with your hands or mind or both - you need it all! Remember, just as is the case with physical exercise (which you should get plenty of), mental exercise gradually makes you mentally stronger, so that you can eventually do easily things that at first appear insurmountably difficult. You can learn to learn three to ten times as fast as you did in high school, to have more fun while doing it, and to gain tremendous reasoning capabilities along the way just by trying to learn to learn more efficiently instead of continuing to use learning strategies that worked (possibly indifferently) back in elementary and high school.

The next section, at long last, will make a very specific set of suggestions for one very good way to study physics (or nearly anything else) in a way that maximally takes advantage of your own volitional biology to make learning as efficient and pleasant as it is possible to be.

## How to Do Your Homework Effectively

By now in your academic career (and given the information above) it should be very apparent just where homework exists in the grand scheme of (learning) things. Ideally, you attend a class where a warm and attentive professor clearly explains some abstruse concept and a whole raft of facts in some moderately interactive way that encourages engagement and "being earnest". Alas, there are too many facts to fit in short term/immediate memory and too little time to move most of them through into long term/working memory before finishing with one and moving on to the next one. The material may appear to be boring and random so that it is difficult to pay full attention to the patterns being communicated and remain emotionally enthusiastic all the while to help the process along. As a consequence, by the end of lecture you've already forgotten many if not most of the facts, but if you were paying attention, asked questions as needed, and really cared about learning the material you would remember a handful of the most important ones, the ones that made your brief understanding of the material hang (for a brief shining moment) together.

This conceptual overview, however initially tenuous, is the skeleton you will eventually clothe with facts and experiences to transform it into an entire system of associative memory and reasoning where you can work intellectually at a high level with little effort and usually with a great deal of pleasure associated with the very act of thinking. But you aren't there yet.

You now know that you are not terribly likely to retain a lot of what you are shown in lecture without engagement. In order to actually learn it, you must stop being a passive recipient of facts. You must actively develop your understanding, by means of discussing the material and kicking it around with others, by using the material in some way, by teaching the material to peers as you come to understand it.

To help facilitate this process, associated with lecture your professor almost certainly gave you an assignment. Amazingly enough, its purpose is not to torment you or to be the basis of your grade (although it may well do both). It is to give you some concrete stuff to $d o$ while thinking about the material to be learned, while discussing the material to be learned, while using the material to be learned to accomplish specific goals, while teaching some of what you figure out to others who are sharing this whole experience while being taught by them in turn. The assignment is much more important than lecture, as it is entirely participatory, where real learning is far more likely to occur. You could, once you learn the trick of it, blow off lecture and do fine in a course in all other respects. If you fail to do the assignments with your entire spirit engaged, you are doomed.

In other words, to learn you must do your homework, ideally at least partly in a group setting. The only question is: how should you do it to both finish learning all that stuff you sort-of-got in lecture and to re-attain the moment(s) of clarity that you then experienced, until eventually it becomes a permanent characteristic of your awareness and you know and fully understand it all on your own?

There are two general steps that need to be iterated to finish learning anything at all. They are a lot of work. In fact, they are far more work than (passively) attending lecture, and are more important than attending lecture. You can learn the material with these steps without ever attending lecture, as long as you have access to what you need to learn in some media or human form. You in all probability will never learn it, lecture or not, without making a few passes through these steps. They are:
a) Review the whole (typically textbooks and/or notes)
b) Work on the parts (do homework, use it for something)
(iterate until you thoroughly understand whatever it is you are trying to learn).
Let's examine these steps.
The first is pretty obvious. You didn't "get it" from one lecture. There was too much material. If you were lucky and well prepared and blessed with a good instructor, perhaps you grasped some of it for a moment (and if your instructor was poor or you were particularly poorly prepared you may not have managed even that) but what you did momentarily understand is fading, flitting further and further away with every moment that passes. You need to review the entire topic, as a whole, as well as all its parts. A set of good summary notes might contain all the relative factoids, but there are relations between those factoids - a temporal sequencing, mathematical derivations connecting them to other things you know, a topical association with other things that you know. They tell a story, or part of a story, and you need to know that story in broad terms, not try to memorize it word for word.

Reviewing the material should be done in layers, skimming the textbook and your notes, creating a new set of notes out of the text in combination with your lecture notes, maybe reading in more detail to understand some particular point that puzzles you, reworking a few of the examples presented. Lots of increasingly deep passes through it (starting with the merest skim-reading or reading a summary of the whole thing) are much better than trying to work through the whole text one line at a time and not moving on until you understand it. Many things you might want to understand will only come clear from things you are exposed to later, as it is not the case that all knowledge is ordinal, hierarchical, and derivatory.

You especially do not have to work on memorizing the content. In fact, it is not desireable to try to memorize content at this point - you want the big picture first so that facts have a place to live in your brain. If you build them a house, they'll move right in without a fuss, where if you try to grasp them one at a time with no place to put them, they'll (metaphorically) slip away again as fast as you try to take up the next one. Let's understand this a bit.

As we've seen, your brain is fabulously efficient at storing information in a compressed associative form. It also tends to remember things that are important - whatever that means - and forget things that aren't important to make room for more important stuff, as your brain structures work together in understandable ways on the process. Building the cognitive map, the "house", is what it's all about. But as it turns out, building this house takes time.

This is the goal of your iterated review process. At first you are memorizing things the hard way, trying to connect what you learn to very simple hierarchical concepts such as this step comes before that step. As you do this over and over again, though, you find that absorbing new information takes you less and less time, and you remember it much more easily and for a longer time without additional rehearsal. Sometimes your brain even outruns the learning process and "discovers" a missing part of the structure before you even read about it! By reviewing the whole, well-organized structure over and over again, you gradually build a greatly compressed representation of it in your brain and tremendously reduce the amount of work required to flesh out that structure with increasing levels of detail and remember them and be able to work with them for a long, long time.

Now let's understand the second part of doing homework - working problems. As you can probably guess on your own at this point, there are good ways and bad ways to do homework problems. The worst way to do homework (aside from not doing it at all, which is far too common a practice and a bad idea if you have any intention of learning the material) is to do it all in one sitting, right before it is due, and to never again look at it.

Doing your homework in a single sitting, working on it just one time fails to repeat and rehearse the material (essential for turning short term memory into long term in nearly all cases). It exhausts the neurons in your brain (quite literally - there is metabolic energy consumed in thinking) as one often ends up working on a problem far too long in one sitting just to get done. It fails to incrementally build up in your brain's long term memory the structures upon which the more complex solutions are based, so you have to constantly go back to the book to get them into short term memory long enough to get through a problem. Even this simple bit of repetition does initiate a learning process. Unfortunately, by not repeating them after this one sitting they soon fade, often without a discernable trace in long term memory.

Just as was the case in our experiment with memorizing the number above, the problems almost invariably are not going to be a matter of random noise. They have certain key facts and ideas that are the basis of their solution, and those ideas are used over and over again. There is plenty of pattern and meaning there for your brain to exploit in information compression, and it may well be very cool stuff to know and hence important to you once learned, but it takes time and repetition and a certain amount of meditation for the "gestalt" of it to spring into your awareness and burn itself into your conceptual memory as "high order understanding".

You have to give it this time, and perform the repetitions, while maintaining an optimistic, philosophical attitude towards the process. You have to do your best to have fun with it. You don't get strong by lifting light weights a single time. You get strong lifting weights repeatedly, starting with light weights to be sure, but then working up to the heaviest weights you can manage. When you do build up to where you're lifting hundreds of pounds, the fifty pounds you started with seems light as a feather to you.

As with the body, so with the brain. Repeat broad strokes for the big picture with increasingly deep and "heavy" excursions into the material to explore it in detail as the overall picture emerges. Intersperse this with sessions where you work on problems and try to use the material you've figured out so far. Be sure to discuss it and teach it to others as you go as much as possible, as articulating what you've figured out to others both uses a different part of your brain than taking it in (and hence solidifies the memory) and it helps you articulate the ideas to yourself! This process will help you learn more, better, faster than you ever have before, and to have fun doing it!

Your brain is more complicated than you think. You are very likely used to working hard to try to make it figure things out, but you've probably observed that this doesn't work very well. A lot of times you simply cannot "figure things out" because your brain doesn't yet know the key things required to do this, or doesn't "see" how those parts you do know fit together. Learning and discovery is not, alas, "intentional" - it is more like trying to get a bird to light on your hand that flits away the moment you try to grasp it.

People who do really hard crossword puzzles (one form of great brain exercise) have learned the following. After making a pass through the puzzle and filling in all the words they can "get", and maybe making a couple of extra passes through thinking hard about ones they can't get right away, looking for patterns, trying partial guesses, they arrive at an impasse. If they continue working hard on it, they are unlikely to make further progress, no matter how long they stare at it.

On the other hand, if they put the puzzle down and do something else for a while - especially if the something else is go to bed and sleep - when they come back to the puzzle they often can immediately see a dozen or more words that the day before were absolutely invisible to them. Sometimes one of the long theme answers (perhaps 25 characters long) where they have no more than two letters just
"gives up" - they can simply "see" what the answer must be.
Where do these answers come from? The person has not "figured them out", they have "recognized" them. They come all at once, and they don't come about as the result of a logical sequential process.

Often they come from the person's right brain 22 . The left brain tries to use logic and simple memory when it works on crosswork puzzles. This is usually good for some words, but for many of the words there are many possible answers and without any insight one can't even recall one of the possibilities. The clues don't suffice to connect you up to a word. Even as letters get filled in this continues to be the case, not because you don't know the word (although in really hard puzzles this can sometimes be the case) but because you don't know how to recognize the word "all at once" from a cleverly nonlinear clue and a few letters in this context.

The right brain is (to some extent) responsible for insight and non-linear thinking. It sees patterns, and wholes, not sequential relations between the parts. It isn't intentional - we can't "make" our right brains figure something out, it is often the other way around! Working hard on a problem, then "sleeping on it" (to get that all important hippocampal involvement going) is actually a great way to develop "insight" that lets you solve it without really working terribly hard after a few tries. It also utilizes more of your brain - left and right brain, sequential reasoning and insight, and if you articulate it, or use it, or make something with your hands, then it exercieses these parts of your brain as well, strengthening the memory and your understanding still more. The learning that is associated with this process, and the problem solving power of the method, is much greater than just working on a problem linearly the night before it is due until you hack your way through it using information assembled a part at a time from the book.

The following "Method of Three Passes" is a specific strategy that implements many of the tricks discussed above. It is known to be effective for learning by means of doing homework (or in a generalized way, learning anything at all). It is ideal for "problem oriented homework", and will pay off big in learning dividends should you adopt it, especially when supported by a group oriented recitation with strong tutorial support and many opportunities for peer discussion and teaching.

## The Method of Three Passes

Pass 1 Three or more nights before recitation (or when the homework is due), make a fast pass through all problems. Plan to spend 1-1.5 hours on this pass. With roughly $10-12$ problems, this gives you around 6-8 minutes per problem. Spend no more than this much time per problem and if you can solve them in this much time fine, otherwise move on to the next. Try to do this the last thing before bed at night (seriously) and then go to sleep.

Pass 2 After at least one night's sleep, make a medium speed pass through all problems. Plan to spend 1-1.5 hours on this pass as well. Some of the problems will already be solved from the first pass or nearly so. Quickly review their solution and then move on to concentrate on the still unsolved problems. If you solved $1 / 4$ to $1 / 3$ of the problems in the first pass, you should be able to spend 10 minutes or so per problem in the second pass. Again, do this right before bed if possible and then go immediately to sleep.

Pass 3 After at least one night's sleep, make a final pass through all the problems. Begin as before by quickly reviewing all the problems you solved in the previous two passes. Then spend fifteen minutes or more (as needed) to solve the remaining unsolved problems. Leave any "impossible" problems for recitation - there should be no more than three from any given assignment, as a general rule. Go immediately to bed.

[^0]This is an extremely powerful prescription for deeply learning nearly anything. Here is the motivation. Memory is formed by repetition, and this obviously contains a lot of that. Permanent (long term) memory is actually formed in your sleep, and studies have shown that whatever you study right before sleep is most likely to be retained. Physics is actually a "whole brain" subject - it requires a synthesis of both right brain visualization and conceptualization and left brain verbal/analytical processing - both geometry and algebra, if you like, and you'll often find that problems that stumped you the night before just solve themselves "like magic" on the second or third pass if you work hard on them for a short, intense, session and then sleep on it. This is your right (nonverbal) brain participating as it develops intuition to guide your left brain algebraic engine.

Other suggestions to improve learning include working in a study group for that third pass (the first one or two are best done alone to "prepare" for the third pass). Teaching is one of the best ways to learn, and by working in a group you'll have opportunities to both teach and learn more deeply than you would otherwise as you have to articulate your solutions.

Make the learning fun - the right brain is the key to forming long term memory and it is the seat of your emotions. If you are happy studying and make it a positive experience, you will increase retention, it is that simple. Order pizza, play music, make it a "physics homework party night".

Use your whole brain on the problems - draw lots of pictures and figures (right brain) to go with the algebra (left brain). Listen to quiet music (right brain) while thinking through the sequences of events in the problem (left brain). Build little "demos" of problems where possible - even using your hands in this way helps strengthen memory.

Avoid memorization. You will learn physics far better if you learn to solve problems and understand the concepts rather than attempt to memorize the umpty-zillion formulas, factoids, and specific problems or examples covered at one time or another in the class. That isn't to say that you shouldn't learn the important formulas, Laws of Nature, and all of that - it's just that the learning should generally not consist of putting them on a big sheet of paper all jumbled together and then trying to memorize them as abstract collections of symbols out of context.

Be sure to review the problems one last time when you get your graded homework back. Learn from your mistakes or you will, as they say, be doomed to repeat them.

If you follow this prescription, you will have seen every assigned homework problem a minimum of five or six times - three original passes, recitation itself, a final write up pass after recitation, and a review pass when you get it back. At least three of these should occur after you have solved all of the problems correctly, since recitation is devoted to ensuring this. When the time comes to study for exams, it should really be (for once) a review process, not a cram. Every problem will be like an old friend, and a very brief review will form a seventh pass or eighth pass through the assigned homework.

With this methodology (enhanced as required by the physics resource rooms, tutors, and help from your instructors) there is no reason for you do poorly in the course and every reason to expect that you will do well, perhaps very well indeed! And you'll still be spending only the 3 to 6 hours per week on homework that is expected of you in any college course of this level of difficulty!

This ends our discussion of course preliminaries (for nearly any serious course you might take, not just physics courses) and it is time to get on with the actual material for this course.

## Mathematics

Physics, as was noted in the preface, requires a solid knowledge of all mathematics through calculus. That's right, the whole nine yards: number theory, algebra, geometry, trigonometry, vectors, differential calculus, integral calculus, even a smattering of differential equations. Somebody may have


[^0]:    ${ }^{22}$ Note that this description is at least partly metaphor, for while there is some hemispherical specialization of some of these functions, it isn't always sharp. I'm retaining them here (oh you brain specialists who might be reading this) because they are a valuable metaphor.

