EP440: ENGINEERING ELECTROMAGNETICS, EXAM #2
Fall 2012, J. B. Snively

Solve Problem 1, then choose 3 others. Please put a box around all final answers! If attempting more than 4, write "Do Not Grade" on the problem that you do not want to have graded. (Sorry, there is no "best of" option.)

For each problem, show as much work as possible on the pages provided (and the backs of the pages if necessary). You may find the following tables useful.

$$
\begin{aligned}
k_e &= \frac{1}{4\pi\epsilon_o} \simeq 9\times 10^9 \ \ [\mathrm{N\cdot m^2/C^2}] \\
\epsilon_o &= 8.854\times 10^{-12} \ \ [\mathrm{C^2/N\cdot m^2}] \text{ or } [\mathrm{F/m}] \\
\mu_o &= 4\pi\times 10^{-7} \ \ [\mathrm{N/A^2}] \text{ or } [\mathrm{H/m}] \\
c &= 2.99792458\times 10^8 \simeq 3\times 10^8 \ \ [\mathrm{m/s}] \\
m_e &= 9.109\times 10^{-31} \ \ [\mathrm{kg}] \\
m_p &= 1.673\times 10^{-27} \ \ [\mathrm{kg}] \\
q_e &= 1.602\times 10^{-19} \ \ [\mathrm{C}]
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{x} &= lnx \\
\int e^{ax} &= \frac{1}{a}e^{ax} \\
\int \frac{dx}{\sqrt{a^2-x^2}} &= \arcsin\frac{x}{a} \\
\int \frac{dx}{\sqrt{x^2+a^2}} &= \ln\left(x+\sqrt{x^2+a^2}\right) \\
\int \frac{xdx}{\sqrt{x^2+a^2}} &= \sqrt{x^2+a^2} \\
\int \frac{dx}{x^2+a^2} &= \frac{1}{a}\arctan\frac{x}{a} \\
\int \frac{dx}{(x^2+a^2)^{3/2}} &= \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}} \\
\int \frac{xdx}{(x^2+a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2+a^2}}
\end{aligned}
$$

$$\mathbf{A\cdot B\times C = B\cdot C\times A = C\cdot A\times B}$$
$$\mathbf{A\times(B\times C) = B(A\cdot C) - C(A\cdot B)}$$
$$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$$
$$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$$
$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$$
$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$
$$\nabla\cdot\nabla V = \nabla^2 V$$
$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$
$$\nabla\times\nabla V = 0$$
$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$
$$\int_v \nabla\cdot\mathbf{A}\,dv = \oint_s \mathbf{A}\cdot d\mathbf{s} \qquad \text{(Divergence theorem)}$$
$$\int_s \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} \qquad \text{(Stokes's theorem)}$$

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x \frac{\partial V}{\partial x} + \mathbf{a}_y \frac{\partial V}{\partial y} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \mathbf{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \mathbf{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r \frac{\partial V}{\partial r} + \mathbf{a}_\phi \frac{\partial V}{r \partial \phi} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r}(r A_r) + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r} \begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \mathbf{a}_r \left(\frac{\partial A_z}{r \partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \mathbf{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \mathbf{a}_z \frac{1}{r} \left[\frac{\partial}{\partial r}(r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R \frac{\partial V}{\partial R} + \mathbf{a}_\theta \frac{\partial V}{R \partial \theta} + \mathbf{a}_\phi \frac{1}{R \sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R \sin\theta} \frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin\theta} \begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R \sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin\theta) A_\phi \end{vmatrix} = \mathbf{a}_R \frac{1}{R \sin\theta} \left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \mathbf{a}_\theta \frac{1}{R} \left[\frac{1}{\sin\theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(R A_\phi) \right] + \mathbf{a}_\phi \frac{1}{R} \left[\frac{\partial}{\partial R}(R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

1. This problem is another variant of an EP440 "classic", developed by Dr. Zettergren.

a. Derive the differential "point" form for the following equation of magnetostatics, while explaining each step in the process (with just a few words).

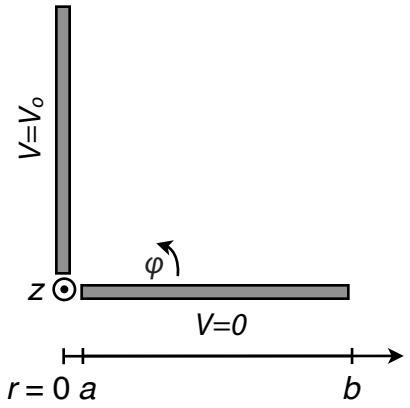
$$\oint_c \mathbf{B} \cdot d\mathbf{l} = \mu_o I$$

b. Show that (and explain why) a static **B** field may be expressed in terms of magnetic vector potential **A**.

c. On the basis of your expressions for (a) and (c), derive Poisson's equation for vector potential **A**.

2. Consider the following "perpendicular-plate" capacitor, where the two plates are infinite in the *z*-direction (out of page). For all of your calculations, assume that the plates are long enough in the radial direction that fringing field effects can be neglected (which is a rather bold assumptions – your solutions will be thus be approximate).

a. Determine the potential *V* as a function of φ between the two plates.

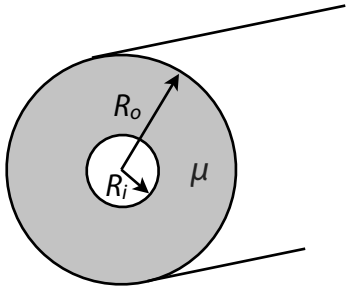


b. Determine the electric field **E** as a function *r* between the two plates.

c. Determine the capacitance *C* per unit length (along *z*) of the system.

3. An conductor of radius R_i is surrounded by (but insulated from) a magnetic material, which has permeability $\mu>\mu_o$ (assume μ_o elsewhere). The conductor carries an evenly-distributed current I_i, directed out of the page.

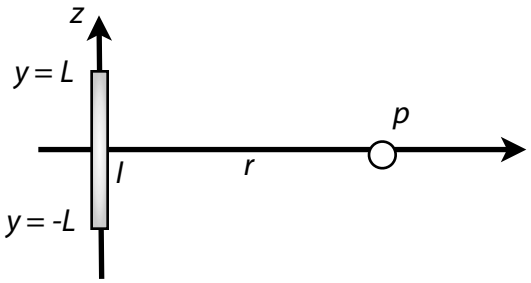
a. Find **B**, **H**, and **M** *everywhere* ($r<R_i$, $R_i<r<R_o$, $r>R_o$). *Box each expression!*



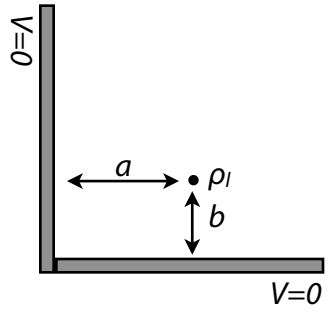
b. Find $\mathbf{J}_{ms}$ for $r=R_i$ and $r=R_o$.

c. Find $\mathbf{J}_m$ for $R_i<r<R_o$.

4. For the finite line current *I,* directed upward along the *z*-axis, find magnetic vector potential **A** *and* the magnetic flux density **B** at point "*p*", using *any* methods you prefer.



5. Find the force on the infinite line charge ρ_l placed near two infinite, perpendicular, grounded, conducting plates. For full credit, mark on the figure the location of the three image *line* charges, and their respective signs.



How would this solution differ if the left plate were held at nonzero potential? (For example, in the perpendicular-plate capacitor!)