

Nonlinear Equations: Root Finding

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Root Finding

For the continuous nonlinear function $f(x)$,
find the value $x=a$ such that $f(a)=0$.

Or, transform to solve $x=g(x)$, and find $x=a$,
such that $a=g(a)$ and $f(a)=0$.

Root Finding

Two Step Process:

- 1) *Bound* the solution
- 2) *Refine* the solution

Trial and Error Methods

Closed Domain Methods

- Interval Halving
- False Position

Open Domain Methods

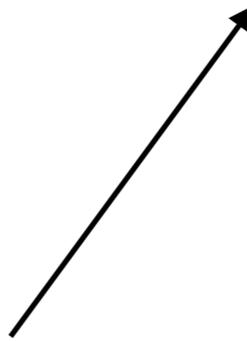
- Fixed-point iteration
- Newton's Method
- Secant Method

Root Finding

Two Step Process:

- 1) *Bound* the solution
- 2) *Refine* the solution

Trial and Error Methods



Trial and Error

a = minimum guess value

δa = guess increment

while $|f(a)| > \text{tol}$ do (not converged)

$a = a + \delta a$

end.

We advance a until $f(a)$ is sufficiently close to 0.

Trial and Error

a = minimum guess value

δa = guess increment

while $|f(a)| > \text{tol}$ do (not converged)

$a = a + \delta a$

end.

Advance a until $f(a)$ is satisfactorily close to 0.

Try for textbook problem (in *degrees*):

$$f(\phi) = \frac{5}{3} \cos(40) - \frac{5}{2} \cos(\phi) + \cos(40 - \phi) = 0$$

Let $a = 30^\circ$ to start, $\text{tol} = 0.0001$.

Root Finding

Two Step Process:

1) *Bound* the solution

2) *Refine* the solution

Closed Domain Methods

- Interval Halving
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Bisection Method

```
a = lower bound guess value
b = upper bound guess value
while |f(c)| > tol do (while not converged)
  c = (a + b) / 2
  if f(a) * f(c) < 0 then (if different sign)
    b = c
  else
    a = c
end.
```

We advance a until $f(a)$ is sufficiently close to 0, or until $(b-a) > \text{tol}$, i.e., we can define tolerance in terms of how $f(a)$ is to the root or how close a and b are to a single value.

Let $a = 30^\circ$, $b = 40^\circ$ to start, $\text{tol} = 0.0001$.

False Position Method

```
 $a$  = lower bound guess value  
 $b$  = upper bound guess value  
while  $|f(c)| > \text{tol}$  do (while not converged)  
   $c = b - f(b)(b-a) / (f(b) - f(a))$   
  if  $f(a) * f(c) < 0$  then (if different sign)  
     $b = c$   
  else  
     $a = c$   
end.
```

We advance a until $f(a)$ is sufficiently close to 0, or until $(b-a) > \text{tol}$, i.e., we can define tolerance in terms of how $f(a)$ is to the root or how close a and b are to a single value.

Here, c is a midpoint calculation defined by a *linear interpolation*!