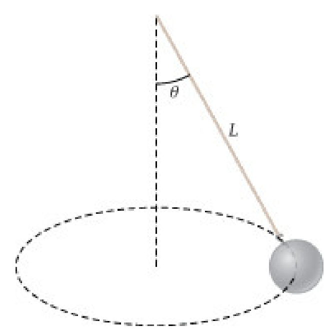
Modern Physics Warmup Problems

Modern Physics (or as I like to call it, Physics 4) is the "capstone" course for introductory physics. You will use all the physics and math you have learned over the past two years and apply it to the very small and very fast. At the end you should have a fairly complete picture of our current understanding of the universe. In order to do this successfully, however, your math and physics skills must be finely honed. To help you prepare, here are some fun physics and math problems that require only the knowledge that you have (hopefully) learned in Physics 1-2-3 and Calculus 1-2-3. Try to recall as much as possible without looking in your old textbook or online (this "retrieval practice" is an important part of learning), but by all means work with your friends and discuss your solution methods.

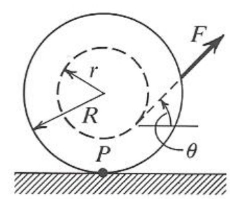
Physics

1. A particle of mass m is constrained to move in the x direction, and it is subject to the time-dependent force $F_x(t) = ae^{-bt}$. Obtain the position x and velocity v_x as functions of time, given that the initial velocity is $v_x(0) = v_0$ and the initial position is $x(0) = x_0$. Sketch both x and v_x versus t.

2. Consider the "conical pendulum" in the figure, where a mass m is attached to a string of length L and it is swung in a horizontal circle such that the angle the string makes with the vertical is θ. (a) Calculate the tension in the string. (b) Obtain the angular velocity ω as a function of the angle θ. (c) Show that in the limit $\theta \to 0$, ω reduces to the well-known planar pendulum angular frequency, $\sqrt{g/\ell}$.



3. A yo-yo rests on a table and the free end of its string is gently pulled at an angle θ to the horizontal. If $\theta \approx \pi/2$, then the yo-yo will accelerate to the left, but if $\theta \approx 0$, then the yo-yo will accelerate to the right. (a) Find the critical angle θ_c such that the yo-yo remains stationary, even though it is free to roll. (b) Find the acceleration of the yo-yo as a function of the angle θ.



4. Einstein's general relativity is a modification of Newton's theory of gravitation. For a satellite of mass m orbiting a central object of mass M, the lowest order correction (due to the weak gravitational field) can be expressed as a modification of Newton's Law of Universal Gravitation

$$F = \frac{GMm}{r^2}\left(1 + 6\frac{v^2}{c^2}\right).$$

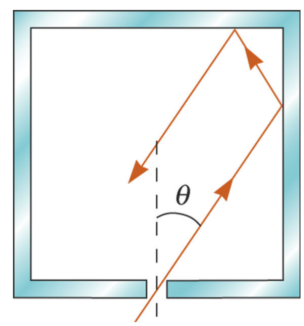
(a) Show that if the satellite's orbital period under the pure Newtonian force GMm/r^2 is T_0, then the modified period T is approximately

$$T \approx T_0\left(1 - \frac{12\pi^2 r^2}{c^2 T_0^2}\right).$$

HINT: Treat the relativistic correction as a small fractional increase in G and use the value of v corresponding to the Newtonian orbit. You will need to use Kepler's laws and elementary kinematics. (b) Show that, in each revolution, a satellite in a circular orbit would travel through an angle greater than under the Newtonian force by an amount $24\pi^3 r^2/c^2 T_0^2$. (c) Finally, show that this angle is expressible as $6\pi GM/c^2 r$. Apply this result to the planet Mercury, and, using the numerical values for Mercury and the Sun, verify that the accumulated advance in angle amounts to 43 seconds of arc per century. This is the calculation that Einstein made with his new theory that explained the longstanding observations of Mercury.

5. A moving particle of mass M collides perfectly elastically with a stationary particle of mass $m < M$. Show that the maximum possible angle through which the incident particle can be deflected is $\arcsin(m/M)$.

6. A ray of light enters a square enclosure though a small hole in the center of one side. All the interior surfaces are mirrored. It is obvious that if $\theta = 45°$ the ray will exit the hole after being reflected once by each of the three mirrors. Determine a formula for θ that will give all the other angles that will result in the ray exiting through the hole after an arbitrary number of reflections from the interior mirrors.



Mathematics

7. An "n-sphere" is the set of points in $(n+1)$-dimensional Euclidean space that are at a fixed distance a from the origin. You know that the volume enclosed by a 2-sphere (the ordinary sphere) is $4\pi a^3/3$, and the "volume" enclosed by a 1-sphere (the ordinary circle) is πa^2. Calculate the volume of the 3-sphere by evaluating the integral

$$V_3 = \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2-z^2}}^{\sqrt{a^2-x^2-y^2-z^2}} du\; dz\; dy\; dx.$$

I suggest that you start with the simpler problems of confirming (via direct integration) the area of a circle

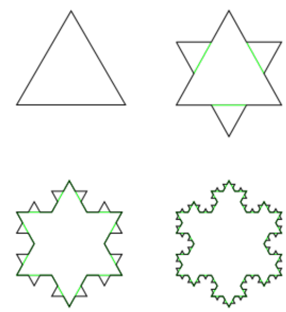
$$V_1 = \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\; dx = \pi a^2$$

and the volume of a sphere

$$V_2 = \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dz\; dy\; dx = \frac{4}{3}\pi a^3.$$

HINT: In the V_2 integral, switch to polar coordinates after the first (trivial) integration over z, and in the V_3 integral, switch to spherical coordinates after the first (trivial) integration over u.

8. An interesting curve that is a "fractal" can be created as follows. Draw an equilateral triangle whose sides are length ℓ, then recursively alter each line segment as follows: 1) divide each line segment into three segments of equal length, 2) draw an equilateral triangle that has the middle segment from step 1 as its base and points outward, 3) remove the line segment that is the base of the triangle from step 2, 4) repeat. The first 4 iterations are shown in the figure. The perimeter diverges, i.e., is infinite, because at each step the perimeter increases by a factor of 4/3. However, the area enclosed by the curve as the number of iterations goes to infinity remains finite. Calculate this area.



9. Three blue socks and six red socks are in a drawer. If two socks are drawn at random from the drawer, what is the probability the two form a matching pair?

10. In your sock drawer, you have some red socks and some blue socks. What is the smallest number of each type of sock that must be in the drawer so that if you remove two socks to wear, the probability that you get two red socks is 50%?