EP501: NUMERICAL METHODS FOR ENGINEERS AND SCIENTISTS

Fall 2017, J. B. Snively

ERAU Daytona Beach

Homework Assignment Project #7, Due: Thursday 5PM, 12/7.

For this assignment... Understand and implement numerical methods for PDEs.

Submission Instructions: Submit to my mailbox, on paper, your work done by hand and printouts of the results with discussion ("Publish", in as few pages as possible).
Email your .m file to **snivelyj@erau.edu**, with the subject "EP501: HW7 Last Name, First Name".

1) As part of our example "bvp.m", a direct solution was performed by solving a linear system. Taking inspiration from this example, construct and demonstrate a similar numerical solution in Matlab, instead using the *compact 4th-order method* described for Example 9.7 (Check your work against the resulting matrix on p. 560 — this is a good opportunity to practice your Matlab sparse matrix initializations!).

2) In Matlab (using example codes as needed), implement and compare the Lax-Wendroff single-step and second-order upwind methods, as presented in Figure 11.14 and 11.22 in your text *for only the c=0.5 cases*. Plot results for both methods at t=0, 5, and 10 seconds on the same figure. How do the errors associated with the second-order upwind method differ (in a few sentences) from the Lax-Wendroff method that we discussed in class?

3) The Lax-Wendroff "Richtmyer" method is usually written in a two-step form somewhat different from that shown in our text, where the second step instead reaches an intermediate time n+1/2 at an intermediate position i+1/2:

$$\begin{aligned} f_{i+1/2}^{n+1/2} &= \frac{1}{2}(f_i^n + f_{i+1}^n) - \frac{u\Delta t}{2\Delta x}(f_{i+1}^n - f_i^n) \\ f_i^{n+1} &= f_i^n - \frac{u\Delta t}{\Delta x}(f_{i+1/2}^{n+1/2} - f_{i-1/2}^{n+1/2}) \end{aligned}$$

Show algebraically (on paper) that this method is equivalent to the single-step approach.

4) Solve the wave equation using (a) the one-step Lax-Wendroff method and (b) the two-step Lax-Wendroff method, to reproduce the results shown in Example 11.9 / Figure 11.29 for only the c=0.5 and 1.0 cases. Show the results of the two methods on two separate figures, since they should be *very* similar!

5) Apply a von Neumann stability analysis on the BTCS method for Convection (Advection) to obtain equation 11.61 and confirm its unconditional stability. Show (on paper) that equation 11.59 is consistent with a tridiagonal problem.