

EP501: NUMERICAL METHODS FOR ENGINEERS AND SCIENTISTS
Fall 2016, J. B. Snively

ERAU Daytona Beach
Homework Assignment Project #8, Due: Friday 5PM, 12/9.

For this assignment... Understand and implement numerical methods for PDEs.

Submission Instructions: Submit to my mailbox, on paper, your work done by hand and printouts of the results with discussion (“Publish”, in as few pages as possible).

Email your .m file to snivelyj@erau.edu, with the subject “EP501: HW8 Last Name, First Name”.

- 1) As part of our example “bvp.m”, a direct solution was performed by solving a linear system. Taking inspiration from this example, construct and demonstrate a similar numerical solution in Matlab, instead using the *compact 4th-order method* described for Example 9.7 (Check your work against the resulting matrix on p. 560 — this is a good opportunity to practice your Matlab sparse matrix initializations!).
- 2) Using the BTCS method, implemented in Matlab, create your own version of Figure 10.19 for this textbook example problem (10.4) to test the method’s performance for this problem at small and large time steps. (Feel free to refer to the code “heat1d.m”, but note that modifications will be needed.)
- 3) In Matlab (using example codes as needed), implement and compare the Lax-Wendroff single-step and second-order upwind methods, as presented in the examples 11.14 and 11.22 in your text *for only the $c=0.5$ cases*. Plot results for both at $t=0, 5$, and 10 seconds on the same figure. How do the errors associated with the second-order upwind method differ (in a few sentences) from the Lax-Wendroff method that we discussed in class?
- 4) The Lax-Wendroff “Richtmyer” method is usually written in a two-step form somewhat different from that shown in our text, where the second step instead reaches an intermediate time $n+1/2$ at an intermediate position $i+1/2$:

$$f_{i+1/2}^{n+1/2} = \frac{1}{2}(f_i^n + f_{i+1}^n) - \frac{u\Delta t}{2\Delta x}(f_{i+1}^n - f_i^n)$$
$$f_i^{n+1} = f_i^n - \frac{u\Delta t}{\Delta x}(f_{i+1/2}^{n+1/2} - f_{i-1/2}^{n+1/2})$$

Show algebraically (on paper) that this method is equivalent to the single-step approach.

- 5) Apply a von Neumann stability analysis on the BTCS method for Convection (Advection) to obtain equation 11.61 and confirm its unconditional stability. Show (on paper) that equation 11.59 is consistent with a tridiagonal problem.