Nonlinear Equations: Root Finding

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Root Finding

For the continuous nonlinear function $f(x)$, find the value $x=a$ such that $f(a)=0$.

Or, transform to solve $x=g(x)$, and find $x=a$, such that $a=g(a)$ and $f(a)=0$.

Root Finding

Two Step Process:

1) *Bound* the solution

2) *Refine* the solution

Trial and Error Methods

Closed Domain Methods

- Interval Halving
- False Position

Open Domain Methods

- Fixed-point iteration
- Newton's Method
- Secant Method

Root Finding

Two Step Process:

1) *Bound* the solution
2) *Refine* the solution

Trial and Error Methods

Trial and Error

```
a = minimum guess value
δa= guess increment
while |f(a)|>tol do (not converged)
 a=a+δa
end.
```

We advance a until $f(a)$ is sufficiently close to 0.

Trial and Error

```
a = minimum guess value
δa= guess increment
while |f(a)|>tol do (not converged)
 a=a+δa
end.
```

Advance *a* until *f*(*a*) is satisfactorily close to 0.

Try for textbook problem (in *degrees*):

$$\boxed{f(\phi) = \tfrac{5}{3}\cos(40) - \tfrac{5}{2}\cos(\phi) + \cos(40-\phi) = 0}$$

Let *a* = 30° to start, tol = 0.0001.

Root Finding

Two Step Process:

1) *Bound* the solution
2) *Refine* the solution

Closed Domain Methods

- Interval Halving
- False Position

Bisection Method

```
a = lower bound guess value
b = upper bound guess value
while |f(c)|>tol do (while not converged)
 c=(a+b)/2
 if f(a)*f(c)<0 then (if different sign)
  b=c
 else
  a=c
end.
```

We advance *a* until *f*(*a*) is sufficiently close to 0, or until (*b*-*a*)>tol, i.e., we can define tolerance in terms of how f(a) is to the root or how close *a* and *b* are to a single value.

Let $a = 30^{\circ}$, $b = 40^{\circ}$ to start, tol = 0.0001.

False Position Method

```
a = lower bound guess value
b = upper bound guess value
while |f(c)|>tol do (while not converged)
 c=b-f(b)(b-a)/(f(b)-f(a))
 if f(a)*f(c)<0 then (if different sign)
  b=c
 else
  a=c
end.
```

We advance *a* until *f*(*a*) is sufficiently close to 0, or until (*b*-*a*)>tol, i.e., we can define tolerance in terms of how f(a) is to the root or how close *a* and *b* are to a single value.
Here, *c* is a midpoint calculation defined by a *linear interpolation*!