EP711: *Constructing Test Cases: Forcing or Initializing a Model*

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Initiating Model Dynamics

Initial Conditions: i.e.,

- Specified perturbations to one or more state variables.
- Self-consistent features to satisfy a particular solution.

Boundary Conditions: i.e.,

- Constant or time-dependent imposed values.
- Inflow or outflow of mass, momentum, or energy.

Source terms: i.e.,

- Distributed sources for mass, momentum, or energy.
- Body forcing (accelerations), parameterized inputs.

Initiating Model Dynamics: Examples

- Unstable initial states that will naturally tend to evolve.
- Random noise / impulse / step functions to assess responses to general inputs.
- Realistic initial state provided by a larger-scale model or assimilated data products.
- Specified wave packets defined by assumptions of linear dispersion characteristics.
- Time-dependent boundary forcing, to specifically constrain one or more variables.
- Time-dependent oscillatory forcing via a distributed acceleration / body force.

Examples of Initializations…

The boundary conditions are those commonly used in this type of model. The bottom boundary is used to force the primary wave. The wave forcing applied to the vertical velocity, w, is a continuously forced, sinusoidal waveform of a specified frequency and horizontal wavenumber turned on smoothly over two wave periods T:

$$w(x, z = 0, t) = \begin{cases} \sin\left(\frac{2\pi}{8T}t\right)\sin\left(kx - \frac{2\pi}{T}t\right) & 0 < t < 2T \\ \sin\left(kx - \frac{2\pi}{T}t\right) & t > 2T. \end{cases} \quad (1)$$

[*Franke and Robinson*, 1999]

Examples of Initializations…

Case B: *Gravity Wave with No Mean Wind*

A steady forcing at the lower boundary corresponds to a horizontal wavelength of 100 km ($k = 2\pi \times 10^{-5}$) and a period of 30 min with no mean motions. The forcing function, smoothed for the first part of the calculation to reduce sound-wave production, is given by

$$w(0, t) = (1 - e^{(\omega/2)t})\, e^{i\omega t}, \tag{14}$$

where $\omega = 3.4907 \times 10^{-3}\ \mathrm{sec}^{-1}$.

[*Houghton and Jones*, 1969]

Examples of Initializations…

The initial horizontal velocity perturbation is similar to that used by Xu et al. (2003) and Liu et al. (2008),

$$u'(x, z, t = 0) = A \exp\left[-\ln(2) \times \frac{(z - z_0)^2}{2\sigma_z^2}\right] \exp\left(\frac{z - z_0}{2H}\right) \cos\left[k_x x + k_z (z - z_0)\right]. \qquad (7)$$

[*Liu et al.*, 2013]

Selfish Examples…

A vertical body forcing was applied which is centered at the bottom boundary and given by

$$F_z(x,z,t) = Ae^{-0.5((x-x_c)^2/\sigma_x^2+(z-z_c)^2/\sigma_z^2)} \cdot e^{-(t-t_c)^2/2\sigma_t^2} \cos(\omega(t-t_c))\cos(k(x-x_c))$$

[*Heale at al.*, 2014]

A disturbance approximating a series of convective updrafts and downdrafts is imposed as a body force (in units of N/kg $\equiv$ m/s^2):

$$F_s(x,z,t) = \mathcal{F}_0 \exp\left(-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(z-z_0)^2}{2\sigma_z^2} - \frac{(t-t_0)^2}{2\sigma_t^2}\right)\cos(\omega t) \quad (1)$$

where σ_x (or σ_r) and σ_z specify the disturbance size in lateral (or radial) and vertical dimensions, σ_t is the disturbance duration modulated by frequency ω, and $\mathcal{F}_0$ is the amplitude.

[*Marshall and Snively*, 2014]

Modeling Framework: Example Chemical Model

Approach: Separate equations of motion and source terms by forms of resulting PDEs / ODEs and solution methods.

Continuity equation for a chemical species:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = D\nabla^2 n + P - L$$

Here, we have (1) advection, (2) diffusion (coefficient *D*), and (3) production *P* and loss *L* source terms (which form an ODE).

1. Solve via method for hyperbolic problems.
2. Solve via method for parabolic problems.
3. Solve via ODE techniques.

Modeling Framework: Example Dynamical Model

Typical Modeling Challenges:

- **Disparate Time Scales / "Stiff" Source Terms**
  - Operator / Time Splitting over different *dt*?
  - Effects of order of operations? Need for implicit solution?
  - Mixed systems: Hyperbolic, parabolic, elliptic in different regimes?
- **Boundary Conditions**
  - Validity for different physical waves in a system of equations?
  - Numerical vs. physical formulations.
- **Numerical vs. Physical Stability, Dissipation, Dispersion…**
  - Identification of physical versus numerical features?
  - Conscious application/acceptance of dispersion or dissipation?