EP440 Lecture #2

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Divergence Theorem:
Integral Form,
Differential “Point” Form

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Static Maxwell's Equations: Integral Form

Gauss's Law:
(E Field)

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\mathrm{encl}}}{\epsilon_o}$$

Gauss's Law:
(B Field)

$$\oint \vec{B} \cdot d\vec{s} = 0$$

Faraday's Law:
("Kirchhoff's Law")

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o i_c$$

Static Maxwell's Equations:
Differential "Point" Form

Gauss's Law: (E Field)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

Gauss's Law: (B Field)

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's Law:

$$\nabla \times \mathbf{E} = 0$$

Ampere's Law:

$$\nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

Divergence Theorem Visualized

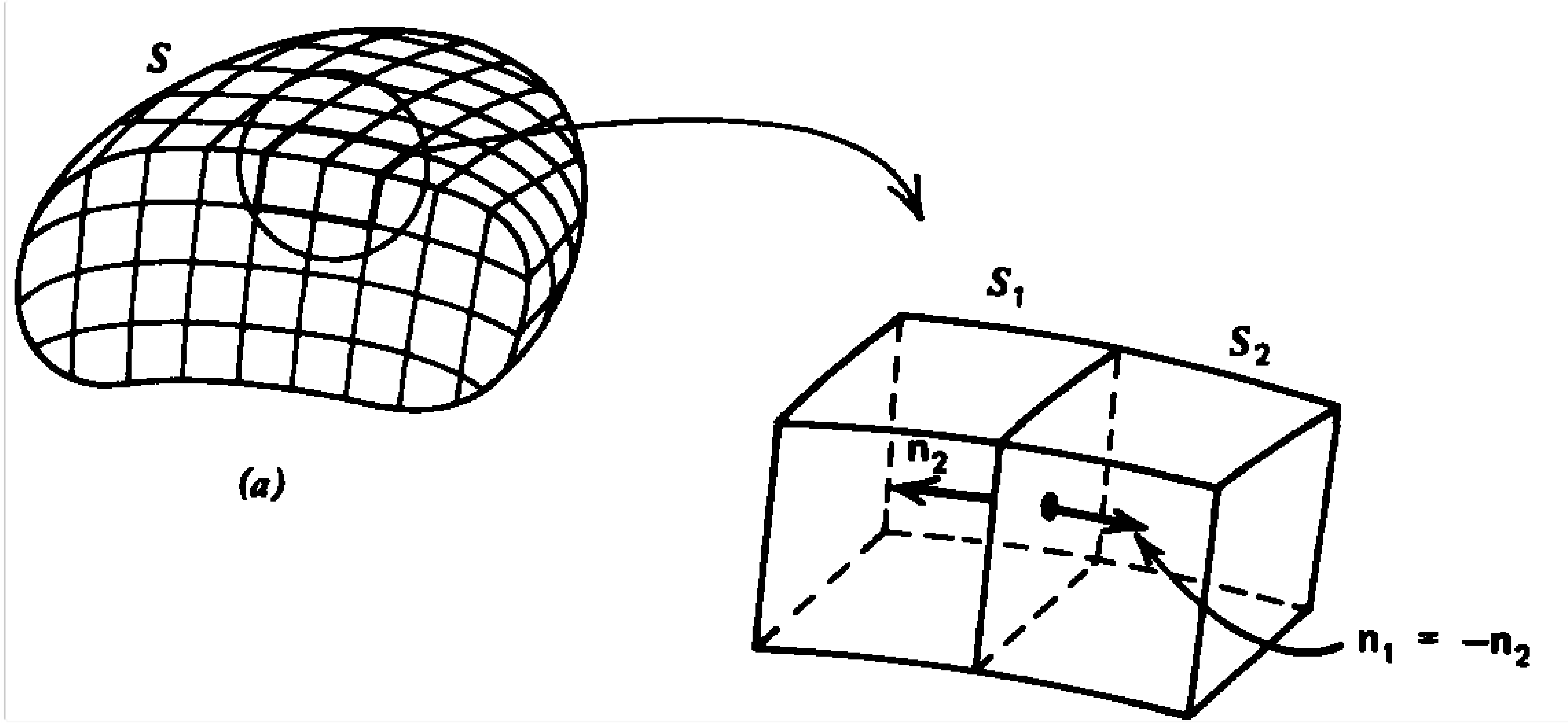

Figure 1-17 Nonzero contributions to the flux of a vector are only obtained across those surfaces that bound the outside of a volume. (a) Within the volume the flux leaving one incremental volume just enters the adjacent volume where (*b*) the outgoing normals to the common surface separating the volumes are in opposite directions.

[Markus Zahn, *Electromagnetic Field Theory*, p. 27]

Divergence Theorem

Accounting for contributions of each infinitesimal volume within the closed surface:

$$\Phi = \oint_S \mathbf{A} \cdot \mathbf{dS} = \lim_{\substack{N\to\infty \\ \Delta V_n \to 0}} \sum_{n=1}^{\infty} (\nabla \cdot \mathbf{A})\,\Delta V_n = \int_V \nabla \cdot \mathbf{A}\, dV \quad (12)$$

[Markus Zahn, *Electromagnetic Field Theory*, p. 27]

Hence the familiar "divergence theorem":

$$\int_V (\nabla \cdot \mathbf{A}) dv = \oint_S \mathbf{A} \cdot d\mathbf{s}$$

Summary

The Divergence Theorem allows us to translate between integral and differential forms of Gauss's Laws.

Hopefully you are reading Chapter 2...