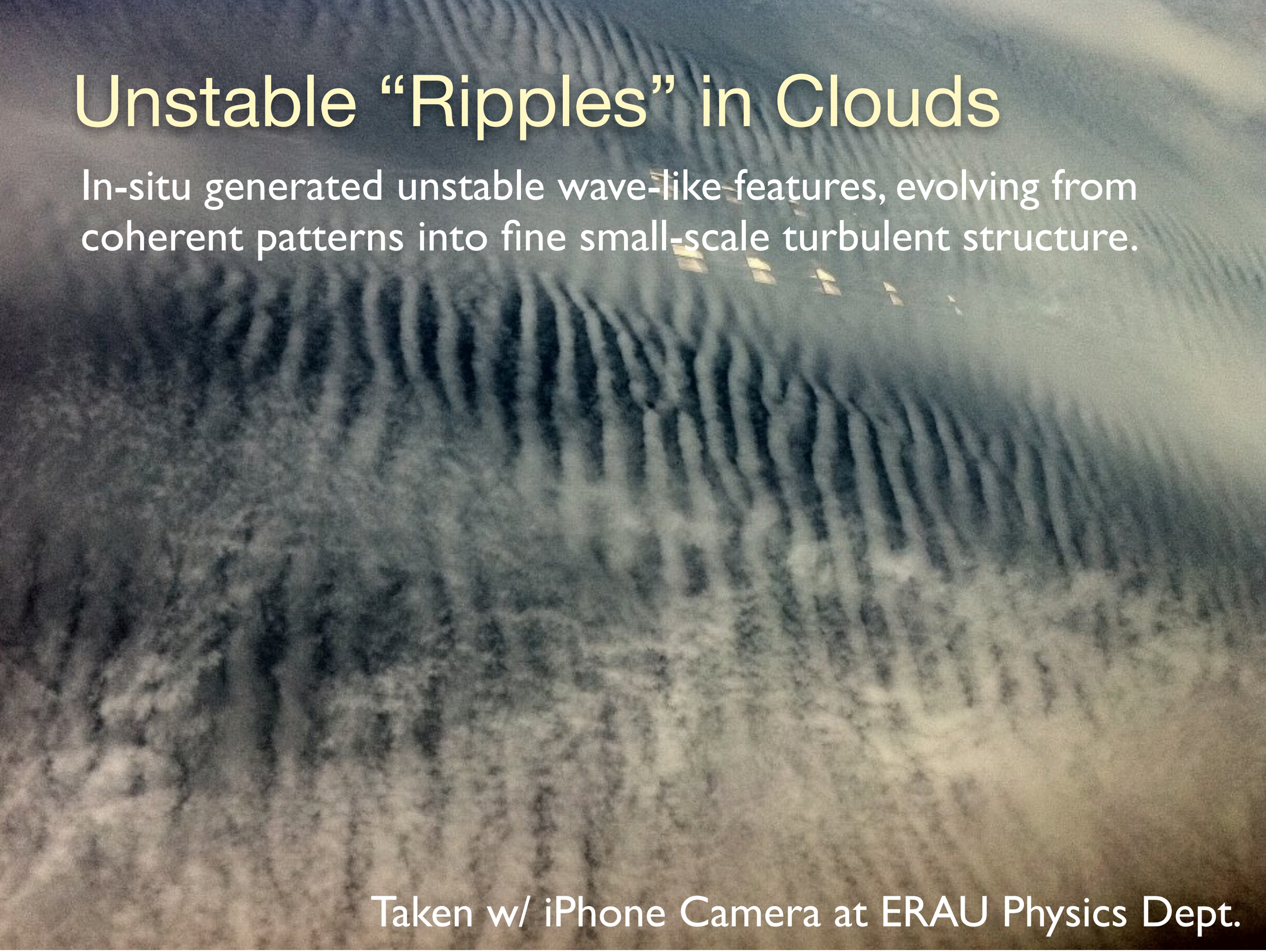
Gravity Waves in Upper-Mesospheric Airglow

... Green emission due to O, red (and near-infrared) emissions due to OH and O_2, yellow emission due to Na.

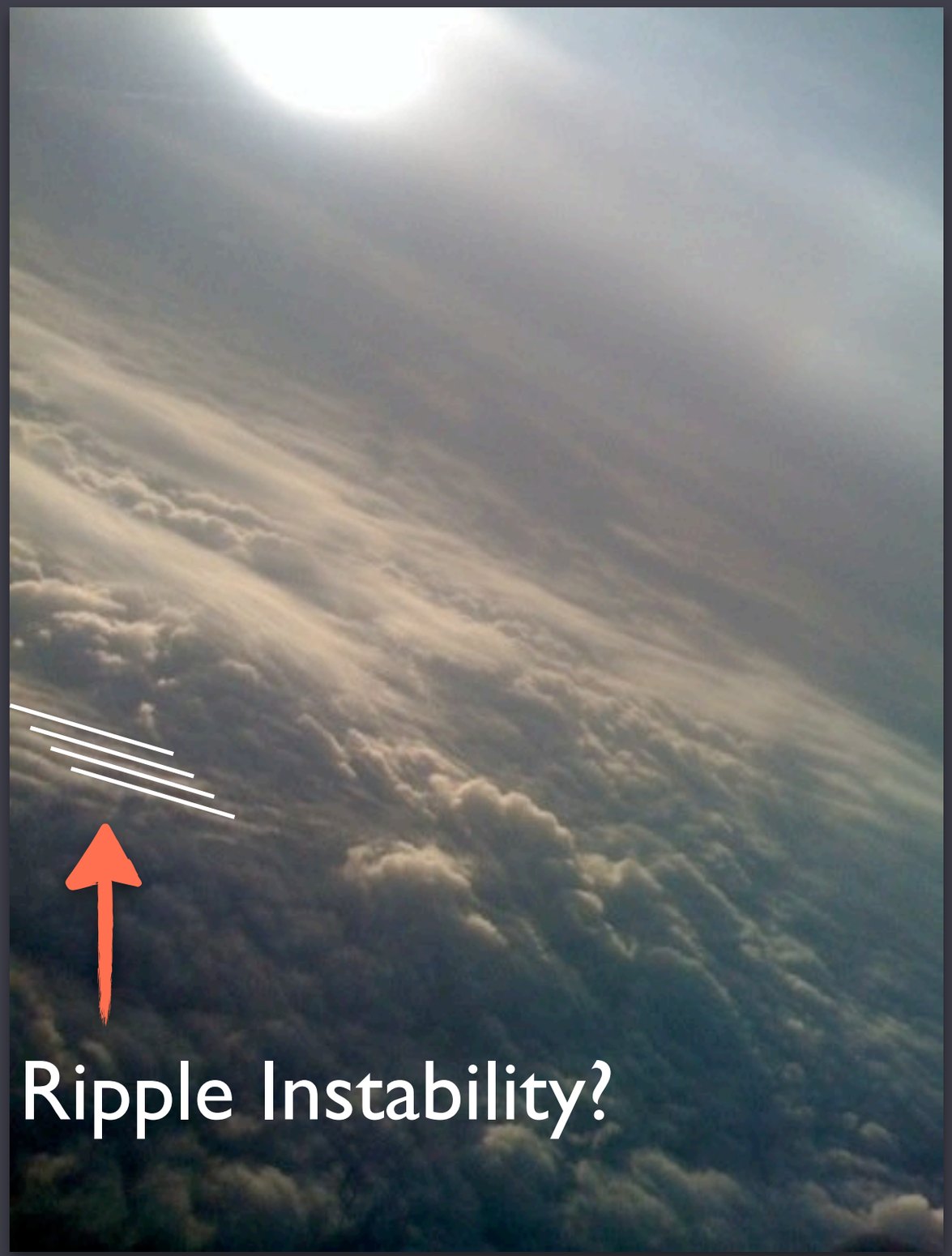
[Mesospheric bore (nonlinear ducted wave) over Texas from *Smith et al.*, 2003]

Unstable “Ripples” in Clouds

In-situ generated unstable wave-like features, evolving from coherent patterns into fine small-scale turbulent structure.



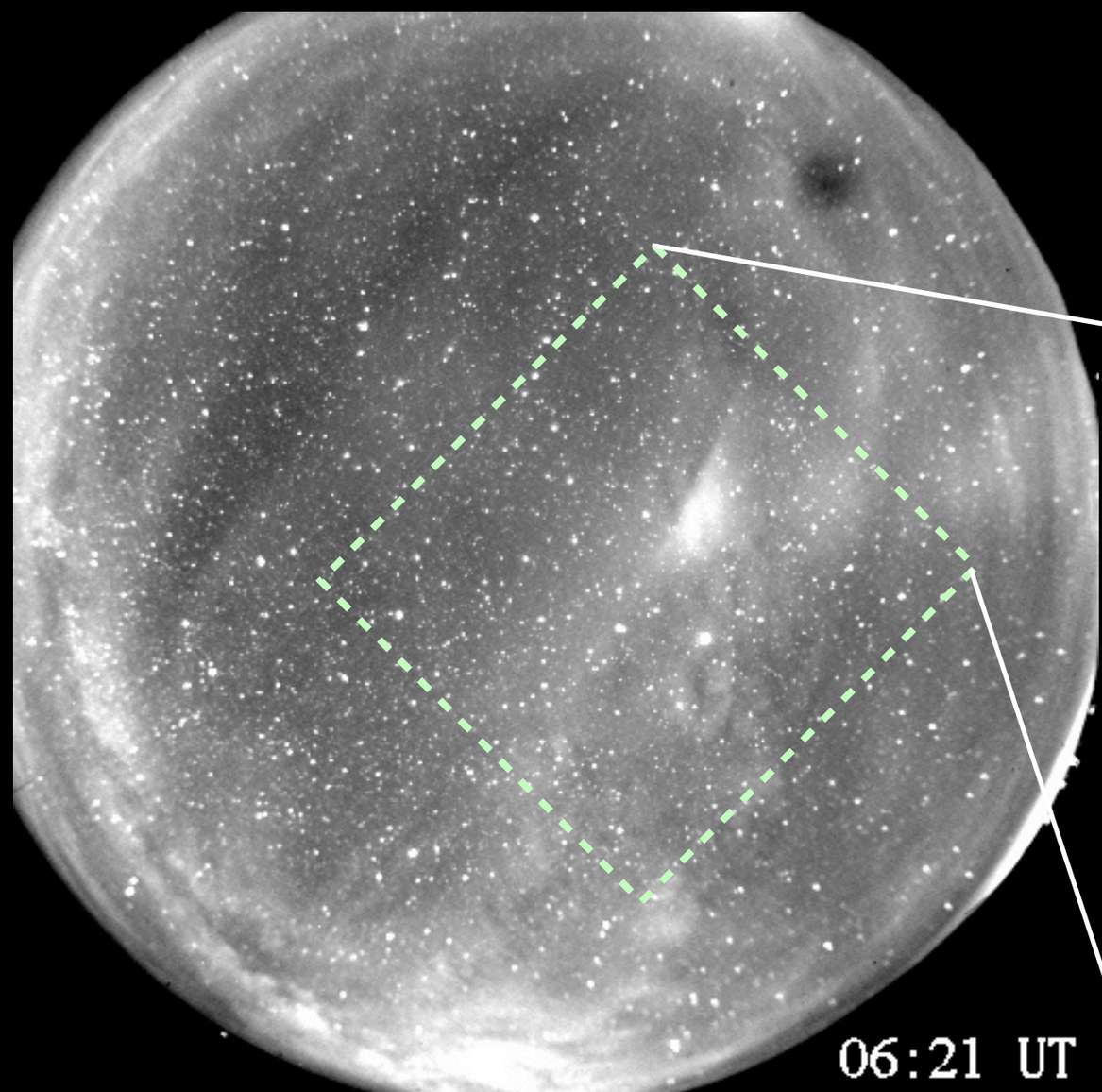
Taken w/ iPhone Camera at ERAU Physics Dept.



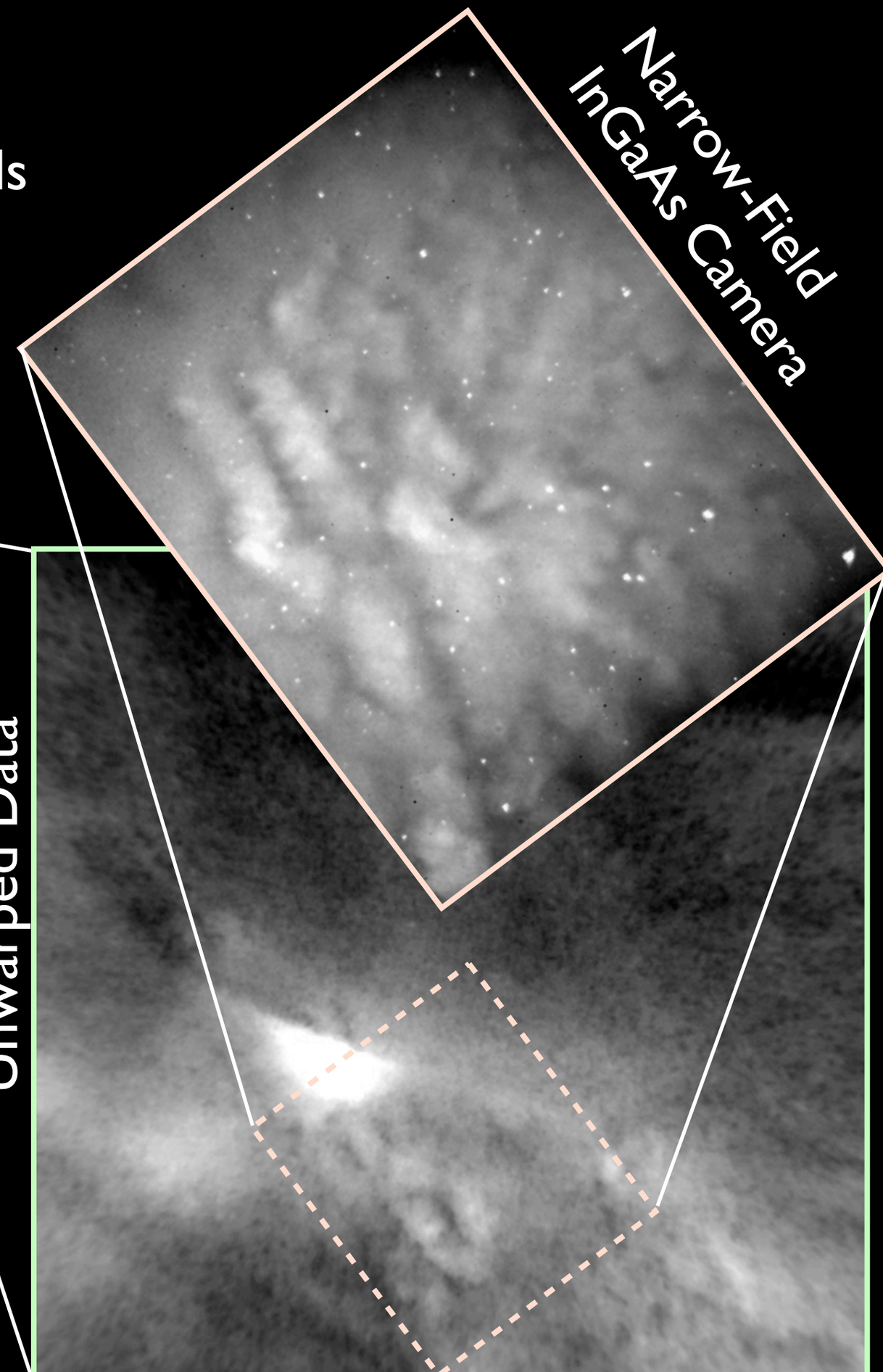
... iPhone pics of wave clouds from an airplane

Observable Scales:

OH airglow data at small and large fields of view, high and low resolutions.



BLO All-Sky Imager



[Courtesy of *M. J. Taylor and P-D. Pautet*]

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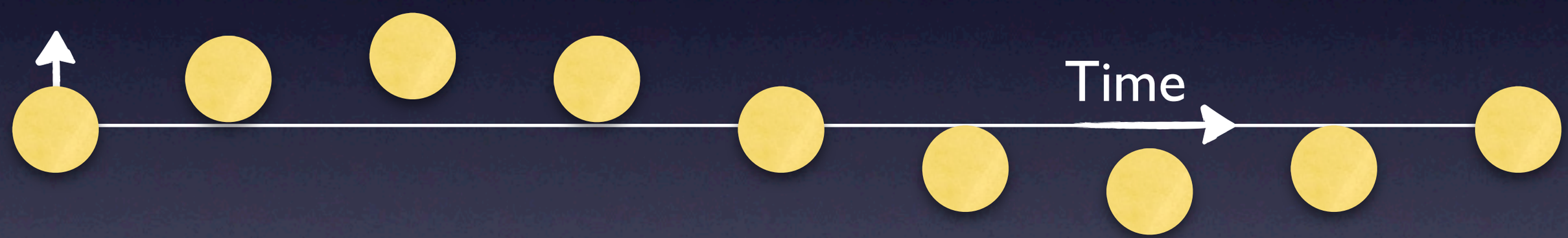
The Brunt-Väisälä Frequency N:

The Earth's atmosphere is stratified, such that gravity and buoyancy maintain equilibrium. Density and pressure decrease exponentially with altitude at a characteristic scale height $H \sim 6\text{-}8\ \mathrm{km}$.

If a parcel is displaced vertically in a gravity-stratified atmosphere,



... it will oscillate at a characteristic frequency N (~0.01-0.02 rad/sec)

$$N^2 = \frac{g}{\theta_o}\frac{d\theta_o}{dz} \simeq \frac{g^2}{c_s}(\gamma - 1)$$

This is the highest frequency at which gravity waves can propagate!

Linearized Equations of Motion:

$$\frac{\partial \tilde{v}_x}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}}{\partial x} \tag{1}$$

$$\frac{\partial \tilde{v}_z}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}}{\partial z} - \frac{\tilde{\rho} g}{\rho_o} \tag{2}$$

$$\frac{\partial \tilde{\rho}}{\partial t} = -\rho_o \frac{\partial \tilde{v}_x}{\partial x} - \rho_o \frac{\partial \tilde{v}_z}{\partial z} - \tilde{v}_z \frac{\partial \rho_o}{\partial z} \tag{3}$$

$$\frac{\partial \tilde{p}}{\partial t} = -\tilde{v}_z \frac{\partial p_o}{\partial z} - \gamma p_o \left(\frac{\partial \tilde{v}_x}{\partial x} + \frac{\partial \tilde{v}_z}{\partial z} \right) \tag{4}$$

Substituting $\tilde{v}_z = (\rho_o/\rho_s)^{-1/2}\tilde{w}_z$, a wave equation for the normalized vertical perturbation velocity $\tilde{w}_z$ is obtained:

$$\boxed{\frac{\partial^2 \tilde{w}_z}{\partial z^2} + \left[\frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2} \right] \tilde{w}_z = 0.}$$

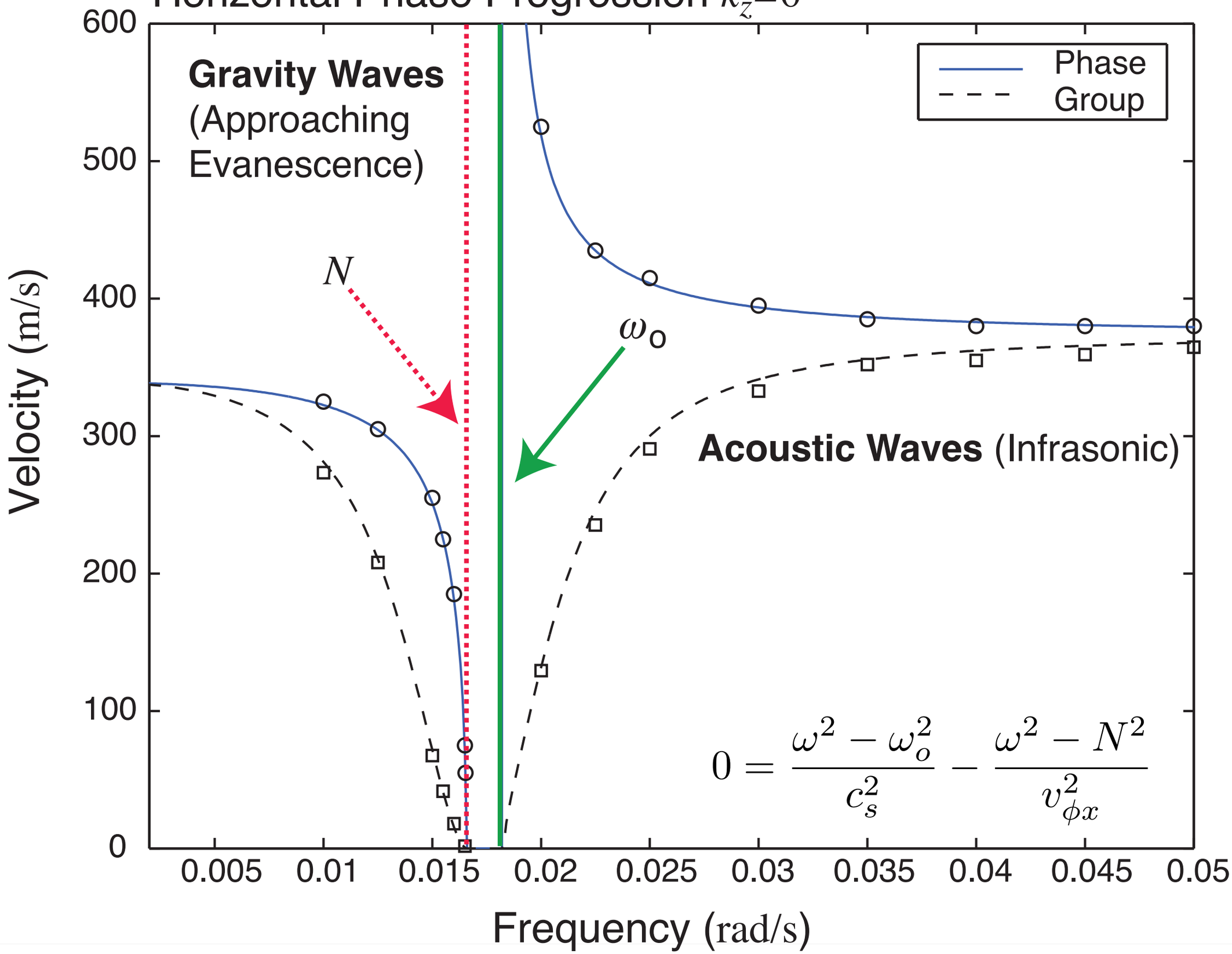
which is governed by the linear dispersion relation

$$\boxed{k_z^2 = \frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2},}$$

where $\omega_o = \frac{g\gamma}{2c_s}$ is the acoustic cut-off frequency and $N = \frac{g}{c_s}\sqrt{\gamma - 1}$ is the Brunt-Väisälä resonance frequency.

Example Phase and Group Velocity:

Horizontal Phase Progression k_z=0



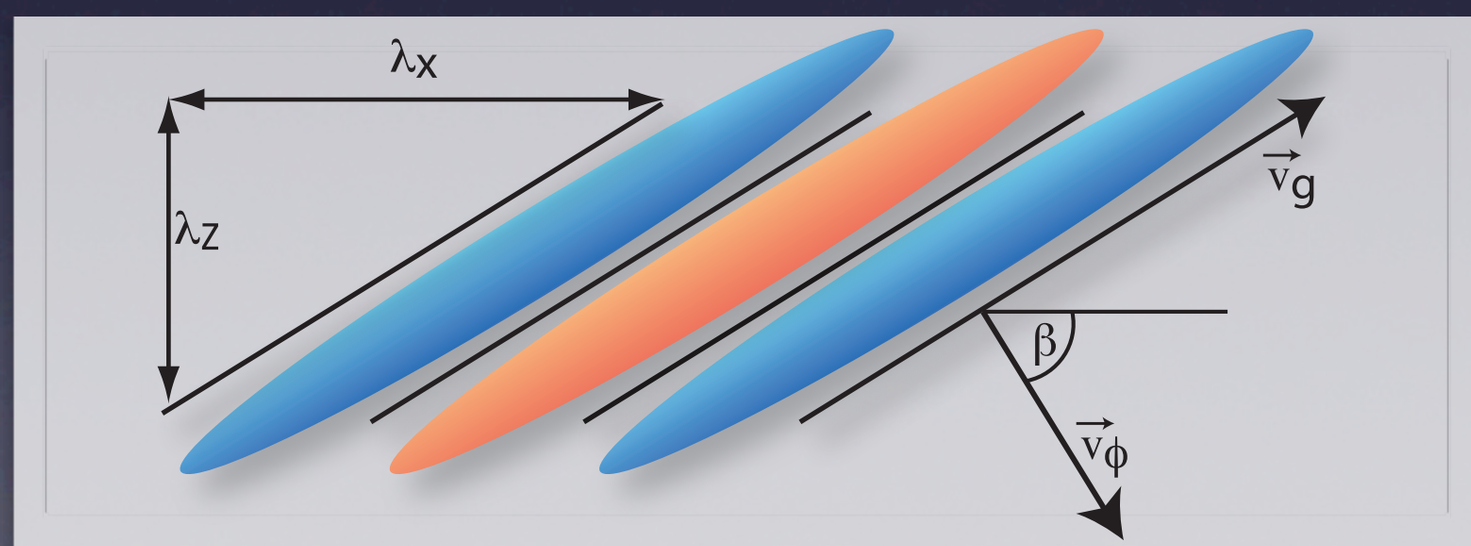
As stated, the compressible dispersion relation is:

$$k_z^2 = \frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2}$$

Gravity waves are transverse wave motions below the Brunt-Väisälä frequency N and the acoustic cut-off frequency ω_o.

Neglecting the acoustic terms, a convenient form for the *in*compressible dispersion relation relates k_x and k_z via the wave vector angle β to the horizontal (see Figure) [e.g., *Nappo*, 2002, p. 32]:

$$\omega = \frac{k_x N}{(k_x^2 + k_z^2)^{1/2}} = N \cos\beta.$$



Group and Phase Velocity are Perpendicular!

Interesting Conclusion: Frequency ω of gravity wave (relative to N) determines λ_x, λ_z, and thus direction of propagation!

Dynamical Model:

- Finite volume method (FVM) solution for nonlinear, compressible, stratified, Euler equations [e.g., *LeVeque*, 1997].
- Viscosity, thermal conduction, and gravity wave sources included via a time-split approach [*Snively and Pasko*, 2003, 2008].
- Coupled solutions for minor species densities (via FVM) for airglow studies [*Snively et al.*, 2005, 2010].

Allow simulation of linear and nonlinear gravity wave dynamics, and perturbations to airglow photochemistry.

Model Equations of Motion:

We model the atmosphere as a non-rotating, fully-nonlinear, compressible gas:

Mass Density:
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{1}$$

Momentum:
$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v}\vec{v}) = -\nabla p - \rho \vec{g} \tag{2}$$

Energy:
$$\frac{\partial E}{\partial t} + \nabla \cdot \{ (E + p)\vec{v} \} = -\rho g v_z \tag{3}$$

where the energy equation and the equation of state for an ideal gas are defined as:

State:
$$E = \frac{p}{(\gamma - 1)} + \frac{1}{2}\rho(\vec{v} \cdot \vec{v}) \tag{4}$$

where ρ is density, p is pressure, $\vec{v}$ is the fluid velocity, along with energy density E. The Euler equations are coupled with equations for viscosity and thermal conduction:

Viscosity:
$$\frac{\partial \vec{v}}{\partial t} = \nu \nabla^2 \vec{v} \tag{5}$$

Conduction:
$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \tag{6}$$

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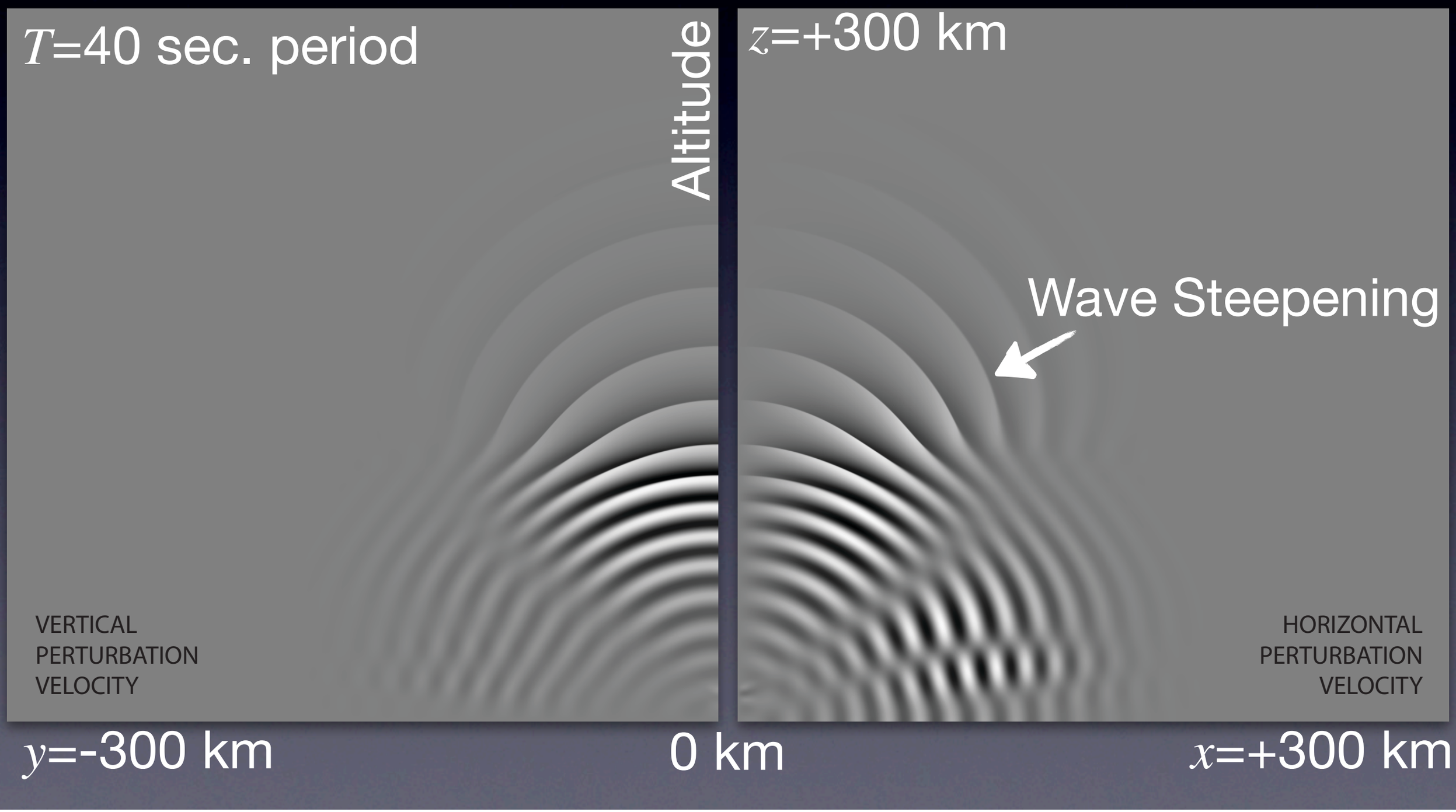
Acoustic Wave Propagation:

Above *N*, gravity waves are not generated

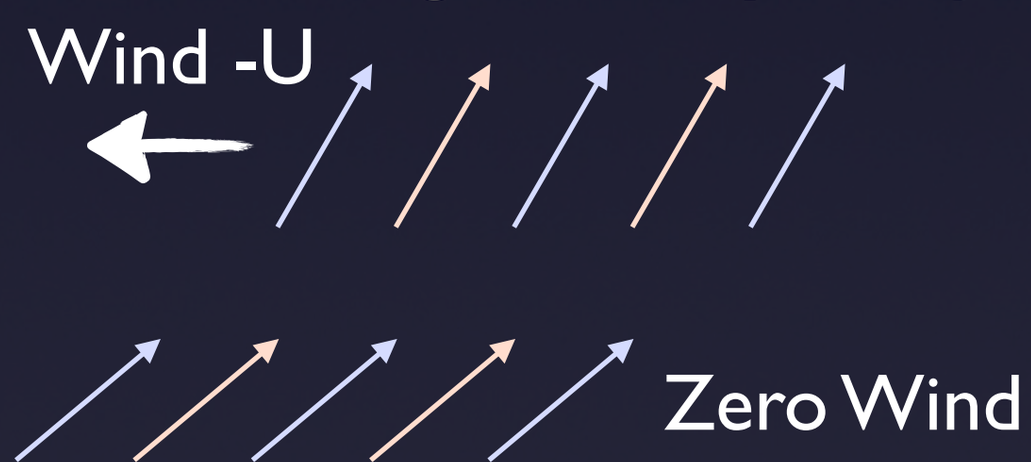


Gravity Wave Propagation:
Effects of Wind and Temperature

Gravity wave vertical wavelength λ_z varies with wind U as it depends on the Doppler-shifted intrinsic frequency $\Omega = \omega - k_x U$, while ground-relative frequency ω and horizontal wavenumber k_x (and wavelength λ_x) remain constant.

Therefore: When wind varies only with altitude, Doppler shifts modify only the intrinsic frequency relative to the flow. Frequency relative to the ground (extrinsic) remains constant!

Shift to higher frequency:



Shift to lower frequency:

Gravity Wave Propagation:

Isothermal, Windless

Wave:

$$\lambda_x = 20\mathrm{km}$$

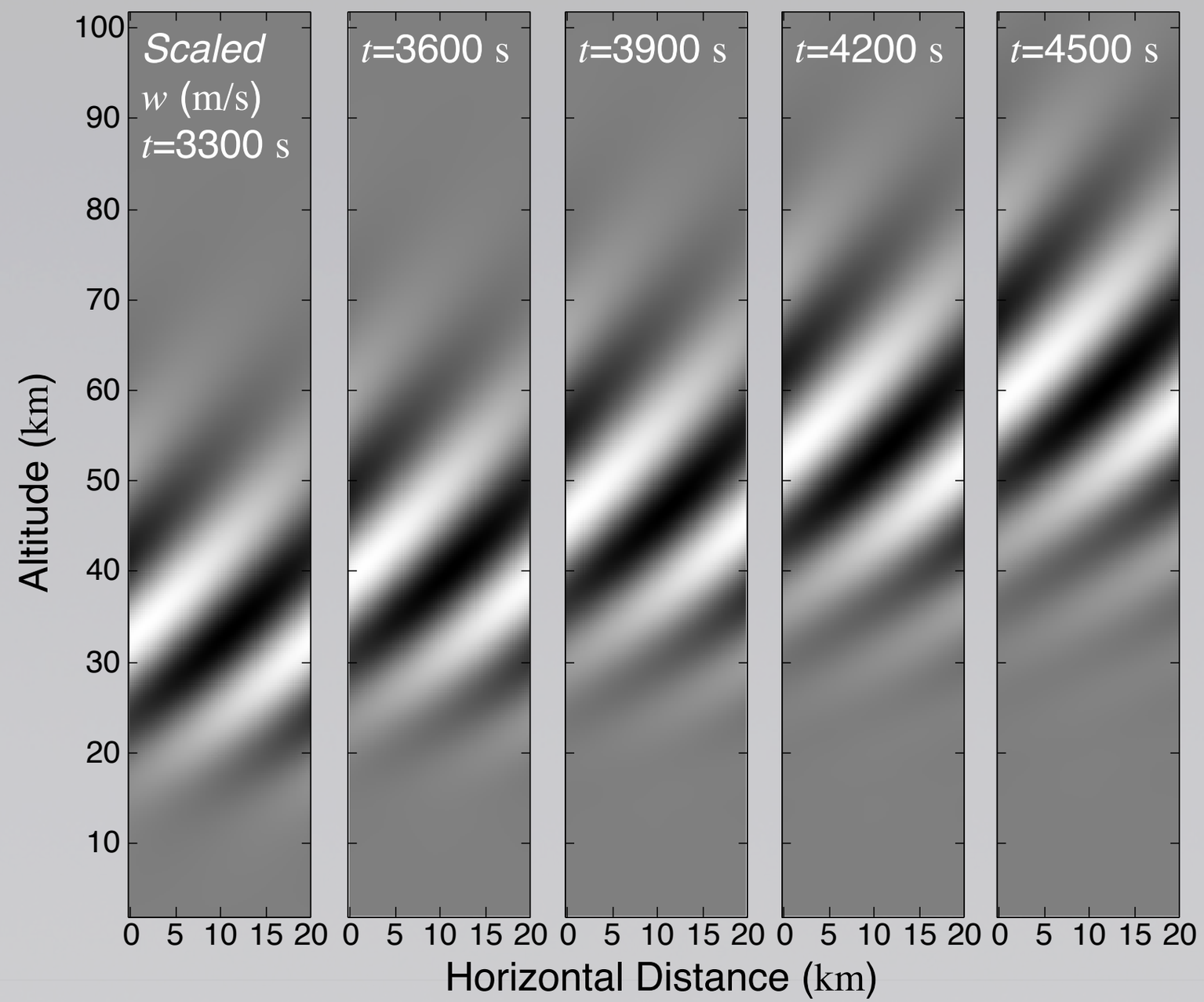
$$\omega = \frac{N}{\sqrt{2}}$$

Wind:

Zero!

Result:

The gravity wave propagates upwards through the horizontally-periodic domain. This simple case never occurs in reality!



Gravity Wave Propagation:
Isothermal, with Positive Wind

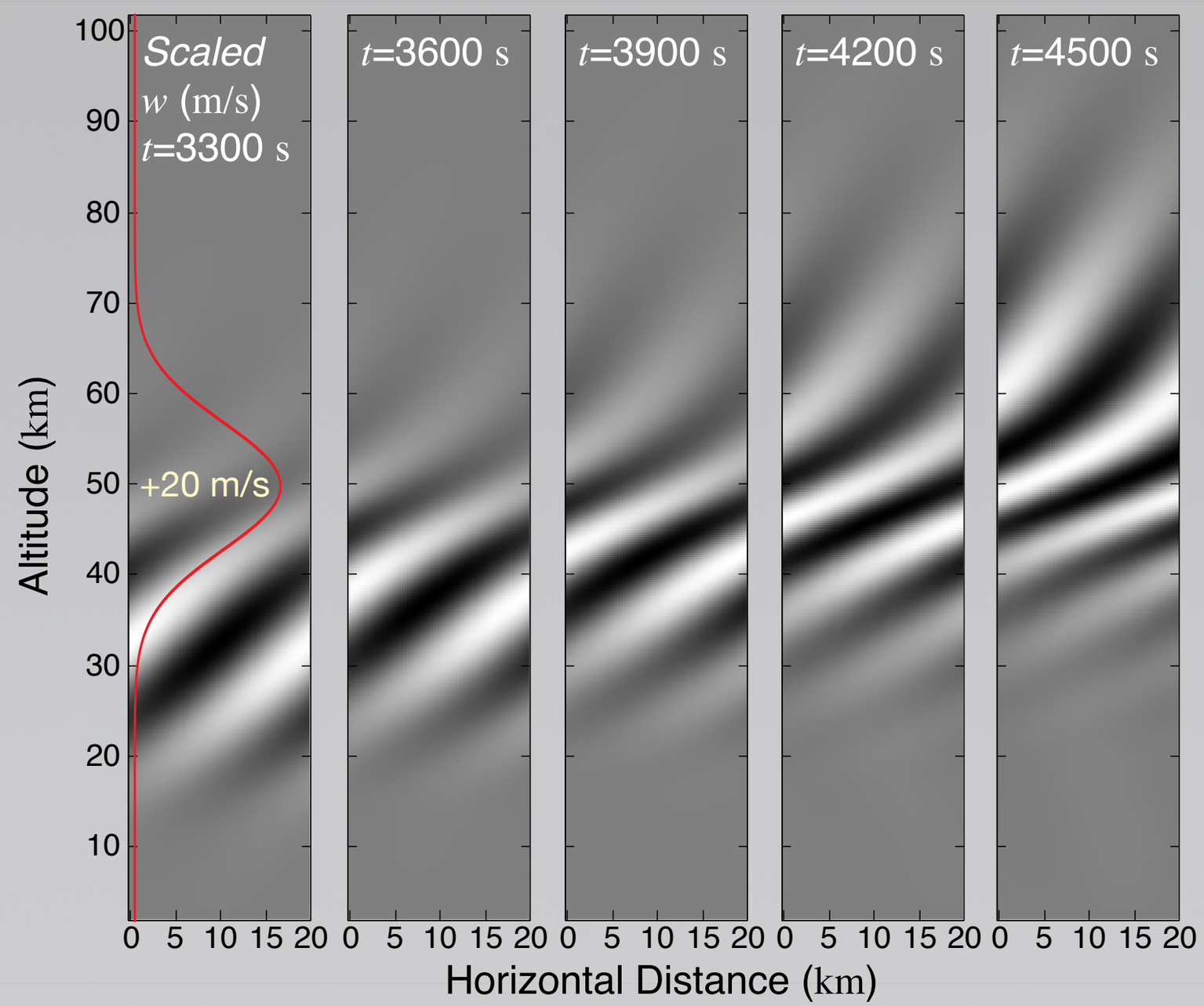
Wave:

$$\lambda_x = 20\mathrm{km}$$

$$\omega = \frac{N}{\sqrt{2}}$$

Wind:
+20 m/s Peak
at 50 km.

Result:
The gravity wave is Doppler-shifted to lower frequency at the wind peak, where vertical wavelength (and group velocity) is reduced.



Gravity Wave Propagation:

Isothermal, Critical Level

Wave:

$$\lambda_x = 20\mathrm{km}$$

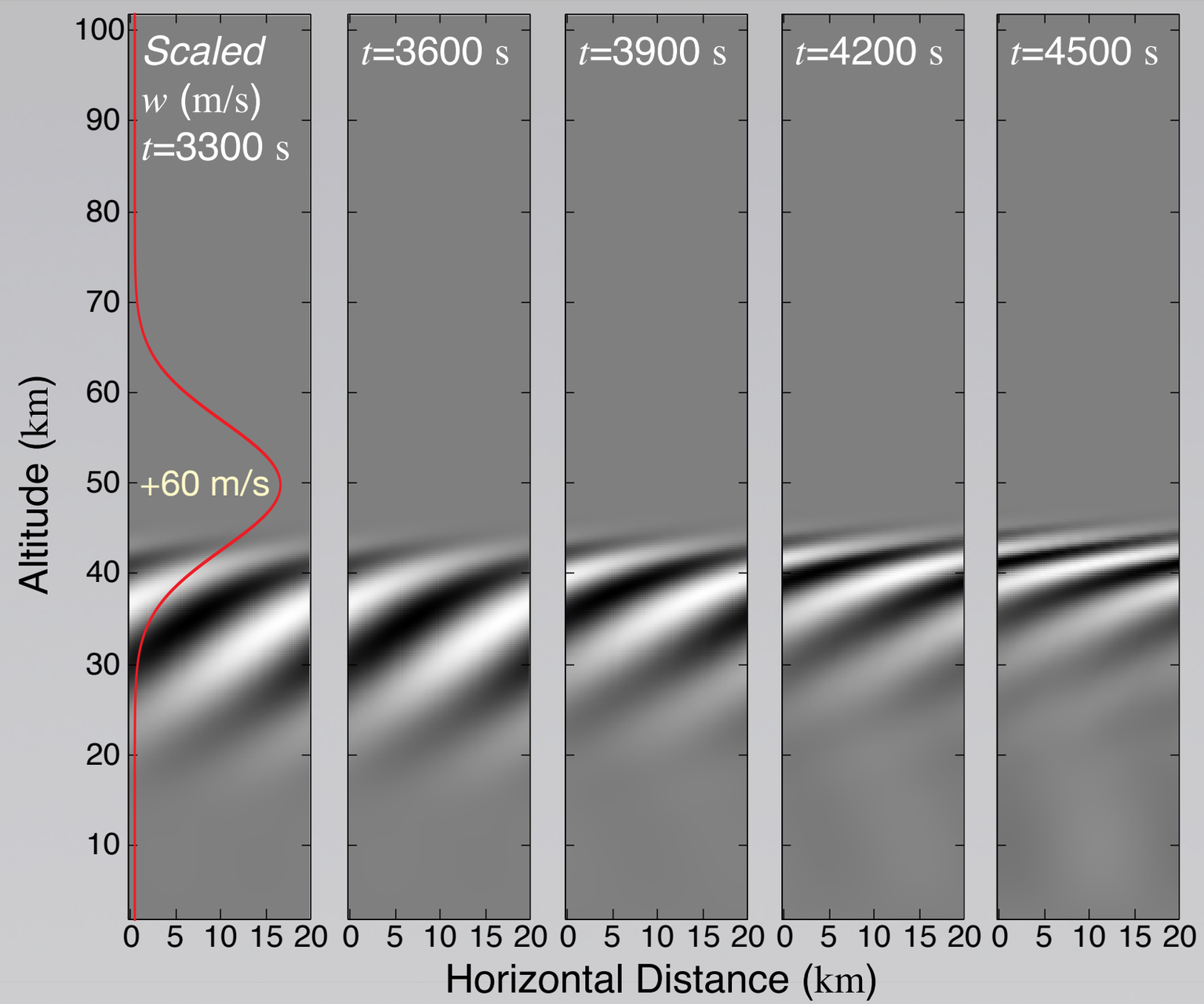
$$\omega = \frac{N}{\sqrt{2}}$$

Wind:

+60 m/s Peak at 50 km.

Result:

The gravity wave is Doppler-shifted towards zero frequency, leading to dissipation and blockage of the wave's passage.



Gravity Wave Propagation:
Isothermal, with Negative Wind

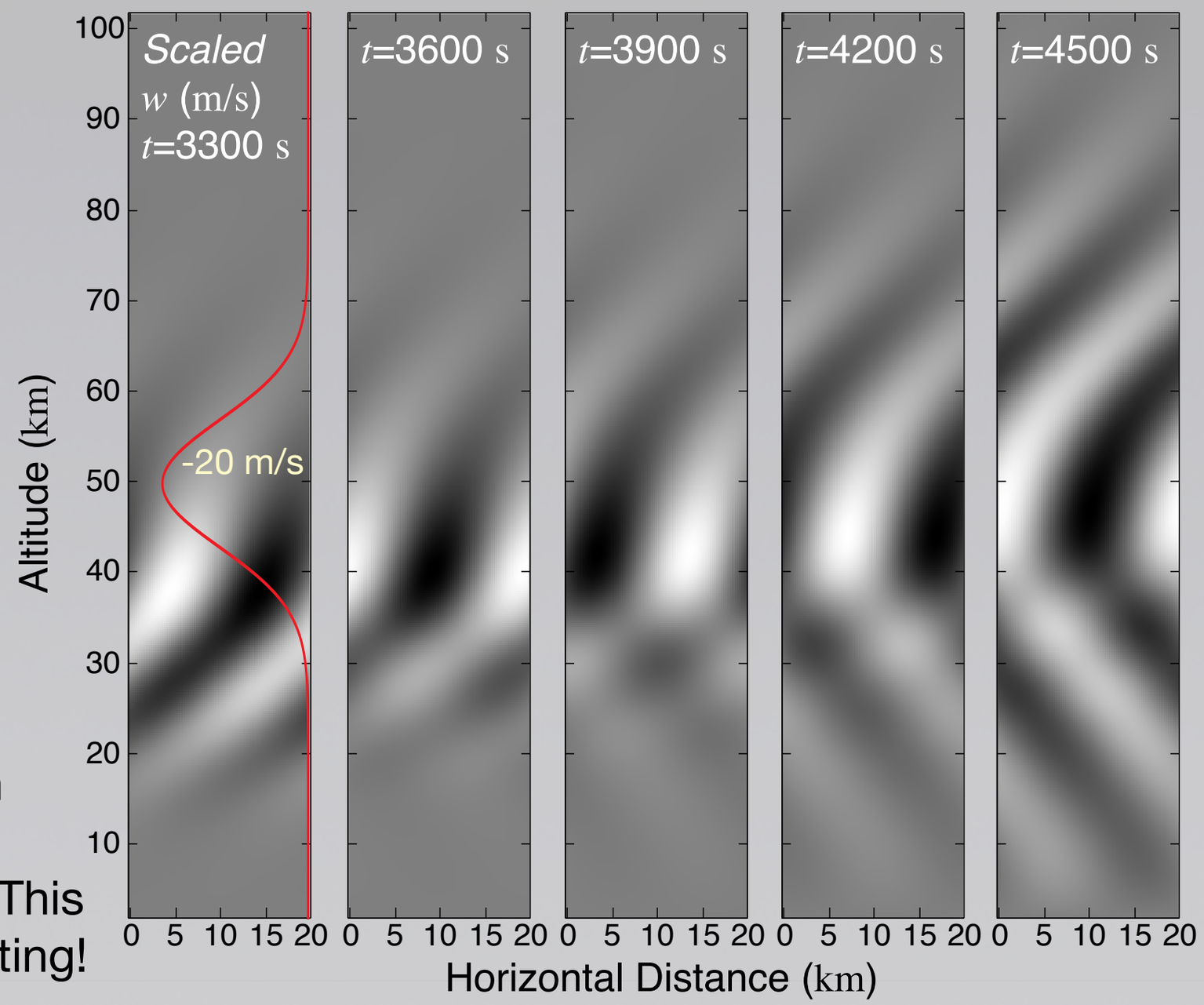
Wave:

$$\lambda_x = 20\mathrm{km}$$

$$\omega = \frac{N}{\sqrt{2}}$$

Wind:
-20 m/s Peak
at 50 km.

Result:
The gravity wave is Doppler-shifted to higher frequency. Vertical wavelength increases, and it is partially reflected. This is the basis for ducting!



Gravity Wave Propagation:
Realistic Thermal Variation

Wave:

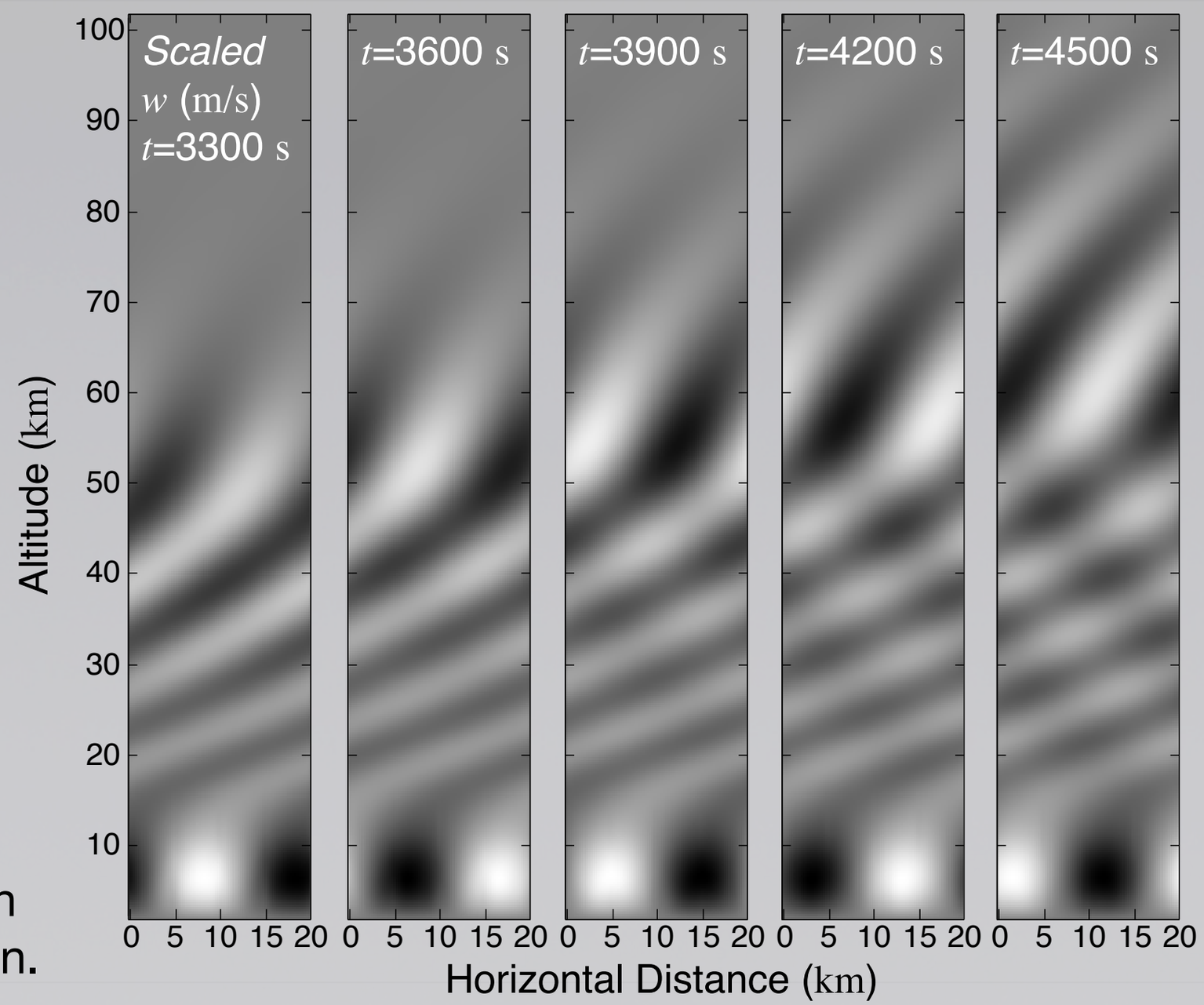
$$\lambda_x = 20\mathrm{km}$$

$$\omega = \frac{N}{\sqrt{2}}$$

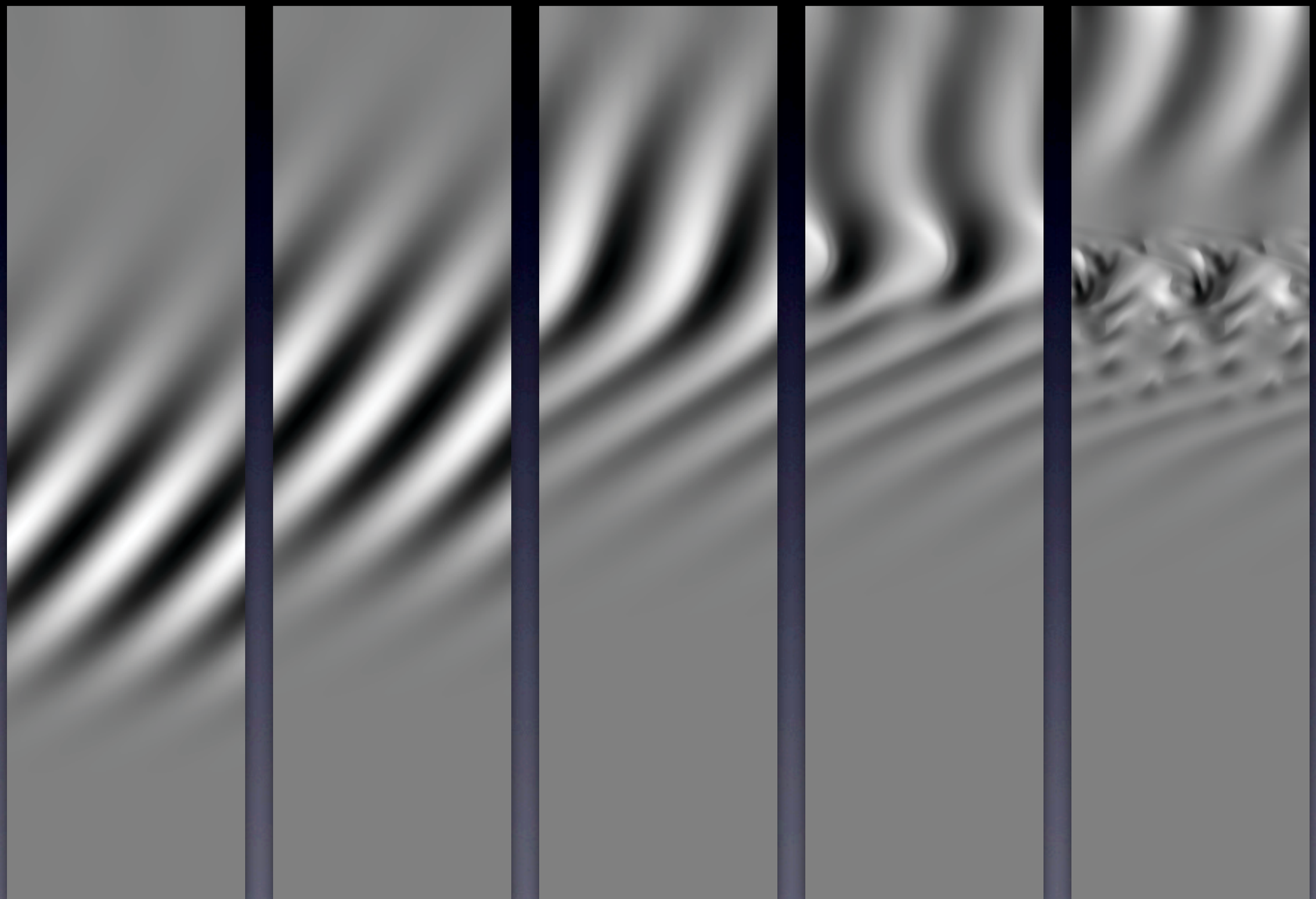
Wind:
Zero!

Temperature:
MSISE90 Profile

Result:
The gravity wave characteristics are modified relative to variations in the Brunt frequency, leading to refraction and partial reflection.



Wave perturbation amplitudes increase with altitude, eventually leading to breaking.



I – Summary

- The gravity wave spectrum is the range of waves for which buoyant effects dominate over compression or rotation, the maximum frequency being the Brunt-Väisälä frequency N.
- Waves of high frequency propagate with nearly vertical trajectories, and are thus most likely to reach high altitudes.
- Gravity wave propagation is strongly influenced by ambient wind and temperature profiles of the middle atmosphere, which may lead to strong refraction, or reflection.
- Upward propagating waves experience growth in amplitude due to decreasing density, and therefore may be subject to breaking.