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Stokes Theorem:

Integral Form,
Differential “Point” Form

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Static Maxwell's Equations: Integral Form

Gauss's Law: (E Field)

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\mathrm{encl}}}{\epsilon_o}$$

Gauss's Law: (B Field)

$$\oint \vec{B} \cdot d\vec{s} = 0$$

Faraday's Law: ("Kirchhoff's Law")

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o i_c$$

Static Maxwell's Equations:
Differential "Point" Form

Gauss's Law: (E Field) $$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

Gauss's Law: (B Field) $$\nabla \cdot \mathbf{B} = 0$$

Faraday's Law: $$\nabla \times \mathbf{E} = 0$$

Ampere's Law: $$\nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

Circulation and Curl Visualized

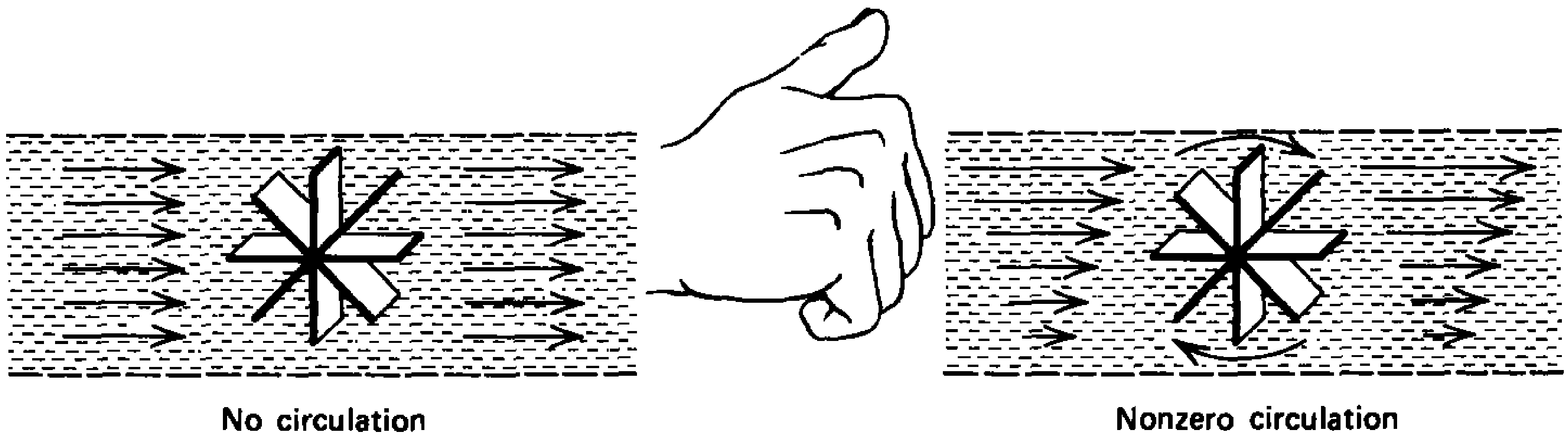


Figure 1-20 A fluid with a velocity field that has a curl tends to turn the paddle wheel. The curl component found is in the same direction as the thumb when the fingers of the right hand are curled in the direction of rotation.

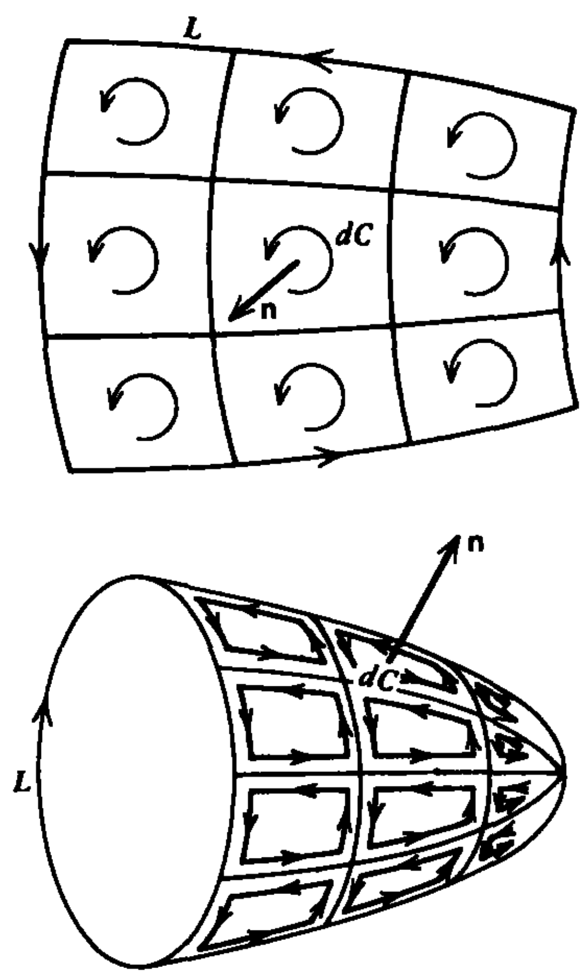
Here, curl is defined for each component as the circulation per unit area around each normal vector direction.

$$(\nabla \times \mathbf{A})_n = \lim_{dS_n \to 0} \frac{\oint_L \mathbf{A} \cdot \mathbf{dl}}{dS_n}$$

[Markus Zahn, *Electromagnetic Field Theory*, p. 31]

Stokes' Theorem Visualized

Figure 1-23 Many incremental line contours distributed over any surface, have nonzero contribution to the circulation only along those parts of the surface on the boundary contour L.



[Markus Zahn, *Electromagnetic Field Theory*, p. 27]

$$\int_S (\nabla \times \mathbf{A}) d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$

The surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector along the contour bounding the surface.

Summary

Stokes' Theorem allows us to translate between integral and differential forms of the curl equations (Ampere's and Faraday's Laws).

Have you started reading Chapter 3 yet?