PS 303
Homework C

<u>Due Wed 1/25</u>

Read Krane, Section 2.6, 2.8

Reynolds:

4. Derive the full Lorentz coordinate transformation by starting with our knowledge of the muon experiment

$$t = \gamma t' + A\frac{x'}{c}$$

$$\frac{x}{c} = u\gamma t' + B\frac{x'}{c}$$

where *A* and *B* are unknown. Now, require that the spacetime interval is invariant

$$t^2 - \frac{x^2}{c^2} = t'^2 - \frac{x'^2}{c^2}$$

and equation the coefficients of both t'^2 and $\left(\frac{x'}{c}\right)^2$ to obtain *A* and *B*.

5. Invert the Lorentz transformation that you obtained above in Problem #4 (which gives the unprimed coordinates in terms of the primed coordinates). Your result should be the transformation quoted in Krane Eqs. (2.23).

6. Complete the derivation of the velocity transformation equation by obtaining expressions for dx/dt' and dt'/dt, plugging them into the expression for v and then solving for v in terms of v'. You should obtain an expression than agrees with Krane's Eq. (2.17).

Extra credit #1 (due 2/1)

Use Figs. 2.8 and 2.9 in Krane to derive (from Einstein's two postulates) the fact that the spacetime interval is invariant. That is, calculate $(\Delta t')^2$ in terms of Δx and Δt. Then, assume there is a third frame, called O'', that is moving *faster* than the O' frame. Some call this the *super*-rocket frame. This time, calculate $(\Delta t')^2$ in terms of $\Delta x''$ and $\Delta t''$. Equating the two expressions gives you the result you want.

Due Fri 1/27

Read Krane, Section 2.7

Reynolds:

7. When Einstein was a boy, he mulled over the following puzzler: A runner looks at himself in a mirror that he holds at arm's length in front of him. If he runs at nearly the speed of light, will he be able to see himself in the mirror? Analyze this question in terms of relativity.

8. A pole that is 20 m long (proper length) is carried through a barn that is only 10 m long (proper length). The pole is moving at a speed such that an observer at rest with respect to the barn measures the pole's length to be contracted to 10 m. Therefore, from this point of view, there is an instant at which the pole just fits in the barn. However, from the point of view of an observer traveling with the pole, the *barn* is length contracted to half its length, 5 m, so that there is no way that the pole fits in the barn. This is the famous 'pole-and-barn' paradox.

Resolve this paradox in the following manner. Make two carefully labeled spacetime diagrams, one in frame O, the barn's frame, and the other in frame O', the pole's frame. In both diagrams, plot the world lines of the following four objects: the front end of the pole, the back end of the pole, and the two barn doors. Take Event #1 to be when the front end of the pole (B) enters the front door of the barn (C) and make this the origin of both spacetime diagrams. The two important events to study are Event #2, when the front end of the pole (B) exits the back door (D), and Event #3, when the back end of the pole (A) enters the front door (C). These last two events are, of course, simultaneous in the O frame, but they are not simultaneous in the O' frame.

You will need to determine the spacetime coordinates of each of these events in both frames using the Lorentz transformations.