PS 320 - Classical Mechanics
Reynolds problems

1. **Rope**

   A uniform rope of length L and mass M rests on a table.

   (a) If you lift one end of the rope upward with a constant speed v, show that the vertical position of the rope's instantaneous center of mass moves upward with constant acceleration, and determine its value. That is, show that the vertical component of the acceleration of the center of mass is constant. NOTE: The shape of the portion of the rope left on the table is unimportant.

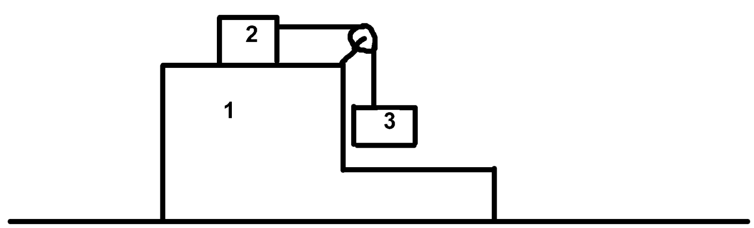
   (b) Next, suppose you hold the rope suspended in air, with its lower end just touching the table. If you now lower the rope with a constant speed v onto the table, show that the rope's center of mass has precisely the same upward, constant acceleration as in part (a).

2. **Rotating spring and bob**

   A bob of mass m is attached to a spring having spring constant k, and the system is whirled around in a horizontal circle at angular velocity ω. In the initial (relaxed) condition the distance of the center of mass of the bob from the axis of rotation is r_0. Find the steady-state distance, r, of the bob from the axis of rotation as a function of the imposed value of ω. Interpret the mathematical result by describing the physical effects predicted.

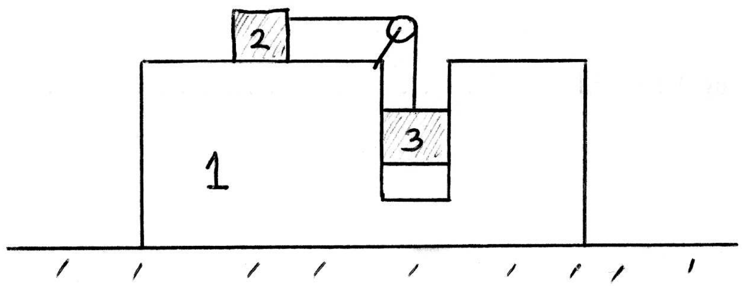
3. **Pedagogical machine I**

   Consider the "pedagogical machine" shown below. Mass 2 and mass 3 are connected by a rope that is hung over a pulley, and mass 1 is free to move horizontally on the table. All the surfaces are frictionless, and mass 3 does not touch mass 1. Find the initial acceleration of mass 1.



4. **Pedagogical machine II**

   Consider the "pedagogical machine" shown below. Mass 2 and mass 3 are connected by a rope that is hung over a pulley, and mass 1 is free to move horizontally on the table. All the surfaces are frictionless, and mass 3 is confined to a frictionless channel in mass 1. Find the acceleration of mass 1.



5. **Traffic**

A motorist is approaching a green traffic light with speed v_0 when the light turns to amber.

(a) If his reaction time is τ, during which he makes his decision to stop and applies his foot to the brake, and if his maximum braking deceleration is a, what is the minimum distance s_{min} from the intersection at the moment the light turns to amber in which he can bring his car to a stop?

(b) If the amber light remains on for a time t before turning to red, what is the maximum distance s_{max} from the intersection at the moment the light turns to amber such that he can continue into the intersection at speed v_0 without running the red light?

(c) Show that if his initial speed v_0 is greater than

$$v_{0,\text{max}} = 2a(t - \tau),$$

there will be a range of distances from the intersection such that he can neither stop in time nor continue through without running the red light.

(d) Make some reasonable estimates of τ, t, and a, and calculate $v_{0,\text{max}}$ in miles per hour. If $v_0 = \frac{2}{3}v_{0,\text{max}}$, calculate s_{min} and s_{max}.

6. **Attractive potential well I**

A particle of mass m has speed $v = \alpha/x$, where x is its displacement. Find the force $F(x)$ responsible for this motion, and the potential energy function $U(x)$ responsible for this force.

7. **Two masses, one swinging**

The equations of motion for two masses, one swinging, are given by Morin on page 15, Eqs. (1.16)

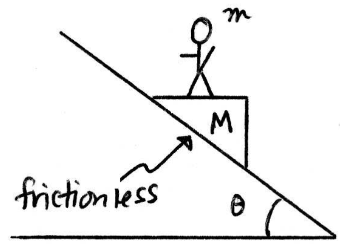
$$2\ddot{r} = r\dot{\theta}^2 - g(1 - \cos\theta),$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} - \frac{g\sin\theta}{r}.$$

Linearize these equations for small θ and small ϵ, where $r = r_0 + \epsilon$ (r_0 is a constant). Retain only linear terms, i.e., only those to the first power in any combination of θ and ϵ and their time derivatives. What type of system does this describe?

8. **Weight on a sliding scale**

A person of mass m stands on a platform (of mass M) that is sliding down a frictionless inclined plane, as in the figure. What is the weight that would be recorded by a scale on the platform? That is, calculate the normal force between the platform and the person. Of course, there must be static friction between the platform and the person in order for the person to remain stationary with respect to the platform. HINT: Draw free body diagrams for the person and the platform, and solve for the acceleration and the various normal forces and friction forces.



9. **Water droplet I**

Solve the equation for the velocity of a raindrop

$$\frac{dv}{dt} = g - kv^2$$

that results from assuming that it collects water at a rate proportional to both its mass and velocity ($\dot{m} = kmv$). This nonlinear, ordinary differential equation is known as a Riccati equation, after the man who developed a method to solve equations of this type. His method is to transform it into a linear equation, solve this linear equation, and finally make the inverse transformation. Use the following transformation

$$z(t) = \frac{1}{v(t) - \sqrt{g/k}}$$

to obtain a differential equation for $z(t)$. It is a linear equation, so you can solve it. Then apply the inverse transformation

$$v(t) = \frac{1}{z(t)} + \sqrt{g/k}$$

to obtain $v(t)$.

10. **Water droplet II**

A water droplet falling in the atmosphere is spherical. Assume that as a spherical water droplet passes through a cloud, it acquires mass at a rate proportional to kA where k is a positive constant and A is its cross-sectional area. Consider a droplet of initial radius r_0 that enters a cloud with velocity v_0. Assume no resistive force. Show (a) that the radius increases linearly with time, and (b) that if r_0 is negligibly small then the speed increases linearly with time within the cloud.

11. **Attractive potential well II**

    Consider a particle moving in the region $x > 0$ under the influence of the potential energy function
    $$U(x) = U_0\left(\frac{a}{x} + \frac{x}{a}\right),$$
    where U_0 and a are positive constants. Plot the potential (*without* your graphing calculator!), find the equilibrium points, and determine whether they are maxima or minima.

12. **Three superballs**

    Solve the superball problem, but this time use *three* balls. If the masses satisfy the inequality
    $$m_1 \gg m_2 \gg m_3,$$
    and if the heaviest ball (m_1) is on the bottom with the lightest ball (m_3) on top, find (approximately) the height that m_3 bounces to. Assume that the balls have negligible radii, and that the initial height is h. State *clearly* all the approximations that you use.

    Extra credit: Can you find the height that the top ball bounces to if there are N superballs? Assume that the masses satisfy the inequality as above.

13. **Rocket**

    A rocket starts from rest in free space by emitting mass. At what fraction of the initial mass is the momentum a maximum?

14. **Chain and scale**

    A chain of mass M and length L is suspended vertically with its lowest end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale (i.e., the force that the scale must exert upward) when a length of chain, x, has fallen (i.e., as a function of time)? (Hint: Consider the change in momentum when a length of chain $\mathrm{d}x$ falls on the scale in a time $\mathrm{d}t$, and don't forget about the portion of the chain that is already lying on the scale. The maximum reading of the scale is $3Mg$.)

15. **Drag on an angled, flat surface**

    Show that the drag force experienced by a flat surface of cross-sectional area A moving through air of density ρ with speed V is given by
    $$F_d = (1 + \cos 2\theta)A\rho V^2,$$
    where the surface is inclined at an angle θ with respect to the direction of motion, i.e., $\theta = 0$ denotes a surface perpendicular to the direction of motion. Assume that the gas particles are initially motionless, and that the collisions between the gas particles and the sphere are perfectly elastic. Note: For each collision there will be a component of $\Delta\mathbf{p}$ perpendicular to the direction of motion, but these will all cancel for a symmetric object. We are only interested in the component of $\Delta\mathbf{p}$ *parallel* to the direction of motion because that is what causes drag.

16. **Drag on a sphere**

    Show that the drag force experienced by a sphere of cross-sectional area A moving through air of density ρ with speed V is given by

    $$F_d = A\rho V^2.$$

    Assume that the gas particles are initially motionless, and that the collisions between the gas particles and the sphere are perfectly elastic. To show this, I suggest using the result that the drag force on an angled flat surface is $F_d = (1 + 2\cos 2\theta)A\rho V^2$, where θ is the angle with respect to the direction of motion, and then breaking up the front surface of the sphere into concentric rings that have the same angle of attack (relative to the direction of motion). Finally, integrating over all concentric rings will result in the total drag force. Note: For each collision there will be a component of $\Delta\mathbf{p}$ perpendicular to the direction of motion, but these will all cancel for a symmetric object. We are only interested in the component of $\Delta\mathbf{p}$ *parallel* to the direction of motion because that is what causes drag.

17. **Neutron star**

    A neutron star is a collection of neutrons bound together by their mutual gravitation with a density comparable to that of an atomic nucleus (approximately 10^{12} g/cm^3). Assuming that the neutron star is a sphere, show that the maximum frequency with which it may rotate (if mass is not to fly off at the equator) is $f = (\rho G/3\pi)^{1/2}$, where ρ is the density. Calculate f for a density of 10^{12} g/cm^3. It is now accepted that pulsars, which emit regular bursts of radiation at repetition rates up to about 30/sec, are rotating neutron stars.

18. **A massive sphere**

    Consider a sphere of mass M where the gravitational field vector $\mathbf{g}$ is *independent* of the the radial position r within the sphere (of course it falls of like r^{-2} outside the sphere). Find the function describing the mass density $\rho(r)$ as a function of radial position. Note: There is spherical symmetry so that ρ does not depend on either of the other two spherical coordinates, θ and ϕ. (answer: $\rho(r) = M/(2\pi R^2 r)$)

19. **Sheet and sphere**

    A uniform solid sphere of mass M and radius R is fixed a distance h above a thin infinite sheet of mass density σ (mass per unit area). With what force does the sphere attract the sheet? (answer: $2\pi GM\sigma$)

20. **Self-gravitating cloud**

    Show that the gravitational self energy (i.e., the energy of assembly piecewise from infinity) of a uniform sphere of mass M and radius R is

    $$U = -\frac{3}{5}\frac{GM^2}{R}.$$

HINT: Calculate the work done to bring a mass dm in from infinity and spread it uniformly over the surface of the mass m that is already assembled. Then integrate over the entire mass M.

21. **The proto-sun and Kelvin**

The gravitational self-energy is the amount of energy released when a cloud of gas and dust contracts (gravitationally) into a protostar. This gravitational energy is ultimately converted into light, and is what allows protostars to shine before the density and temperature at their center are large enough to trigger nuclear reactions. In the mid-nineteenth century, nuclear reactions were not known, and Lord Kelvin (William Thomson) calculated the lifetime of the sun in the following manner.

(a) Given the Sun's current luminosity (rate of energy release in the form of radiation), calculate the rate at which its radius should decrease (assume that no mass is lost). HINT: Find the relation between dU/dt and dR/dt from your result in the "Self-gravitating cloud" problem. Would it be possible for us (with current technologies) to observe such a radius change? You might have to look up some numbers in the library for this part.

(b) Assume that the Sun in the past has been shining with its current luminosity. How long can it have been shining if all that energy had been released during its gravitational collapse from "infinity" (i.e., a very large interstellar cloud). (answer: about 20 million years)

Note: The above estimates are incorrect, because there is a theorem of classical mechanics known as the "virial theorem," which says that half of any gravitational energy released must go into heating the cloud, and the other half can be radiated away. However, the above estimates were good enough to show that the "Kelvin" time scale is much shorter than the geologic time scale, and, since the Earth cannot be older than the Sun, there must be another energy supply for the Sun besides gravitational energy.

"The sun must, therefore, either have beencreated as an active source of heat at some time of not immeasurable antiquity, by an over-ruling decree; or the heat which he has already radiated away, and that which he still possesses, must have been acquired by a natural process, following permanently established laws. Without pronouncing the former supposition to be essentially incredible, we may safely say that it is in the highest degree improbabli, if we can show the latter to be not contradictory to known physical laws. And we do show this and more, by merelypointing to certain actions, going on before us at present, which, if sufficiently abundant at some past time, must have given the sun heat enough to account for all we know of his past radiation and present temperature."

— Lord Kelvin, 1862

22. **Tunnel through the Earth I**

A particle is dropped into a hole drilled straight through the center of the Earth. Neglecting rotational effects and friction, show that the particle's motion is simple harmonic if you assume the Earth is spherical and has a uniform density. Show that the period of oscillation is about 84 minutes.

23. **Tunnel through the Earth II**

    Let's try the same thing as with the previous tunnel problem, but this time we do not require that the tunnel passes through the center of the Earth (although we do require that the tunnel is straight). That is, the tunnel is a straight chord from one spot on the surface of the Earth to another. Assuming that the tunnel's surface is frictionless, show that the period of oscillation is the same as when the tunnel passes through the center of the Earth. If you designed a transportation system with this idea, you could travel from any point on the Earth's surface to any other in only 42 minutes!

24. **Shape of reference geoid**

    Calculate the shape (r as a function of θ) of the Earth (the reference geoid) in the following manner. Start with the expression for the gravitational potential

$$\Phi(r,\theta) = -\frac{GM}{r} + \frac{GMJ_2a^2}{r^3}P_2(\cos\theta) - \frac{1}{2}r^2\omega^2\sin^2\theta,$$

    where a is the Earth's equatorial radius and ω is its angular velocity of rotation. Choose the potential surface of interest to be that which passes through the equatorial radius

$$\Phi_0 = \Phi(a, \pi/2).$$

    Then, setting $\Phi = \Phi_0$, and assuming $r = a(1+\epsilon)$, where $|\epsilon| \ll 1$, solve for $\epsilon(\theta)$. Use your knowledge of Taylor series expansions to retain only the largest terms. Then, compare this shape to that of a true ellipse (i.e., determine $\epsilon(\theta)$ for an ellipse and plot both functions on the same graph).

25. **Chain and peg**

    A flexible chain of length L is hung over a peg with one end slightly longer than the other. Assuming that the chain slides off with no friction, derive and solve the differential equation of motion to show that

$$y(t) = y_0 \cosh\left(t\sqrt{\frac{2g}{L}}\right),$$

    where $2y$ is the difference in length of the two ends, and $y = y_0$ when $t = 0$. (The above solution only is valid, of course, for $0 < y < L/2$.)

26. **Water droplet III**

    We have modeled a falling raindrop gaining mass in two different ways. Let's take a third model, one that I think is more realistic. In this model, the time rate of change of the mass is proportional to both the cross-sectional area A and the velocity v (but not the mass directly)

$$\dot{m} = kAv,$$

where k is a positive constant. This makes more sense because more water vapor will be swept up by a larger cross-sectional area, and, of course, the faster the drop moves the faster it will sweep up water vapor. This assumes that the density of the raindrop and the water vapor is constant.

Derive the dynamical equation for this raindrop, as well as any auxiliary differential equations that might be needed for a *complete* solution. (Make sure that all constants and variables are defined.) Although you do not have to solve it, discuss how you would go about solving it.

27. **Tunnel through the Earth III**

    In Problem 18 we considered a spherically symmetric distribution of mass M whose gravitational field is

    $$\vec{g} = g\hat{r},$$

    where $g = -MG/R^2$ is a negative constant. (Recall that this means the mass density is not constant.) Consider a particle of mass m in a straight frictionless tunnel drilled through a chord of this sphere (i.e., it does not pass through the center, but its closest approach to the center is a distance a, similar to Problem 23). (a) What is the component of the gravitational force on the particle in the direction of its motion? (b) What is the potential energy of the particle as a function of its position in the tunnel? (c) Sketch this potential energy, and describe (qualitatively) the motion of the particle.

28. **Pendulum phase portrait**

    Draw the phase portrait for the pendulum (supported by an inextensible rod)

    $$\ddot{\theta} + \omega^2 \sin\theta = 0.$$

    Indicate the stable equilibrium points as well as the unstable equilibrium points. Indicate which paths represent oscillatory motion and which represent the pendulum executing circular motion. Can you derive the equation of the phase paths?

29. **Cubic potential**

    Construct the phase portrait for the potential energy function $U(x) = -(\lambda/3)x^3$.

30. **Logistic model**

    Consider the "logistic" model for the population of a single species, say rabbits. If the rabbits have a birth rate that is larger than the death rate then their population increases without bound. However, realistically we might assume that as their population grows too large, they start competing with each other for the available food. One such model that "explains" this is the logistic model:

    $$\frac{dx}{dt} = ax - ex^2$$

    where $-ex^2$ represents the decrease in population, and $x(t)$ is the population of the rabbits. (Both a and e are positive.)

(a) Find the equilibrium values of x.

(b) Since this is a Riccati equation, we can solve it using the variable-transformation technique that we used before. Transform the dependent variable from x to y via the following transformation: $x = (1/y) + (a/e)$. Solve for $y(t)$, and then perform the inverse transformation to solve for $x(t)$.

(c) Sketch $x(t)$ for two different initial conditions: $x(0) = 0.1(a/e)$ and $x(0) = 10(a/e)$.

(d) Analyze the equation and the solutions to say something qualitative about the dynamics of the rabbit population.

31. **Predator-prey analysis**

(a) Let $f(x)$ and $g(y)$ have local minima at $x = a$ and $y = b$ respectively. Show that $f(x) + g(y)$ has a minimum at (a, b). (Think of $f(x) + g(y)$ as a function of two variables $z(x, y)$ that represents the height or potential energy as a function of position.) Deduce that there exists a neighborhood of (a, b) in which all solutions of the family of equations

$$f(x) + g(y) = \text{constant}$$

represent closed curves surrounding (a, b).

(b) For the predator-prey problem discussed in class (wolves and rabbits), show using the above result that all solutions are periodic (assuming $x > 0$ and $y > 0$).

The system we considered was

$$\frac{dx}{dt} = ax - cxy$$

$$\frac{dy}{dt} = -by + dxy.$$

32. **Energy in oscillation**

Consider a simple harmonic oscillator. (a) Calculate the *time* averages of the kinetic and potential energies over one cycle, and show that these quantities are equal. (b) Calculate the *spatial* averages of the kinetic and potential energies over one cycle. Discuss both results, and explain why they make sense.

33. **The Spring-Pendulum I**

A spring-pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k, and the mass is free to move in both the vertical and horizontal directions. Write the equations of motions (Newton's second law) in two ways. (a) Use Cartesian coordinates (x and y) where the origin is the equilibrium position of the mass. You should obtain two second order differential equations, one for x and the other for y. Then, linearize the equations for small x and small y (compared to what?), and qualitatively characterize the system. (b) Use polar coordinates (r and θ) where the origin is the point of support of the pendulum. Now your ODEs will be for r and θ as functions of time. Again, linearize the equations and compare and contrast with the results from part (a).

34. **Bobbing in liquid**

    An object of uniform cross-sectional area A and mass density ρ floats in a liquid of density ρ_0 and at equilibrium displaces a volume V. Show that the period of small oscillations about the equilibrium position is give by

$$T = 2\pi\sqrt{V/gA}$$

    where g is the acceleration due to gravity.

35. **Damped oscillator I**

    If the amplitude of a damped oscillator decreases to $1/e$ of its initial value after n periods, show that the frequency of the oscillator must be approximately $[1-(8\pi^2n^2)^{-1}]$ times the frequency of the corresponding undamped oscillator. HINT: Assume that the damping is small and use a Taylor series expansion. The damping parameter must be small compared to what?

36. **Damped oscillator II**

    Express the displacement $x(t)$ and the velocity $\dot{x}(t)$ for the overdamped oscillator in terms of hyperbolic functions.

37. **Quality Factor**

    Show that, if a driven oscillator is only lightly damped and driven near resonance, the Q of the system is approximately

$$Q \approx 2\pi \times \left(\frac{\text{Total energy}}{\text{Energy lost during one period}}\right).$$

    HINT: calculate the work done by the friction force.

38. **Straight Line**

    Using the calculus of variations, show that the shortest distance between two points in a plane is a straight line.

39. **Refraction of Light**

    Consider a medium in which the index of refraction is a function of altitude y (for example, the atmosphere). If light attempts to traverse the atmosphere, it will refract, and we are interested in finding the path $y(x)$ that light takes. Using calculus of variations and Fermat's principle of least time, derive the differential equation that describes the path of the light. HINT: the second form of Euler's equation is useful.

40. **Pendulum on a train**

    Find the frequency of small oscillations of a simple pendulum hanging from the ceiling of a railroad car that has a constant acceleration a in the positive x direction. HINT: be sure to choose your generalized coordinates relative to an inertial frame of reference. Explain why the sign of the acceleration a does not affect the frequency ω (i.e., you should get the same answer if the train were acclerating in the negative x direction).

41. **Sphere in a Cylinder**

A uniform solid sphere of radius ρ and mass m is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius R. Determine the Lagrangian, the equation of constraint, and Lagrange's equation of motion. Find the frequency of small oscillations about the bottom. (Don't forget the elementary physics of rotational motion!)

42. **Double Pendulum**

A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. The lengths of the two rods are ℓ_1 and ℓ_2 and the masses of the bobs are m_1 and m_2 (the rods are massless). Find Lagrange's equations of motion for the system (note that there will be two [coupled] equations, one for each generalized coordinate). Do not assume small angles. Do assume that they are both confined to move in the same plane.

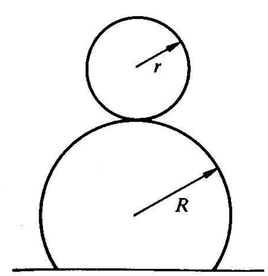
EXTRA CREDIT: If $\ell_1 = \ell_2$ and $m_1 = m_2$ and the displacements from equilibrium are small (i.e., assume small angles), solve the two (coupled) equations of motion for the two normal modes of the system.

43. **The Spring-Pendulum II**

A spring-pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k. (a) Find Lagrange's equations of motion (do not assume small angles). (b) Taylor expand the equations of motion around the equilibrium position (i.e., keep only the linear terms) and determine the frequencies of small oscillations. You should obtain two *uncoupled* linear oscillators.

44. **The hoop on a cylinder**

A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the hoop starts rolling from rest on top of the cylinder, find, by the method of Lagrange undetermined multipliers, the point at which the hoop falls off the cylinder. (You know that moment of inertia of the hoop is mr^2, but it is instructive to solve the problem for a disk of arbitrary moment of inertia I.)



45. **Lagrangian I**

The Lagrangian for a particular physical system can be written as

$$L' = \frac{m}{2}\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - \frac{k}{2}\left(ax^2 + 2bxy + cy^2\right),$$

where a, b, and c are arbitrary constants but subject to the condition that $b^2 - ac \neq 0$. What are the equations of motion? Examine particularly the two cases $a = 0 = c$ and $b = 0$, $c = -a$. What is the physical system described by the above Lagrangian?

46. **Lagrangian II**

A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - V^2(x),$$

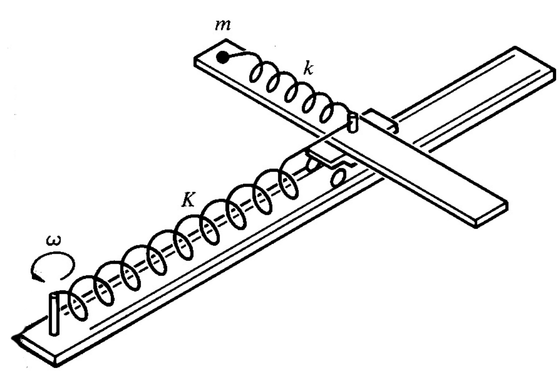
where V is some arbitrary differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation.

47. **Double carriage/spring**

A carriage runs along rails on a rigid beam. The carriage is attached to one end of a spring, equilibrium length r_0 and force constant K, whose other end is fixed on the beam. On the carriage there is another set of rails perpendicular to the first along which a particle of mass m moves, held by a spring fixed on the beam, of force constant k and zero equilibrium length. Beam, rails, springs and carriage are assumed to have zero mass. The whole system is forced to move in a plane about the point of attachment of the first spring, with a constant angular speed ω. The length of the second spring is at all times considered small compared to r_0.

(a) What is the energy of the system? Is it conserved?

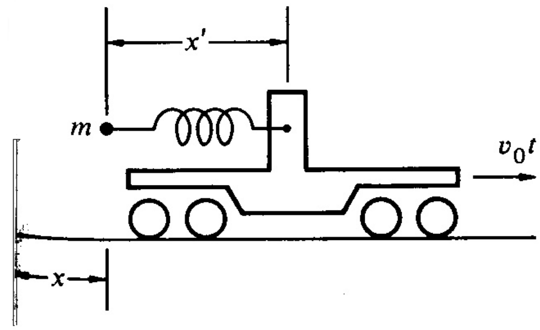
(b) Using generalized coordinates in the laboratory frame, what is the Jacobi integral (i.e., H) for the system? Is it conserved?



48. **Spring on a cart**

Consider a mass m attached to a spring of constant k, the other end of which is fixed on a massless cart that is being moved uniformly by an external force with speed v_0. Determine the equation of motion of the mass.

Investigate the behavior of the Hamiltonian function for the case where x is the generalized coordinate, and the case where x' is the generalized coordinate. Determine, for each of these cases, whether H is constant, and separately whether H is conserved.



49. An underdamped harmonic oscillator is subject to an applied force

$$f(t) = A\,e^{-at}\cos(\omega t).$$

Find a particular solution by expressing f as the real part of a complex exponential function, and looking for a solution for x having the same exponential time dependence. HINT: This means rewriting the equation of motion as

$$Re\left[\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \mathcal{F}(t)\right],$$

where $f = Re(\mathcal{F})$. Now you can solve the complex equation for a complex x, but at the end take the real part. CHECK: In the limit $a \to 0$, you should obtain the standard result for a harmonically driven oscillator.

50. Find the particular solution to the underdamped harmonic oscillator if the driving force is proportional to $\cos^2 \omega t$.

51. A force $A\cos\omega t$ acts on a damped oscillator starting at time $t = 0$ (the external force is zero for $t < 0$). What must the initial values of x and v be in order that there be no transient?

52. In my posted solution to Reynolds # 33, I obtained the following equations of motion in polar coordinates,

$$\ddot{r} = g\cos\theta - \omega_0^2(r - b) + r\dot{\theta}^2,$$
$$r\ddot{\theta} = -g\sin\theta - 2\dot{r}\dot{\theta}.$$

Show that these are identical to Morin's equations (6.12) and (6.13) with the appropriate change of variables.

53. Show that the geodesic on the surface of a right circular cylinder is a helix. A *geodesic* is the shortest path between two points in a curved space.

54. Consider a disk of mass M rolling without slipping down a stationary plane inclined an angle α with respect to the horizontal. Choose reasonable coordinates (say, the distance s down the plane and the angle θ orienting the disk). Obtain the Lagrangian and the equations of motion. What is the constraint equation? What is the force of constraint?