EP711 Lecture

Tuesday, March 20th, 2018

Modeling of Acoustic and Gravity Waves in Earth's Atmosphere (*Constructing Test Cases*)

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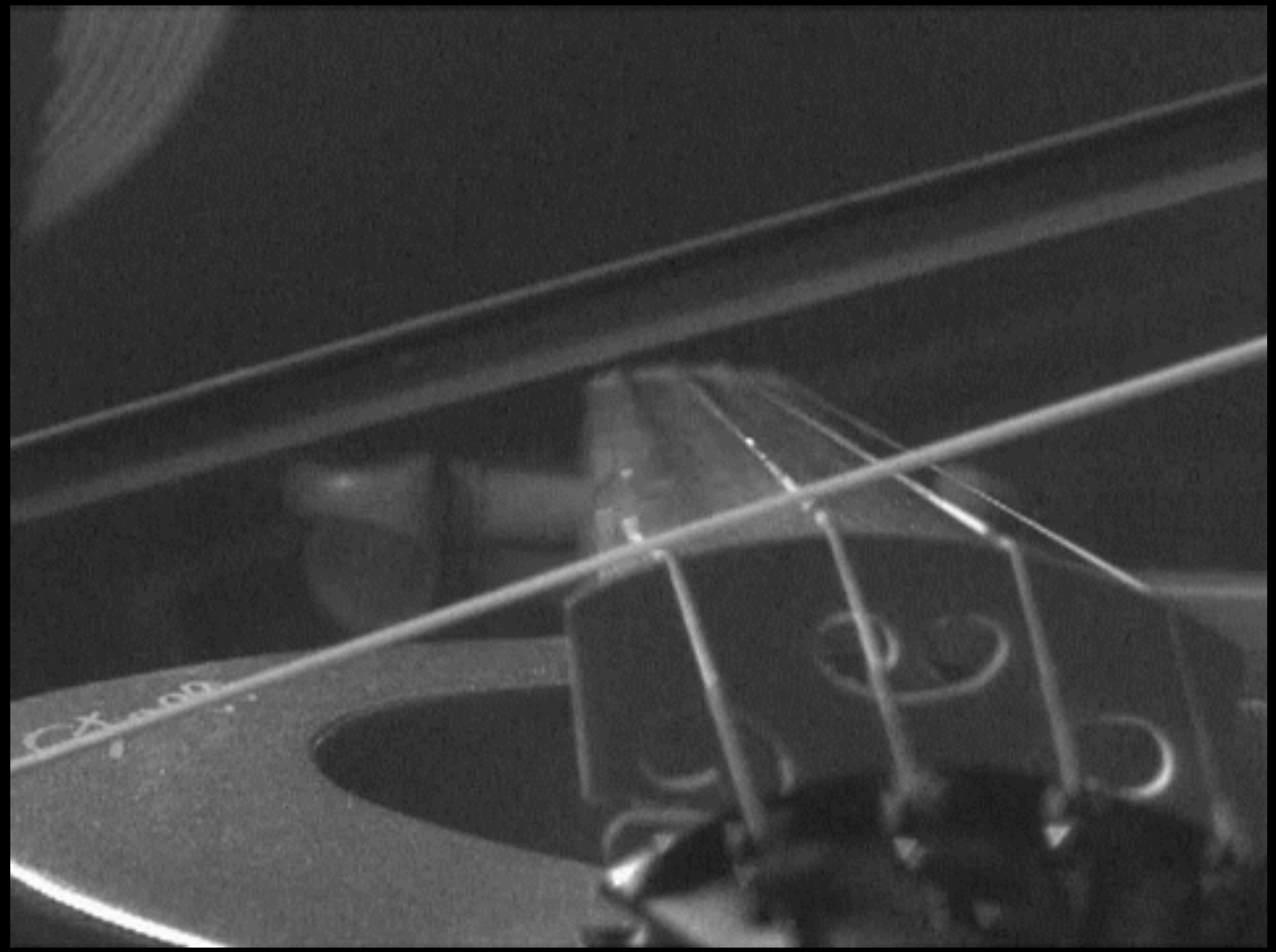
Wave Types

Transverse: Wave oscillations are perpendicular to propagation.



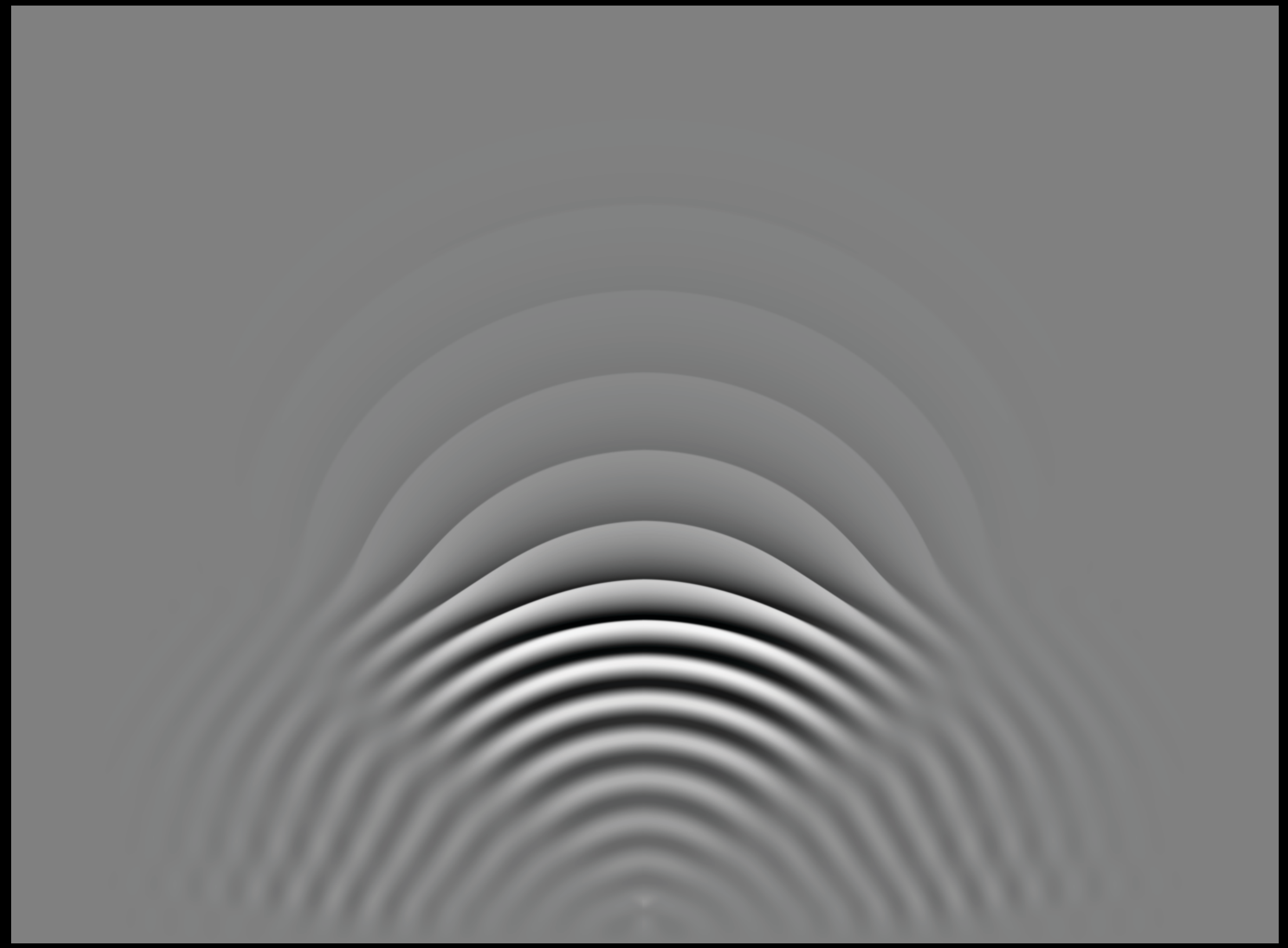
Waves on Strings

Slinky "Wiggles"

Longitudinal: Wave oscillations are parallel to propagation.

Slinky "Compressions"

Acoustic Waves



[Slinky, Violin, and Ripple courtesy of Wikimedia Commons and their Authors]

Wave Types

Mixed: Motion associated with wave oscillations occurs parallel and perpendicular to the direction of propagation.

Ripples on Water
(Due to Surface Tension and Gravity)

"Internal Gravity Waves" in the atmosphere …

… and ocean. Internal because "inside" the fluid.

[Images courtesy of Wikimedia Commons and their Authors.]

“Internal Gravity Waves” in the atmosphere … … and ocean. Internal because “inside” the fluid.

They are called **“Gravity Waves”** because they arise from disturbances to the atmosphere’s stable resting state, which is otherwise held in balance by *buoyancy* and *gravity*.

Parcels of air, when lifted (i.e., by weather, mountains, volcanoes, or ocean waves), will *oscillate* at frequencies no higher than the *Brunt-Väisälä frequency* (periods 5-10 minutes).

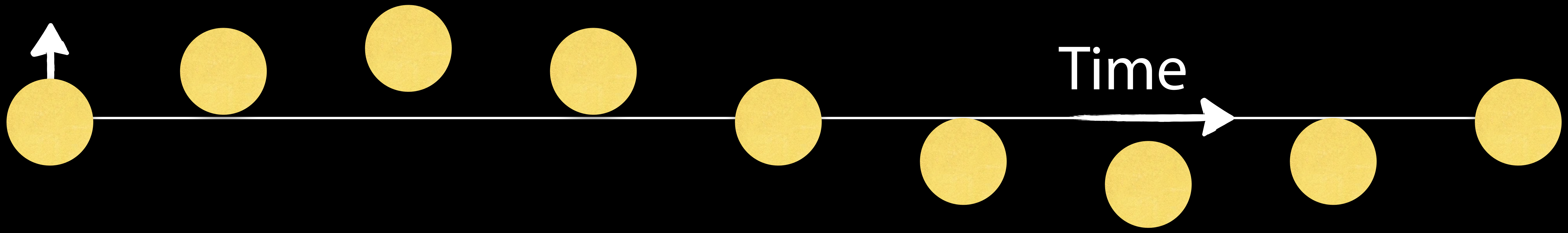
[Images courtesy of Wikimedia Commons and their Authors.]

Acoustic-Gravity Wave Dispersion

The Brunt-Väisälä Frequency N:

The Earth's atmosphere is stratified, such that gravity and buoyancy maintain equilibrium. Density and pressure decrease exponentially with altitude at a characteristic scale height *H*~ 6-8 km.

If a parcel is displaced vertically in a gravity-stratified atmosphere,



... it will oscillate at a characteristic frequency *N* (~0.01-0.02 rad/sec)

$$\boxed{N^2 = \frac{g}{\theta_o}\frac{d\theta_o}{dz} \simeq \frac{g^2}{c_s}(\gamma - 1)}$$

This is the highest frequency at which gravity waves can propagate!

Acoustic-Gravity Wave Dispersion

Linearized Equations of Motion:

$$\frac{\partial \tilde{v_x}}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}}{\partial x} \tag{1}$$

$$\frac{\partial \tilde{v_z}}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}}{\partial z} - \frac{\tilde{\rho} g}{\rho_o} \tag{2}$$

$$\frac{\partial \tilde{\rho}}{\partial t} = -\rho_o\frac{\partial \tilde{v_x}}{\partial x} - \rho_o\frac{\partial \tilde{v_z}}{\partial z} - \tilde{v_z}\frac{\partial \rho_o}{\partial z} \tag{3}$$

$$\frac{\partial \tilde{p}}{\partial t} = -\tilde{v_z}\frac{\partial p_o}{\partial z} - \gamma p_o\left(\frac{\partial \tilde{v_x}}{\partial x} + \frac{\partial \tilde{v_z}}{\partial z}\right) \tag{4}$$

Substituting $\tilde{v}_z = (\rho_o/\rho_s)^{-1/2}\tilde{w}_z$, a wave equation for the normalized vertical perturbation velocity $\tilde{w}_z$ is obtained:

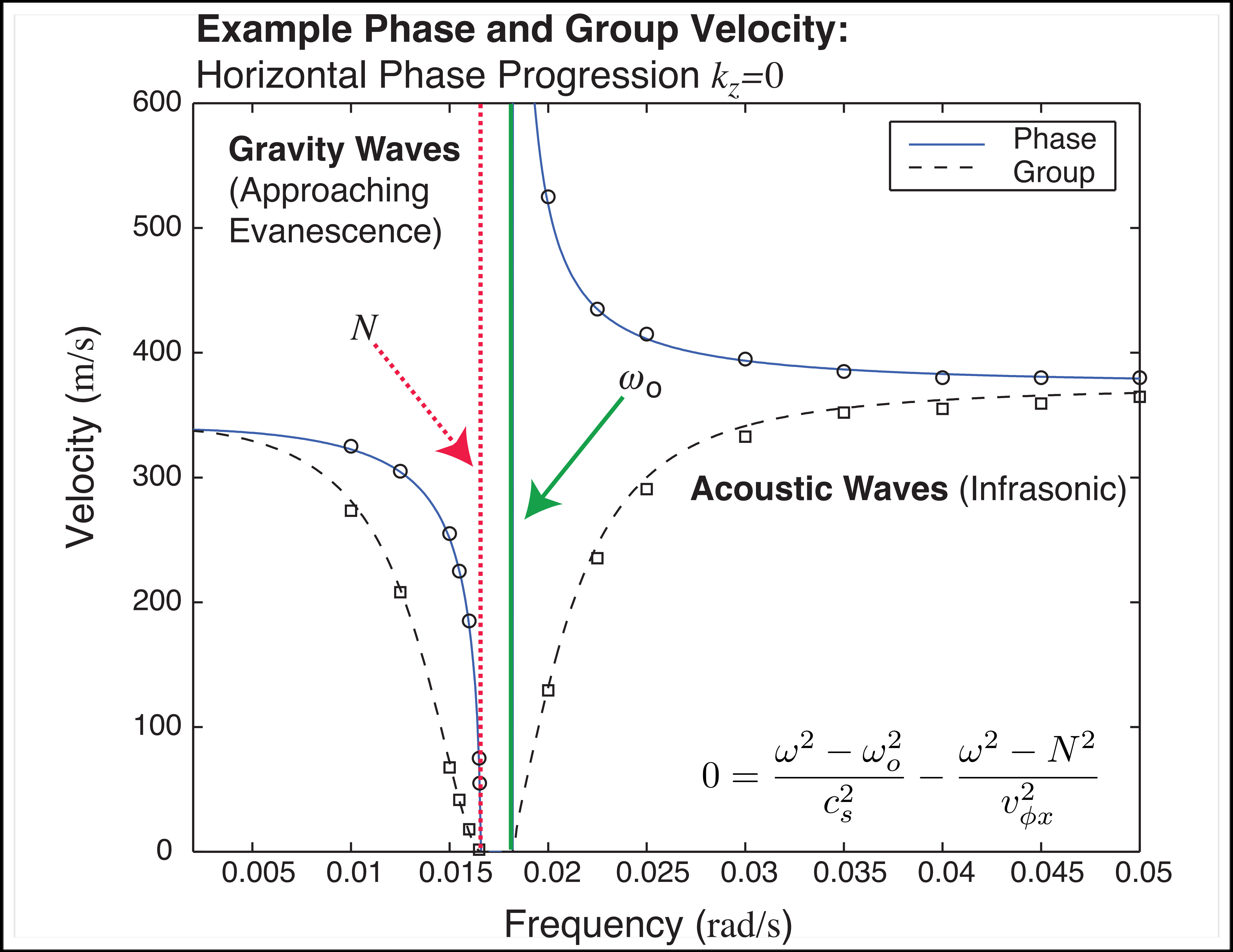
$$\boxed{\frac{\partial^2 \tilde{w}_z}{\partial z^2} + \left[\frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2}\right]\tilde{w}_z = 0.}$$

which is governed by the linear dispersion relation

$$\boxed{k_z^2 = \frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2},}$$

where $\omega_o = \frac{g\gamma}{2c_s}$ is the acoustic cut-off frequency and $N = \frac{g}{c_s}\sqrt{\gamma - 1}$ is the Brunt-Väisälä resonance frequency.

Acoustic-Gravity Wave Dispersion



Acoustic-Gravity Wave Dispersion

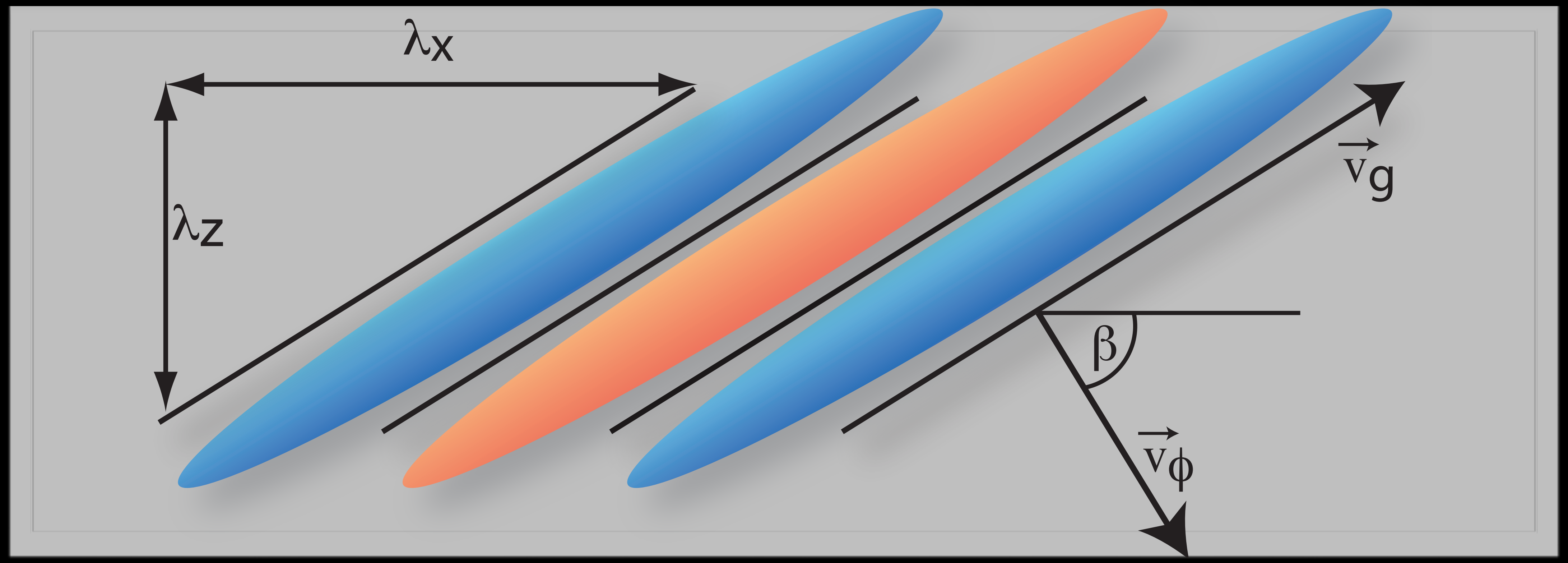
As stated, the compressible dispersion relation is:

$$k_z^2 = \frac{\omega^2 - \omega_o^2}{c_s^2} - \frac{\omega^2 - N^2}{v_{\phi x}^2}$$

Gravity waves are transverse wave motions below the Brunt-Väisälä frequency N and the acoustic cut-off frequency ω_o.

Neglecting the acoustic terms, a convenient form for the *in*compressible dispersion relation relates k_x and k_z via the wave vector angle β to the horizontal (see Figure) [e.g., *Nappo*, 2002, p. 32]:

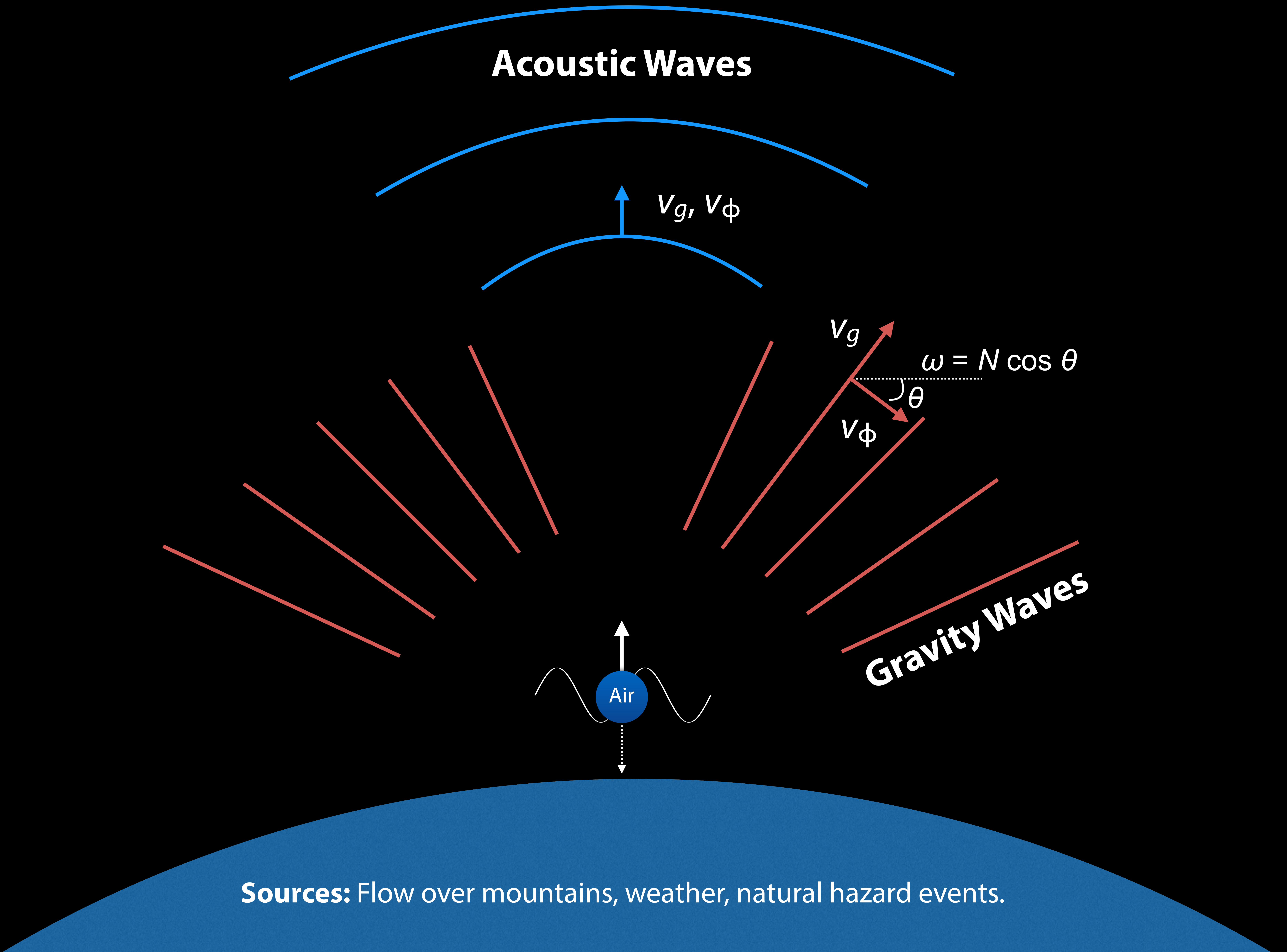
$$\omega = \frac{k_x N}{(k_x^2 + k_z^2)^{1/2}} = N \cos \beta.$$



Group and Phase Velocity are Perpendicular!

Interesting Conclusion: Frequency ω of gravity wave (relative to N) determines λ_x, λ_z, and thus direction of propagation!

Acoustic-Gravity Wave Dispersion



Waves in the *Upper* Atmosphere

Wave Scales (i.e., wavelengths):

Acoustic Waves (In This Room, *c*=340 m/s):

f~20 Hz, λ=17 meters.

(At least that we can hear!)

f~20,000 Hz, λ=1.7 centimeters.

Acoustic Waves (Thermosphere, *c*=700 m/s, very low density):

T_A~4 minutes (f=0.00416 Hz), λ=168 kilometers.

T~10 seconds (f=0.1 Hz), λ=7 kilometers.

Gravity Waves (Mesosphere and Thermosphere):

T~Several hours, λ=1000s of kilometers.

T_B~5 minutes, λ=10s of kilometers.

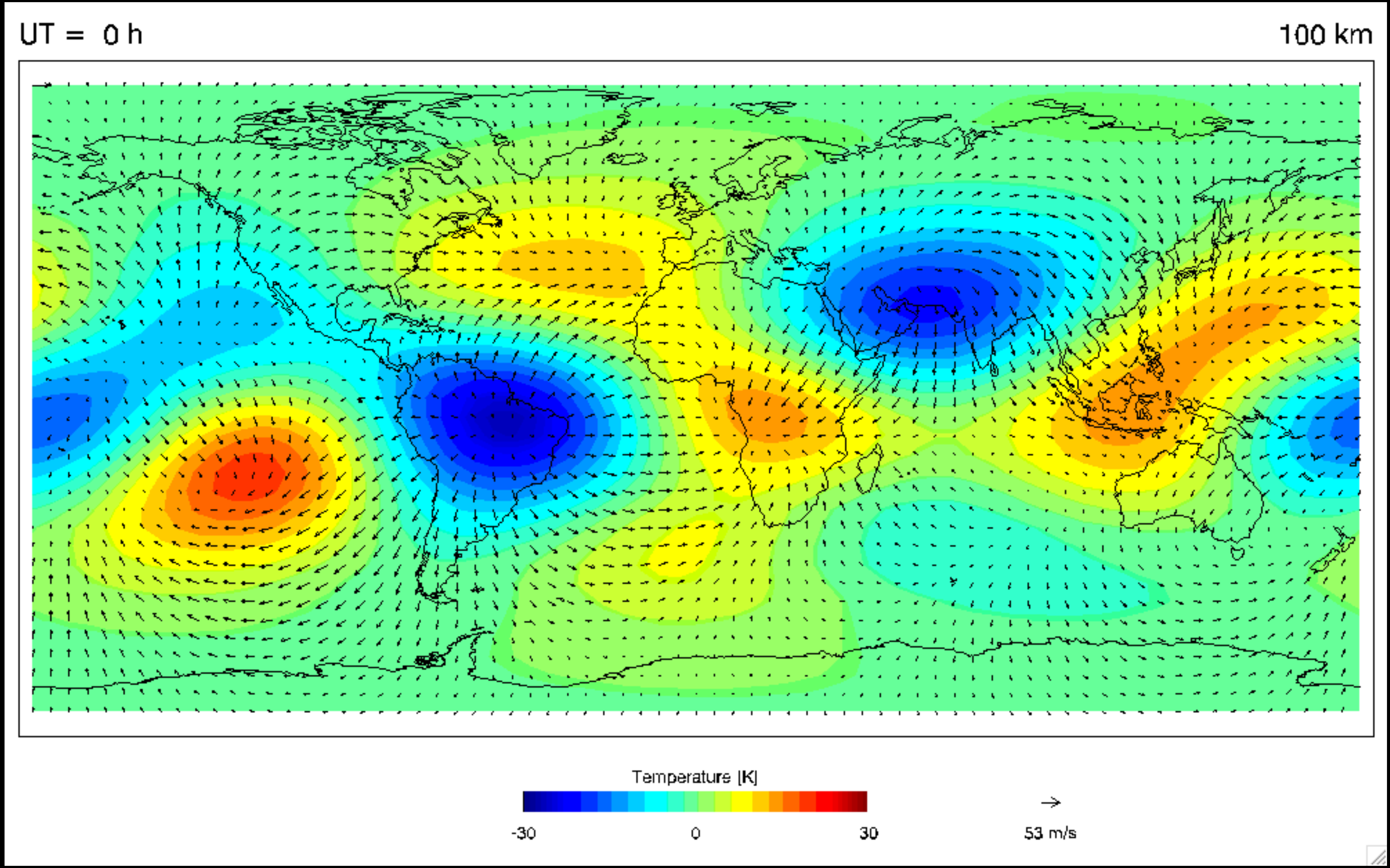
Waves in the *Upper* Atmosphere

Tidal/Planetary

Gravity

Acoustic

Atmospheric (solar-thermal) tides: 12-24 hour periods.



Here reconstructed from global satellite data.

Note: **Lunar tides are very weak in the atmosphere!**

[*Jens Oberheide* / TIMED-SABER / Wikimedia Commons, 2009]

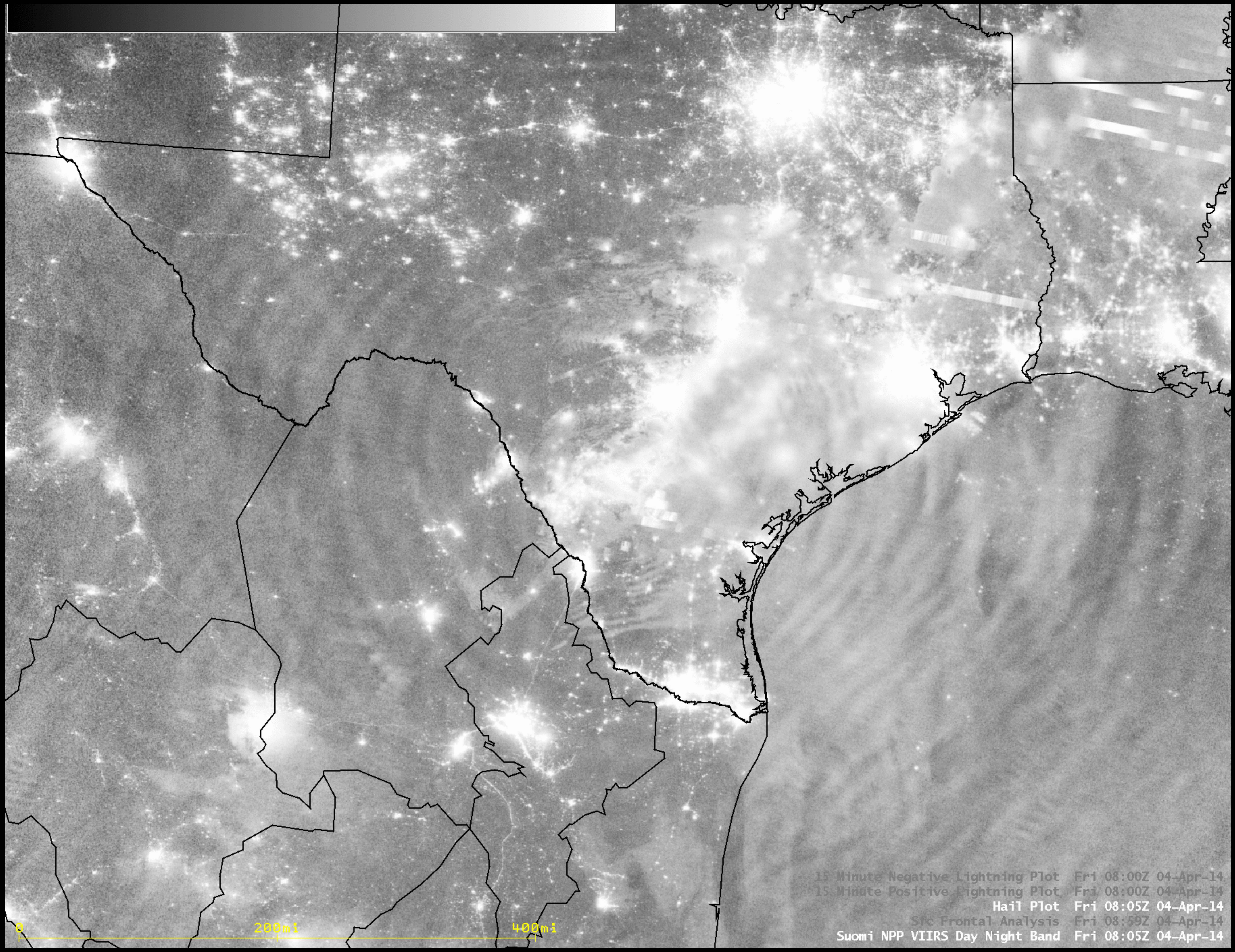
Waves in the *Upper* Atmosphere

Tidal/Planetary

Gravity

Acoustic

Atmospheric Gravity Waves: ~Minutes to hours periods.



Here seen in airglow, over a thunderstorm in Texas.

[*Miller et al.*, Proc. Nat. Acad. Sci., 2015]

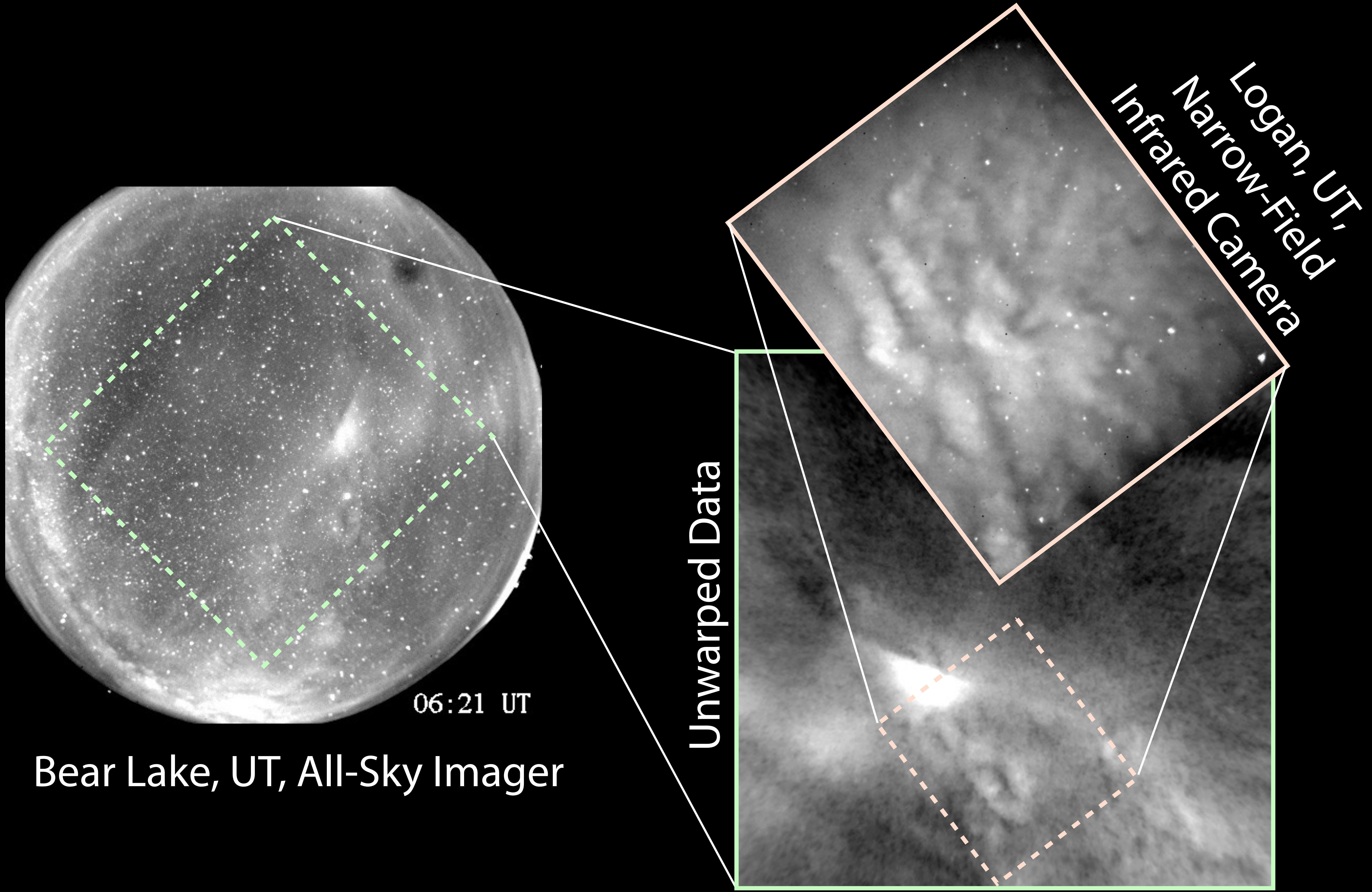
Waves in the *Upper* Atmosphere

Tidal/Planetary

Gravity

Acoustic

Atmospheric Gravity Waves: ~Minutes to hours periods.



[Courtesy of *M. J. Taylor and P-D. Pautet*]

Waves in the *Upper* Atmosphere

Tidal/Planetary

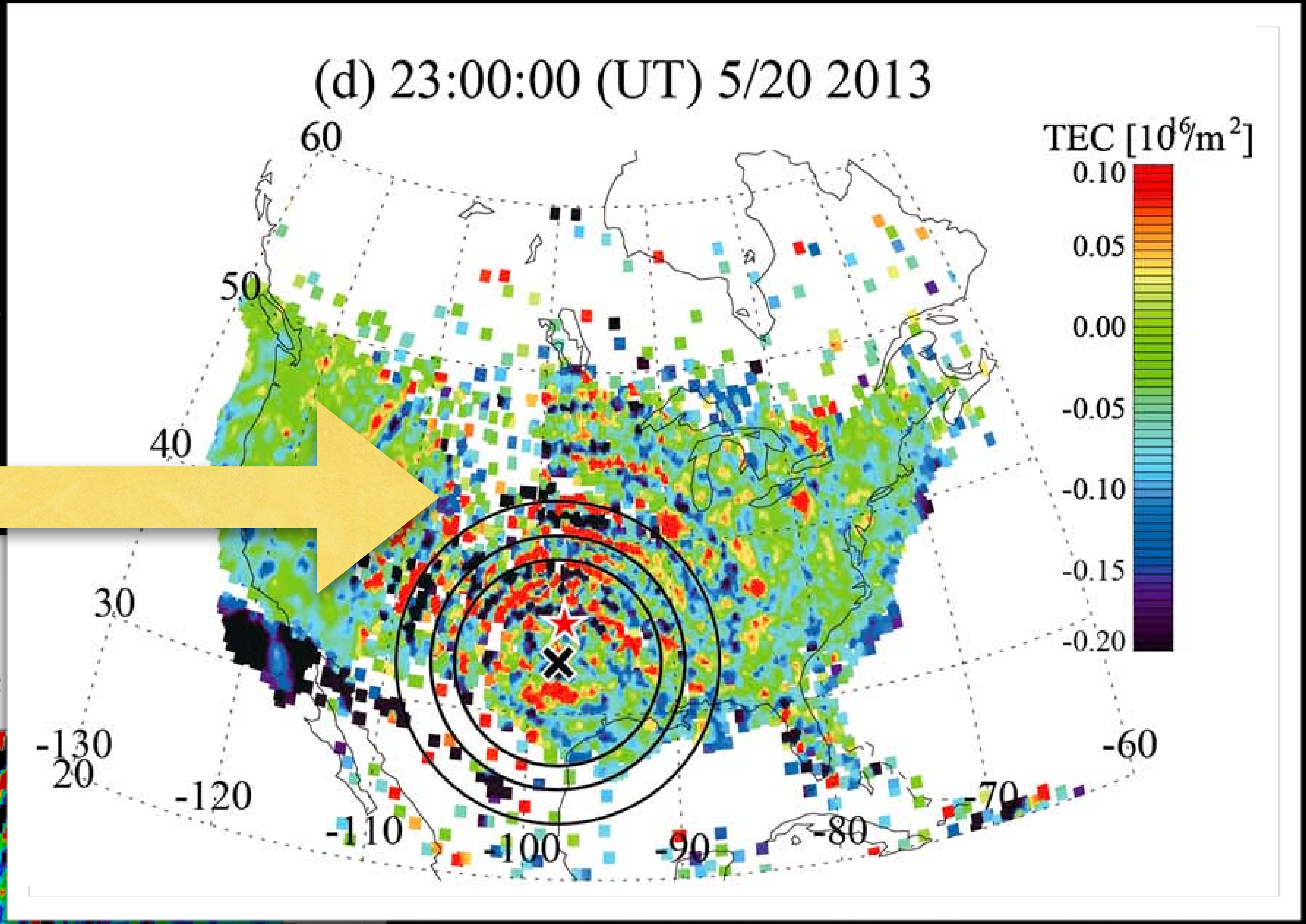
Gravity

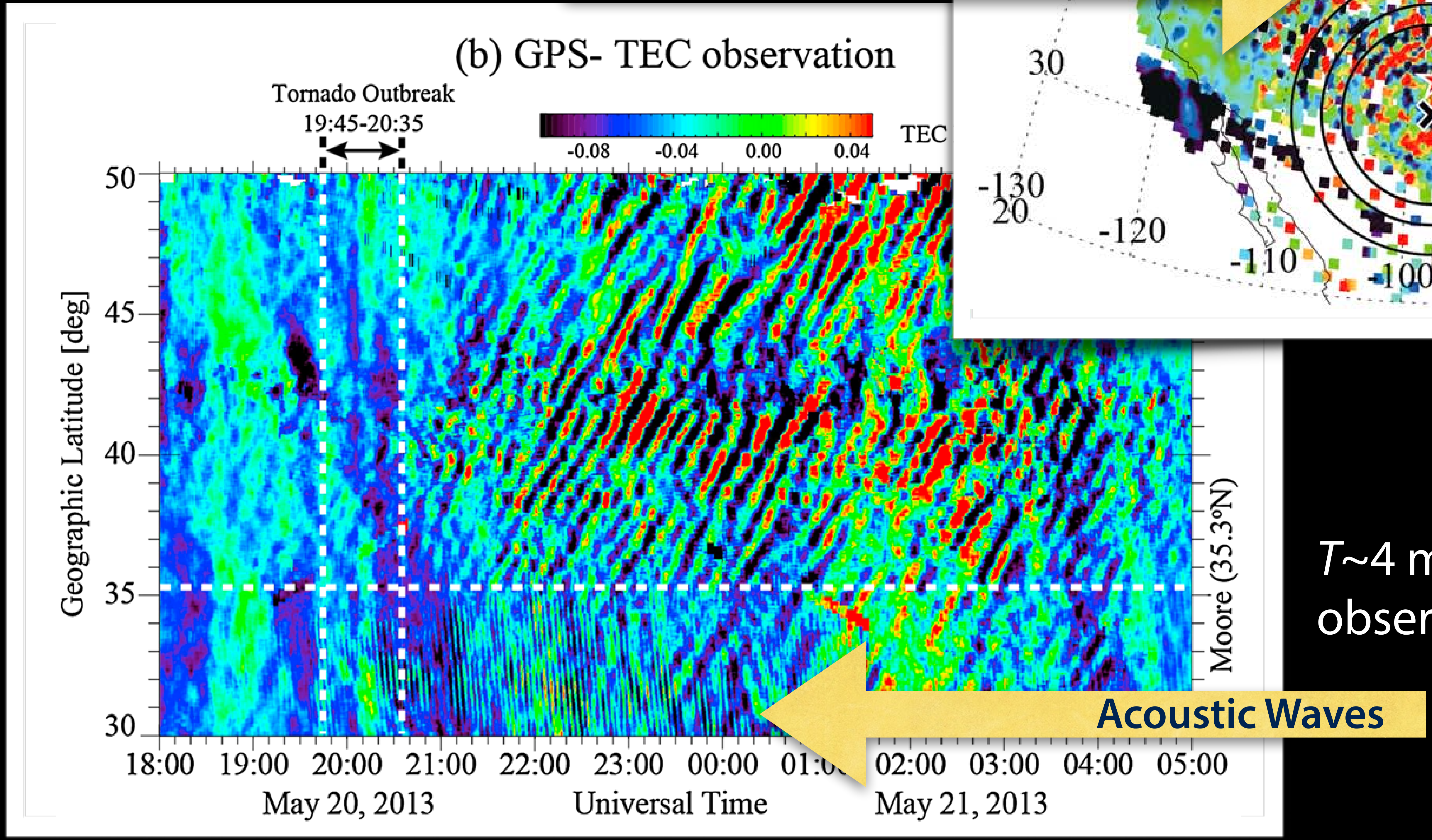
Acoustic

Acoustic and *Gravity Waves:* ~Minutes to hours periods.

Here seen in the ionosphere, over the 2013 Moore, Oklahoma, EF5 Tornado.

T~13 minute period gravity waves in the ionosphere.





T~4 minute acoustic waves observed toward equator.

[*Nishioka et al.*, GRL, 2015]

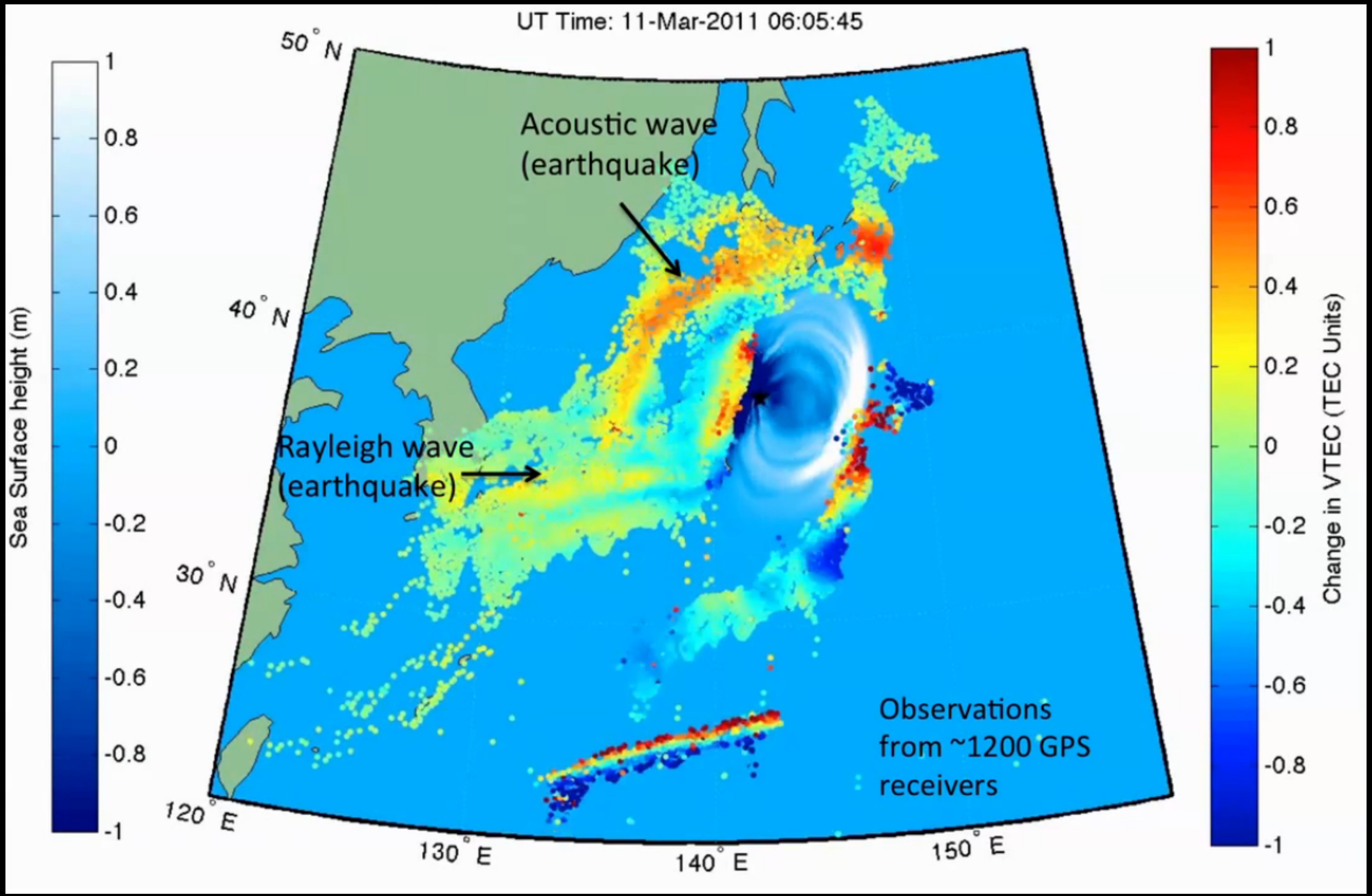
Waves in the *Upper* Atmosphere

Tidal/Planetary

Gravity

Acoustic

Atmospheric Acoustic Waves: ~1-4 minute periods.



Here seen in GPS measurements of the ionosphere (near ~250 km altitude!), following the Tohoku Earthquake and Tsunami.

[*Galvan et al.*, Radio Science, 2012]

What about the Smaller Scales?

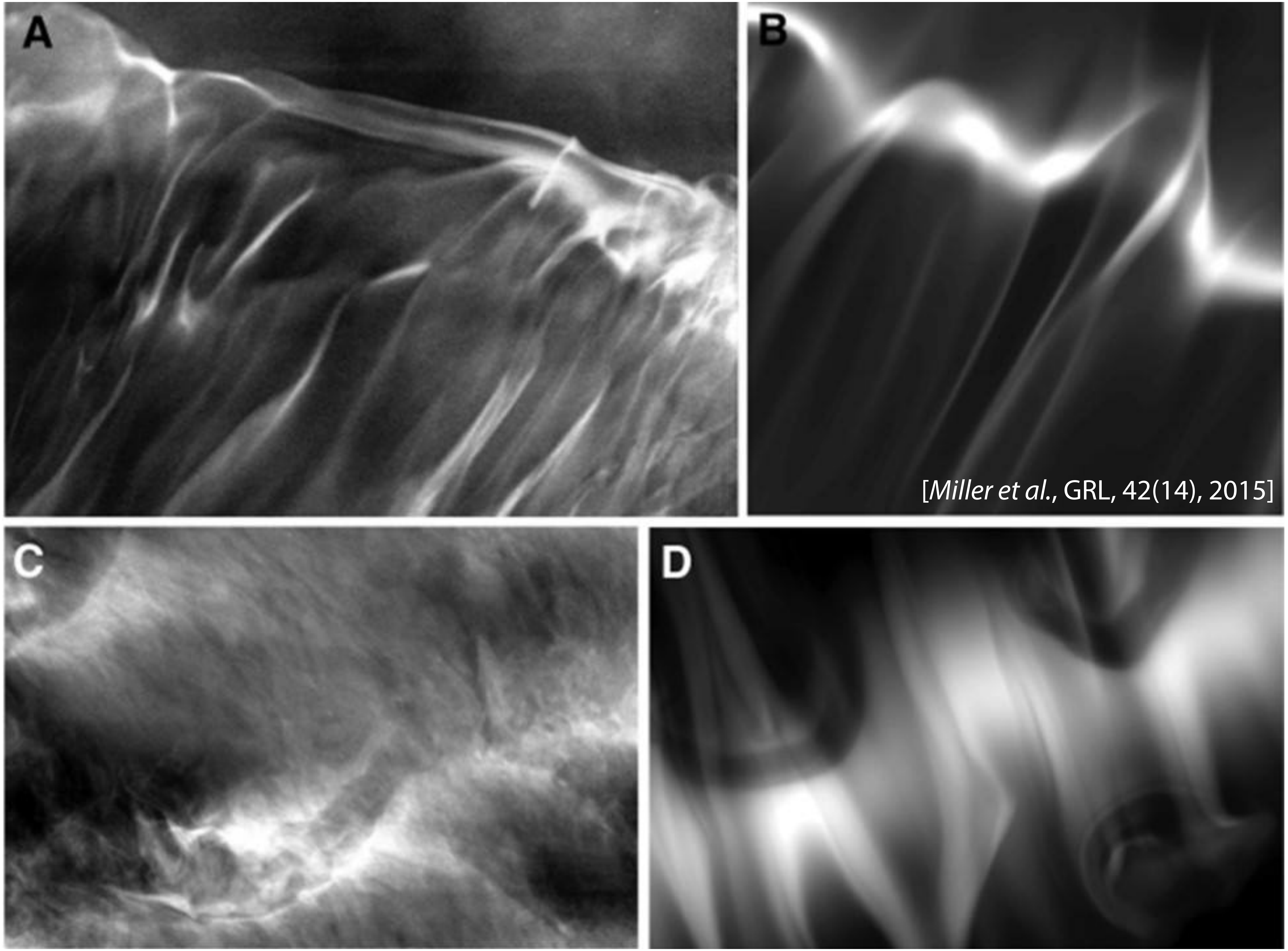
How fine of atmospheric structure and dynamics can we detect from the ground or space? What can we learn about small (but mighty?) waves?

Noctilucent Clouds…

[Image Wikimedia Commons / Johann Enmarc]

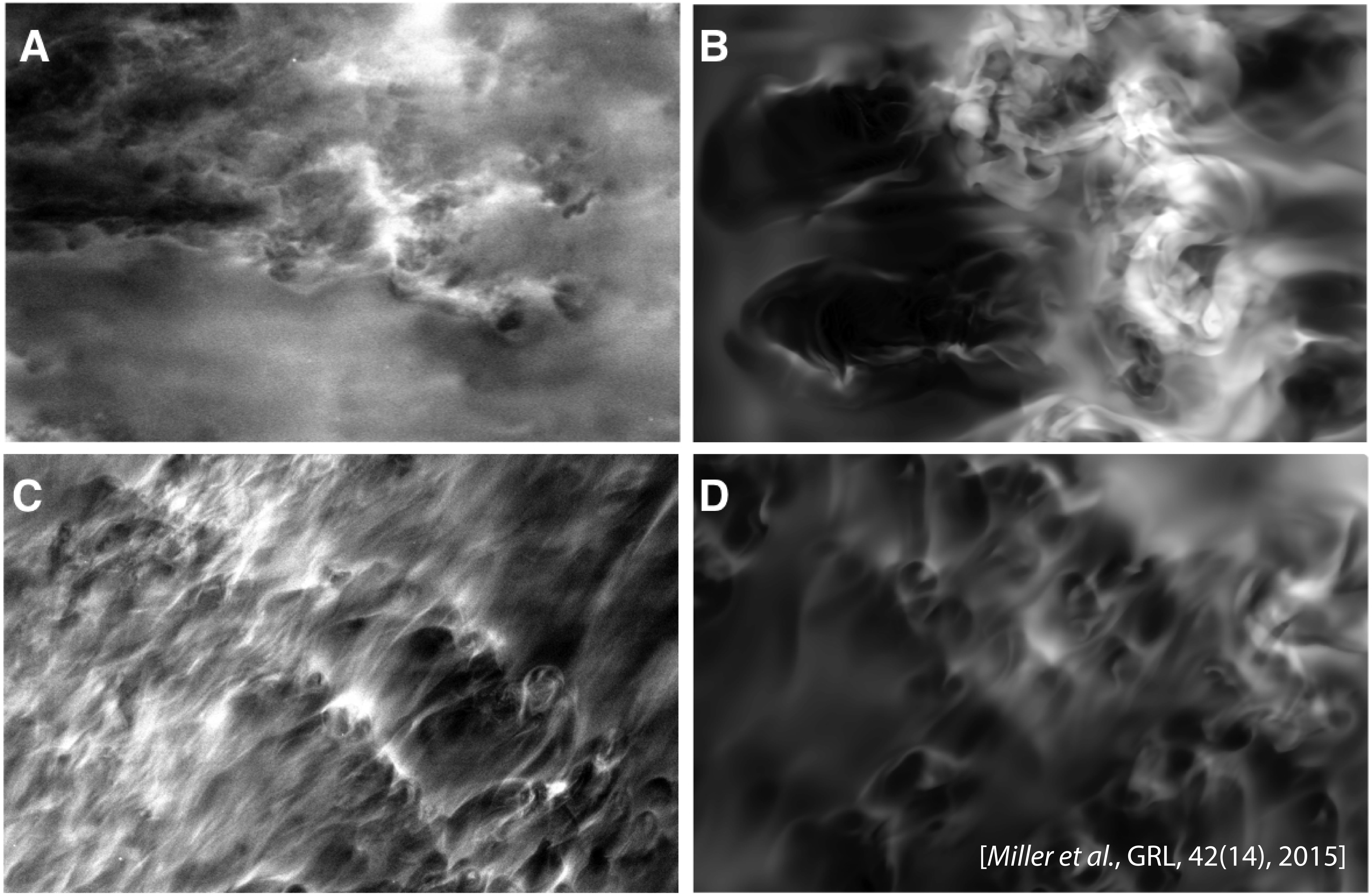
Breaking Waves in Noctilucent Clouds (NLCs)

Captured *fortuitously* during the "EBEX" balloon experiment to measure the cosmic microwave background, via the star-pointing camera!



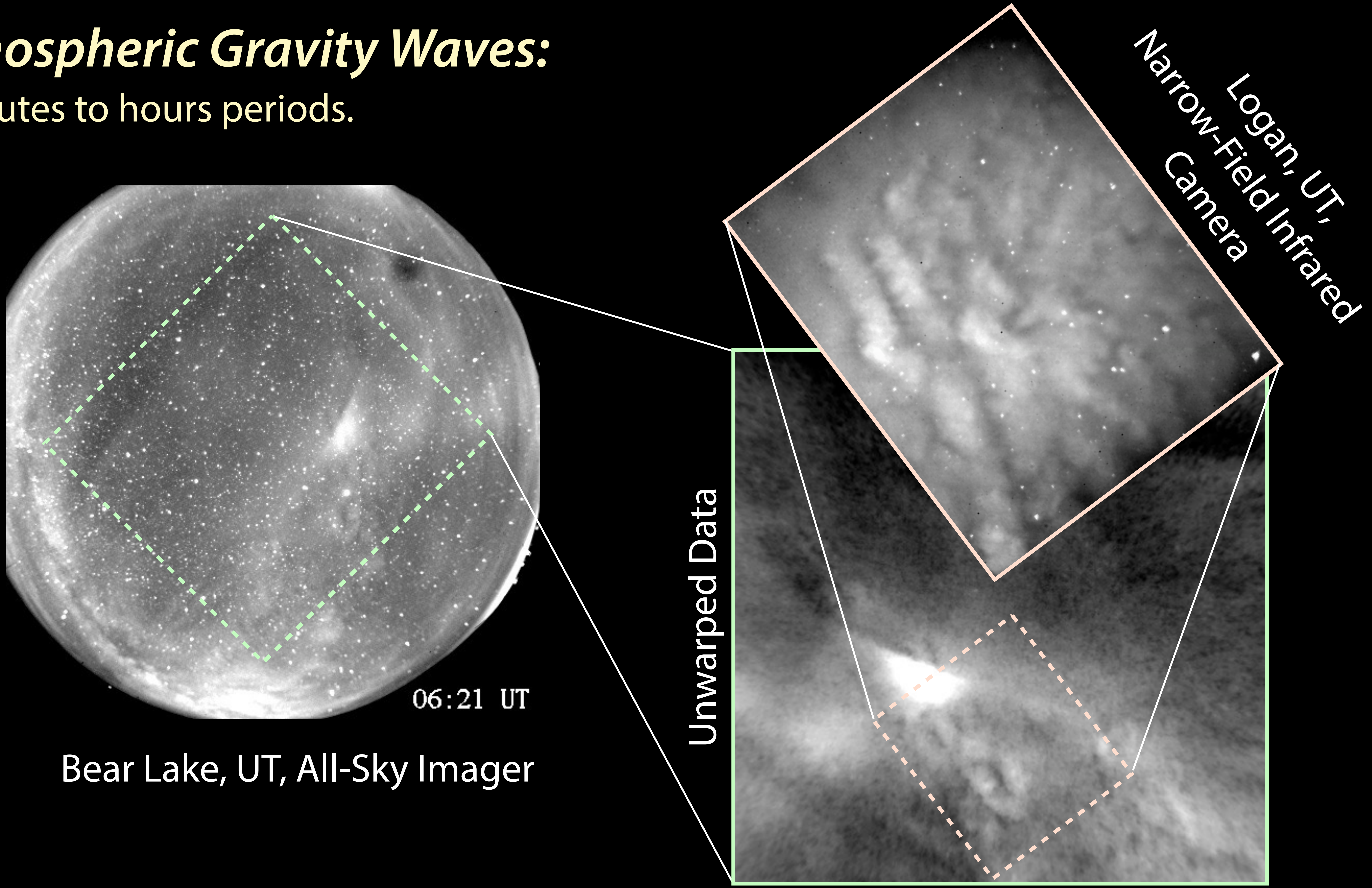
Turbulence in Noctilucent Clouds (NLCs)

Captured *fortuitously* during the "EBEX" balloon experiment to measure the cosmic microwave background, via the star-pointing camera!



Atmospheric Gravity Waves:

~Minutes to hours periods.



Bear Lake, UT, All-Sky Imager

[Courtesy of *M. J. Taylor and P-D. Pautet*]

"MAGIC"

Atmosphere Model

[*Snively and Pasko*, 2008; *Snively*, 2013; *Zettergren and Snively*, 2015]

- **Model for Acoustic & Gravity wave Interactions and Coupling**
- Based on finite volume method (FVM) [*LeVeque*, 2002] for **nonlinear**, **compressible**, **Euler or Navier-Stokes** equations.
- Solves the *complete* Riemann problems via a **flux-difference-splitting (*f-wave*) Roe solver** [e.g., *Bale et al.*, 2002], to capture physics of the Euler equations with gravity (with "transverse" Riemann solvers to avoid dimensional splitting).
- Applies **2nd or *3rd* order (CFL-dependent) flux limiters** [e.g., *Kemm*, 2011] *to each* "Wave", calculated directly from characteristics, to control dispersion and dissipation at small scales and enable "*Implicit* Large Eddy Simulation" (**ILES**) with shock-capturing.
- Incorporates **Navier-Stokes viscosity and thermal conduction** via a fast explicit method (adaptively sub-stepped).

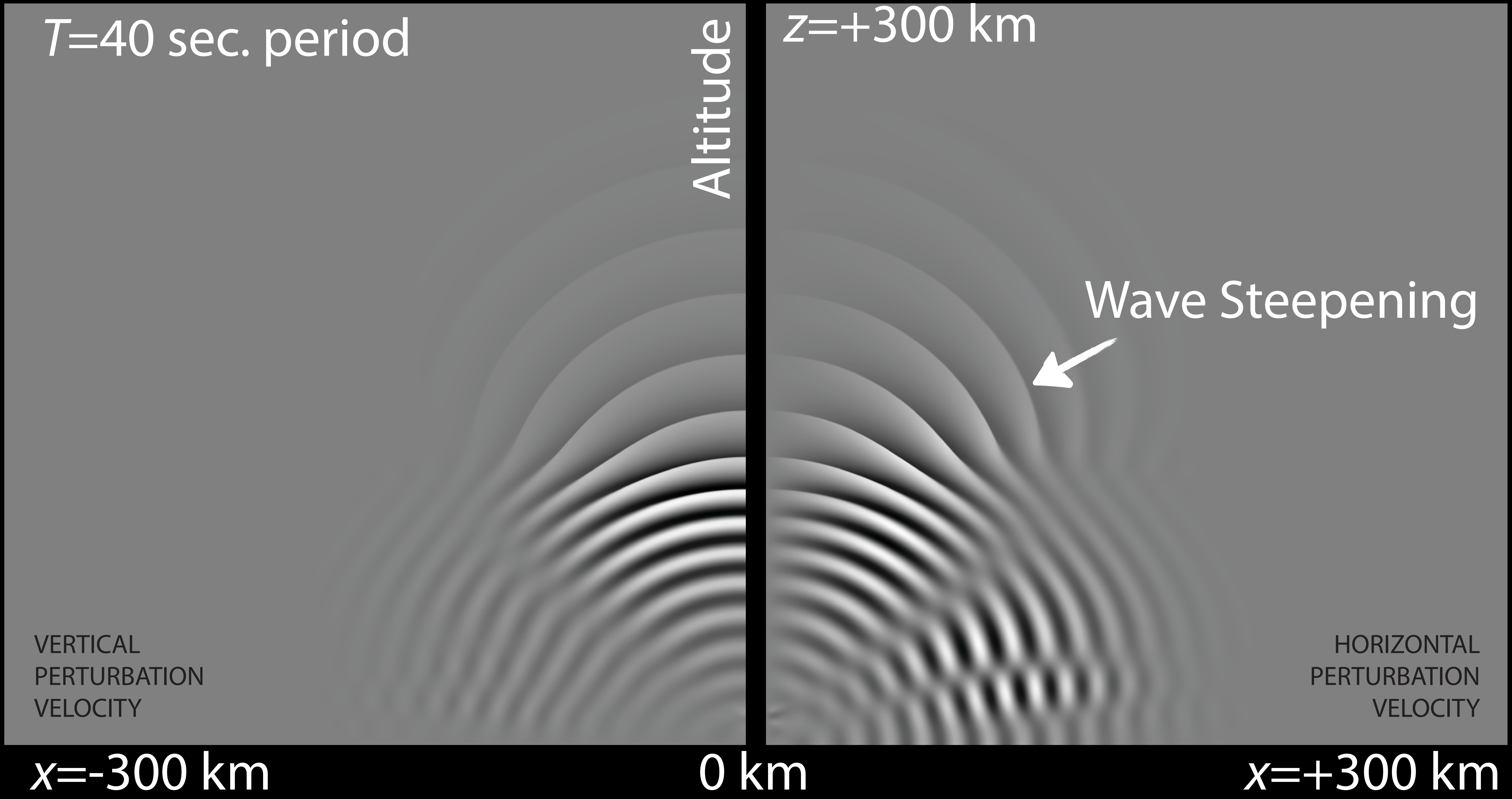
$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + p\mathbf{I}) &= \rho \mathbf{g} + \nabla \cdot \boldsymbol{\tau} \\
\frac{\partial E}{\partial t} + \nabla \cdot \{(E+p)\mathbf{v}\} &= \rho \mathbf{g} \cdot \mathbf{v} + (\nabla \cdot \boldsymbol{\tau}) \cdot \mathbf{v} + \kappa \nabla^2 T
\end{aligned}
$$

$$E = \rho \epsilon + \frac{1}{2}\rho(\mathbf{v} \cdot \mathbf{v})$$

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k} \right)$$

Acoustic Wave Propagation:

At frequencies above *N*, gravity waves are not generated …

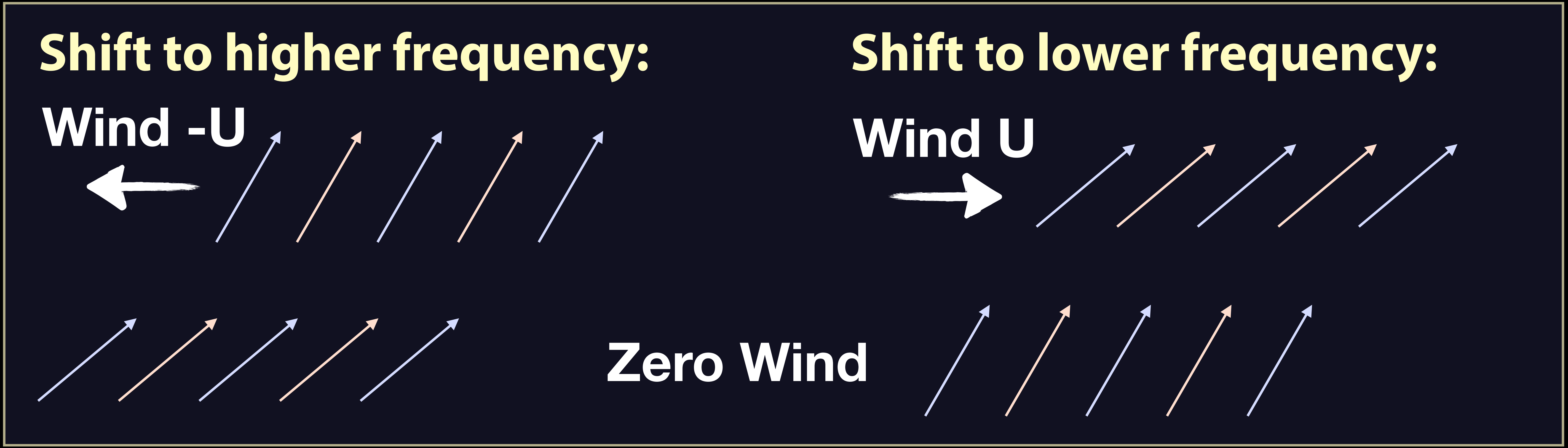


Gravity Wave Propagation:

And refraction through realistic winds …

Gravity wave vertical wavelength λ_z varies with wind U as it depends on the Doppler-shifted intrinsic frequency $\Omega = \omega - k_x U$, while ground-relative frequency ω and horizontal wavenumber k_x (and wavelength λ_x) remain constant.

Therefore: When wind varies only with altitude (not horizontally or with time), Doppler shifts modify only the intrinsic frequency relative to the flow. Frequency relative to the ground (extrinsic) remains constant!



Gravity Wave Propagation:

Isothermal, Windless

Wave:

$$\lambda_x = 20\text{km}$$

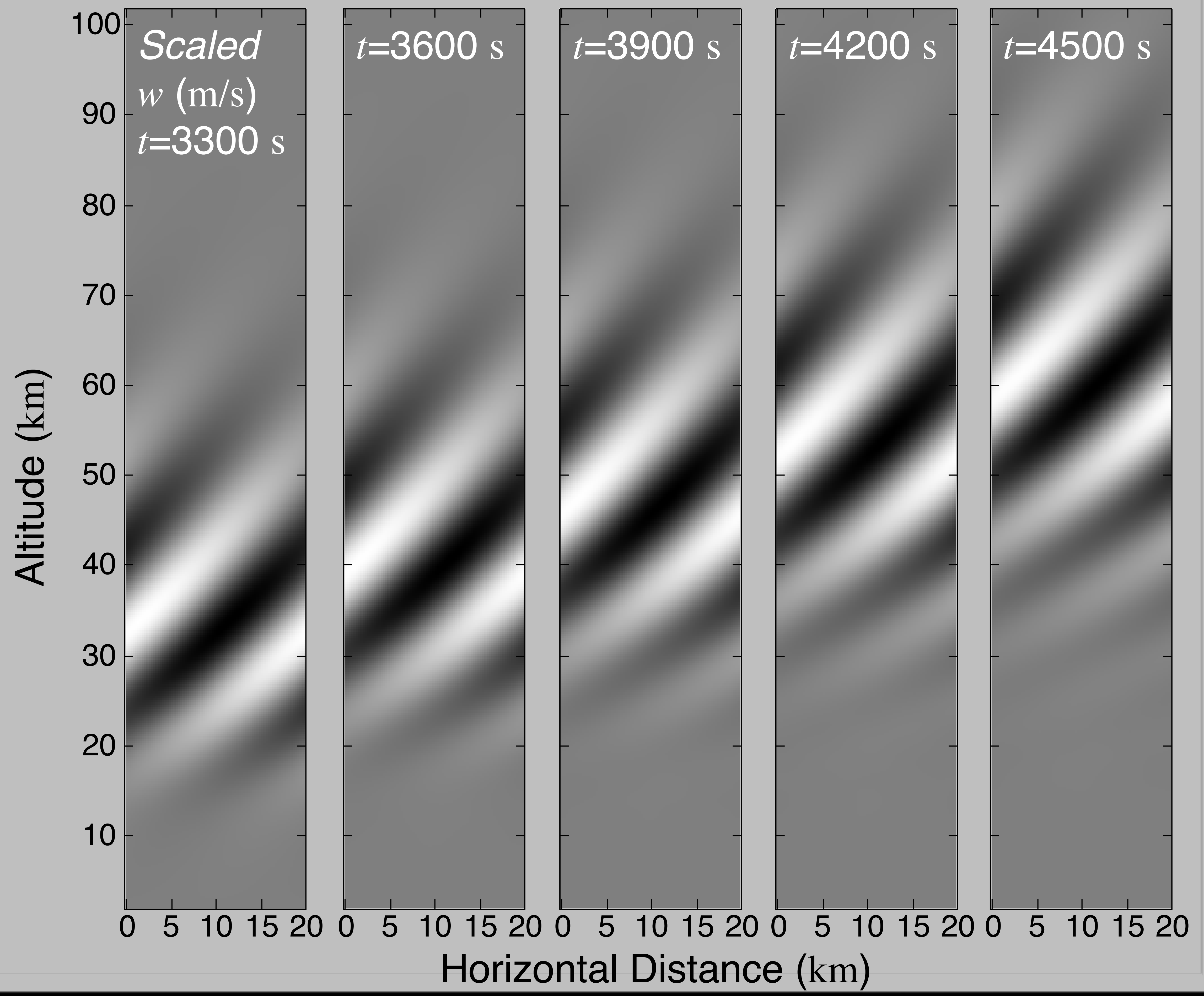
$$\omega = \frac{N}{\sqrt{2}}$$

Wind:

Zero!

Result:

The gravity wave propagates upwards through the horizontally-periodic domain. This simple case never occurs in reality!



Gravity Wave Propagation:

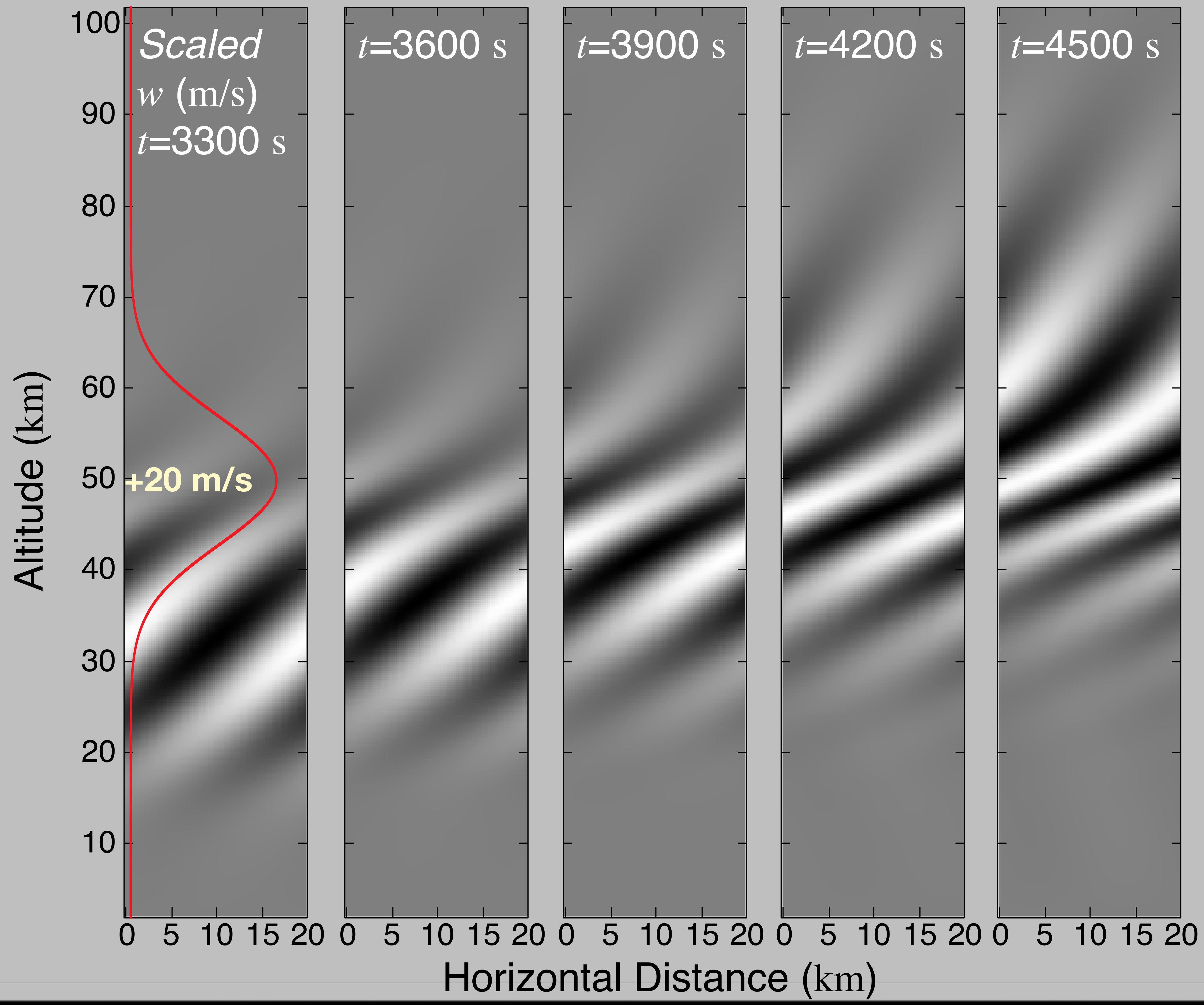
Isothermal, with Positive Wind

Wave:

$$\lambda_x = 20\text{km}$$
$$\omega = \frac{N}{\sqrt{2}}$$

Wind:
+20 m/s Peak at 50 km.

Result:
The gravity wave is Doppler-shifted to lower frequency at the wind peak, where vertical wavelength (and group velocity) is reduced.



Gravity Wave Propagation:

Isothermal, Critical Level

Wave:

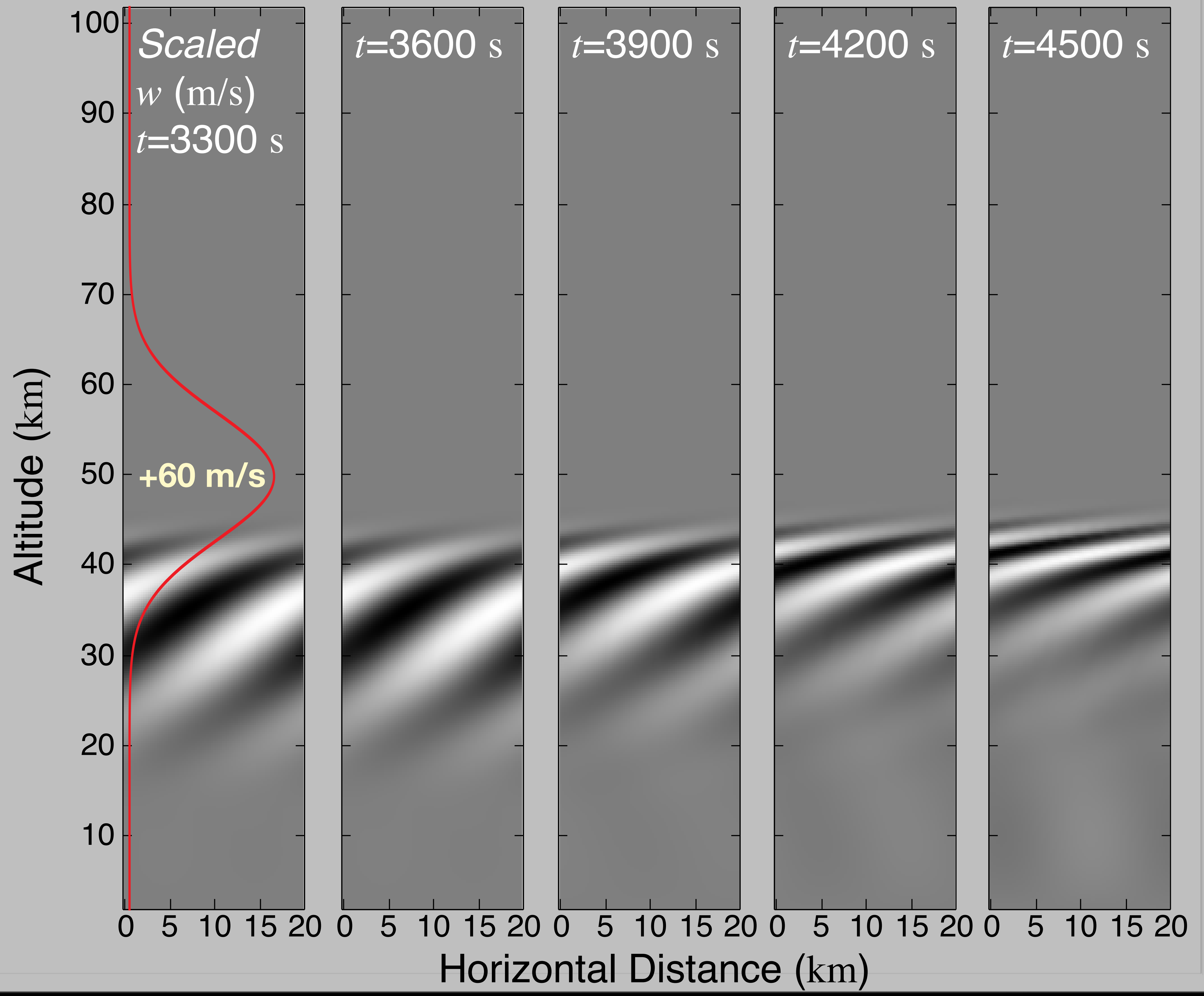
$$\lambda_x = 20\text{km}$$

$$\omega = \frac{N}{\sqrt{2}}$$

Wind:
+60 m/s Peak
at 50 km.

Result:

The gravity wave is Doppler-shifted towards zero frequency, leading to dissipation and blockage of the wave's passage.



Gravity Wave Propagation:

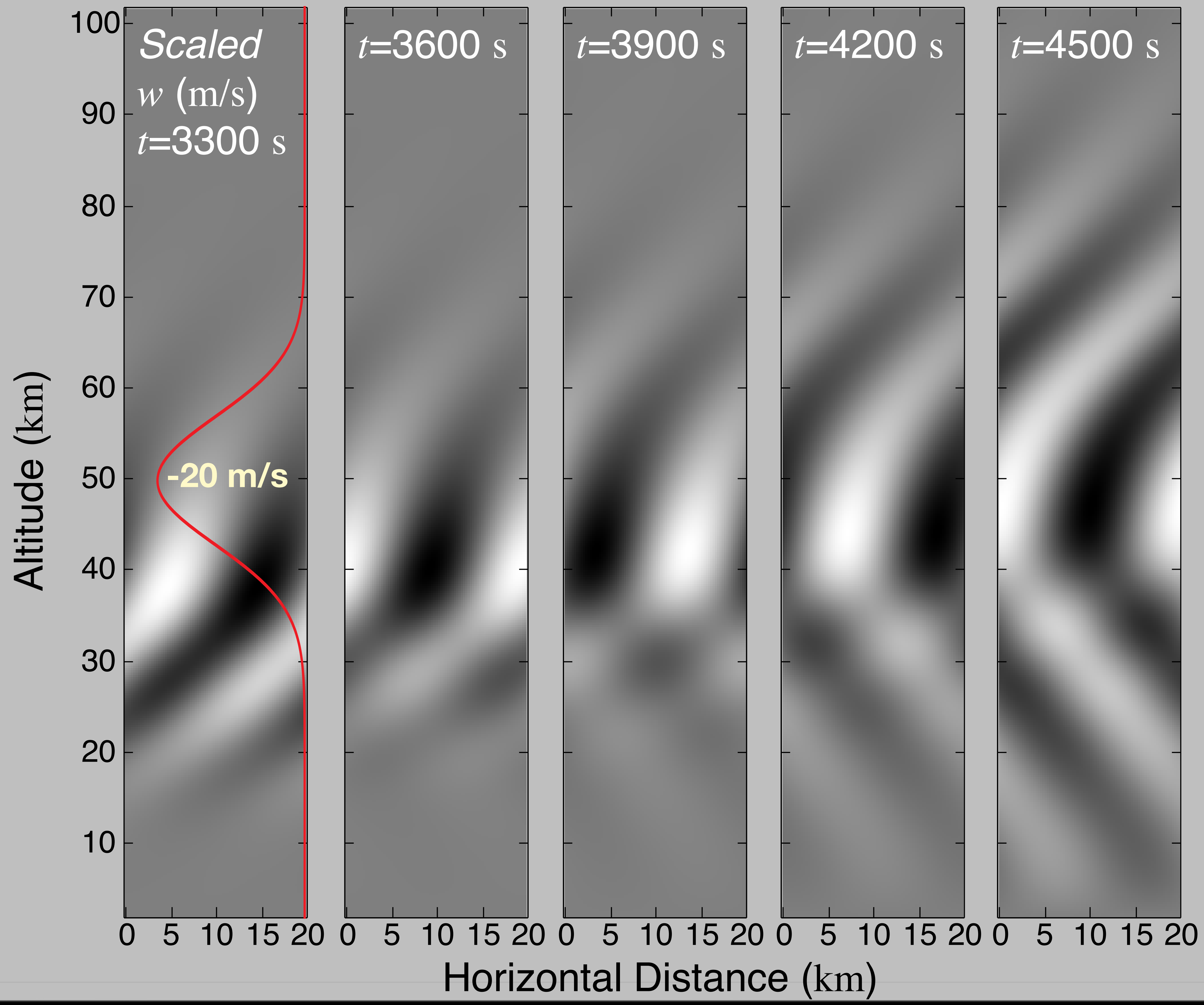
Isothermal, with Negative Wind

Wave:

$$\lambda_x = 20\text{km}$$
$$\omega = \frac{N}{\sqrt{2}}$$

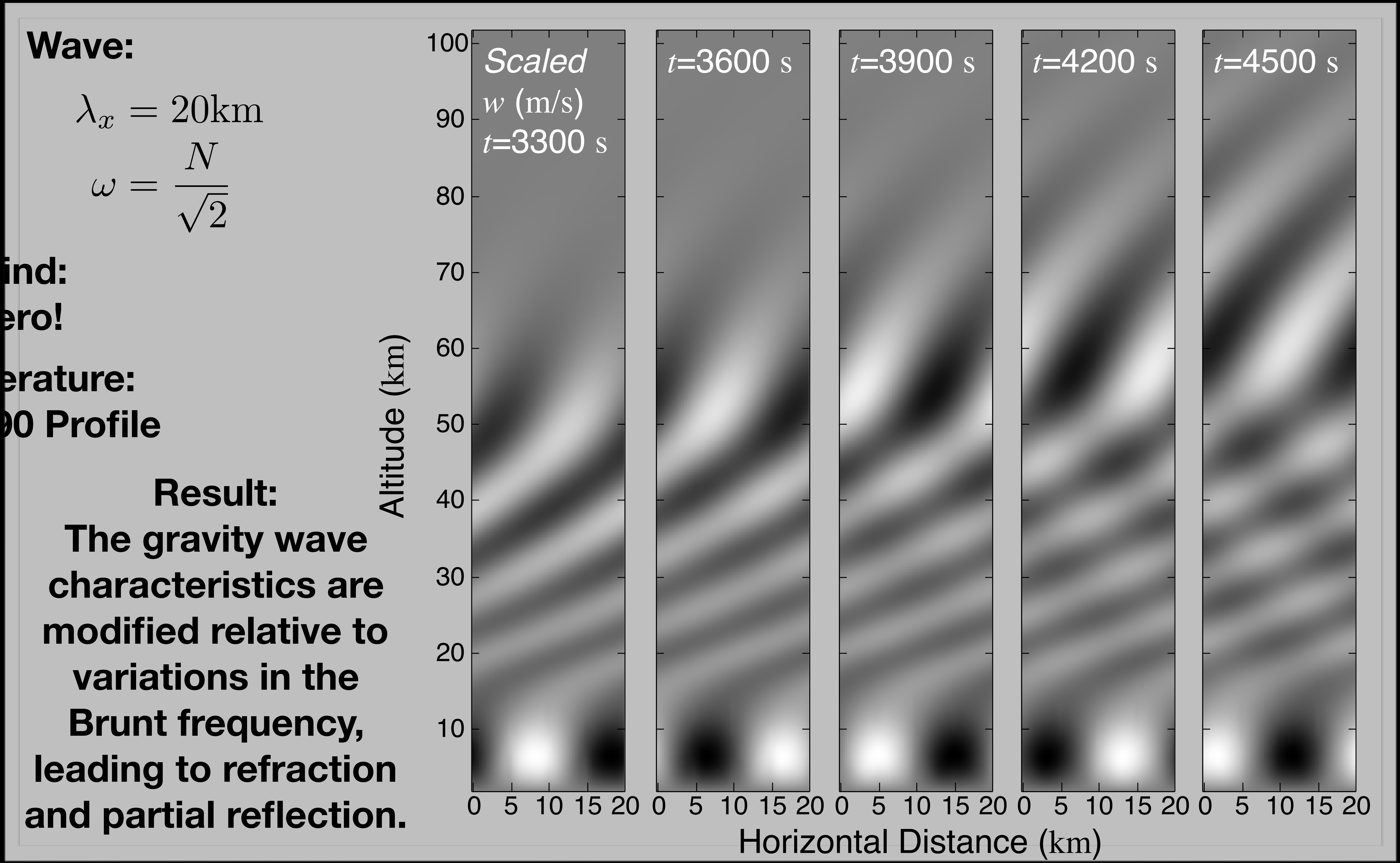
Wind:
-20 m/s Peak at 50 km.

Result:
The gravity wave is Doppler-shifted to higher frequency. Vertical wavelength increases, and it is partially reflected. This is the basis for ducting!



Gravity Wave Propagation:

Realistic Thermal Variation



Wave perturbation amplitudes increase with altitude, eventually leading to breaking.