EP711: COMPUTATIONAL ATMOSPHERIC DYNAMICS EP711

Spring 2018, J. B. Snively

ERAU Daytona Beach Campus
Take-Home Exam

For each problem, provide your solutions, codes, figures, and any discussion in a digital form. Please feel free to provide additional derivations, drawings, or figures in scanned PDF or handwritten (paper) form. Please include a list of references (if used), which may include any course material or provided papers/documents.

1) Investigate gravity wave propagation in a windy atmosphere. (40%)
Use the meridional and zonal wind profiles stored in the following txt file:
`http://pages.erau.edu/~snivelyj/ep711/wind.txt`
And use the 2D Euler Equations code from our class examples:
`http://pages.erau.edu/~snivelyj/ep711/flux2d_euler_gw.m`

Perform simulations, using the wind profiles within the 2D Euler Equations code:
Use the same parameters as provided in the `flux2d_euler_gw.m` file, but with a domain extending to Y_{max} = 150 km altitude, and with dy = 1000 m. Configure the code to perform two separate runs; first, using the meridional wind profile and, second, using the zonal wind profile, both provided as columns in the `wind.txt` file. *Feel free to edit this file for easy read-in by Matlab, and to any tricks necessary to fit these data in the model domain, extrapolating (or making up data) as needed for boundary cells as needed.*

Provide the following:

a) The line plot profiles of Meridional and Zonal Wind Speeds provided by `wind.txt`.
b) A line plot profile of Gradient Richardson Number *Ri*, assuming a constant N (aka Brunt-Väisälä frequency) equal to that in the `flux2d_euler_gw.m` code's initial atmosphere.
c) Two separate model simulation output figures at t~4400 seconds, showing (1) wave propagation into the meridional wind profile and (2) into the zonal wind profile.
d) A **paragraph** explaining your findings, including the differences in meridional (N-S) vs. zonal (E-W) propagation of gravity waves, and likely causes for any model failures.

Perform one additional model simulation for propagation into the zonal wind profile, with the wave source amplitude increased by a factor of five, and provide the following:

e) The model simulation output figure, at t~5000 seconds.
f) The analytical expression that you will to use to calculate N^2 from the model's results.
g) An "imagesc" plot of the model N^2 of the atmosphere at t~5000 seconds, to reveal the effects of the wave on the atmosphere's *stratification*.
h) An "imagesc" plot of the model *Ri* of the atmosphere at t~5000 seconds, to reveal the effects of the wave on the atmosphere's *stability*.
i) Two figures with two subplots each showing "imagesc" plots of the time-evolutions (with t-z axes) taken along the center of the horizontal domain (x~0), for *Ri* and N^2, and for winds u_x and u_y (i.e., u_z). I.e., these show their evolutions over altitude and time at x~0.
j) A **paragraph** explaining your findings, including comment as to whether any regions of negative or <1/4 *Ri* are formed by the GW and winds during the simulation.

3) <u>Investigate the stability for an implicit method for advection.</u> (15%)
The explicit Forward Euler (Forward in Time, Centered in Space) method is *unstable* for advection. The implicit Backward Euler (Backward in Time, Centered in Space) method is *unconditionally stable* for the heat equation. Write a finite difference expression for an implicit method for the constant-coefficient advection equation that is Backward in Time and Centered in Space (BTCS). Perform Von Neumann analysis to demonstrate its stability. Explain in a few sentences why satisfying the stability criteria may not ensure an accurate or useful solution. (Note that you do <u>not</u> need to implement or demonstrate this method.)

4) <u>Confirm that the flux form of Lax-Wendroff does what it claims.</u> (8%)
Recall that "flux form" finite difference and finite volume methods for hyperbolic problems adhere to form given by:

$$Q_j^{n+1} = Q_j^n - \frac{\Delta t}{\Delta x}(F_{j+1/2}^n - F_{j-1/2}^n)$$

A flux function *F* for constant-coefficient advection is given on Slide 11 of Lecture 5.
Confirm algebraically that this reduces to the "standard" Lax-Wendroff method.

5) <u>Graphically determine whether this flux-limiter is TVD.</u> (10%)
The "*Koren"* flux limiter function is given by the following expression:

$$\phi(\theta) = \max[0, \min(2\theta, (1+2\theta)/3, 2)]$$

Graphically determine whether this limiter satisfies TVD criteria, e.g., as defined by Sweby.

6) <u>Reformulate the Energy Equation.</u> (12%)
Starting with total energy conservation law equation on Slide 8 of Lecture 12, derive a nonlinear energy equation for temperature, instead. (This equation thus is for $\partial T/\partial t$, vs. $\partial E/\partial t$.) You may assume an ideal gas equation of state and our usual definition of energy *E*, and may use the momentum equation on the previous Slide 5 as a basis to define the terms describing conservation of kinetic energy.

7) <u>Explain (in words, diagrams, and/or math) the difference between "upwind" and "central" (or "centered") finite volume methods and their relative benefits</u> (15%)
In class, we discussed the concept of upwind methods, which take into account the directions in which "information" propagates through a mesh using characteristic theory, and centered methods, which do not. Explain in any way you wish (within a single page "essay"), the differences between these methods, and why a scientist or engineer may favor one or the other for certain problems.