Paradoxical twins and their special relatives

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We present a variation and extension of the twin paradox recently put forth by Boughn. These additional considerations make a strong case that there is no meaning to the question of "where" the aging difference occurs in the twin paradox. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

A recent article by Boughn,[1] in this journal, presented a relativity parable with great pedagogical power. It is often the case that an important work inspires inferior sequels. In that tradition, we would like to present here additional considerations based on Boughn's twin paradox. Our main point here will be to make the case that the question "where does the asymmetry in aging *really* occur?" is meaningless.

The Boughn paradox involves twins Dick and Jane who start from rest at home (where Mom and Dad remain). They enter their resting rockets which are separated by a distance L along the x axis, and follow identical instructions as they accelerate in the positive x direction. After a while they exhaust rocket fuel, so that they are coasting (not accelerating). Since they have followed identical instructions they must, when they are both coasting, be moving at the same speed relative to Mom and Dad. They therefore have no velocity with respect to each other; they are in the same inertial frame. From the symmetry of the motion, as observed by Mom and Dad, the twins must be the same age when they are coasting. But the twins are now in an inertial frame, call it S', which is moving, say at velocity v, with respect to the frame of Mom and Dad. What is simultaneous to Mom and Dad is not simultaneous in frame S'. As Boughn points out, the twins' birthday—which is simultaneous to Mom and Dad—is observed in frame S' to occur with a time difference $\Delta t' = \gamma v L \equiv vL/\sqrt{1-v^2}$ (where we are using units in which $c=1$). If Jane's position is at larger x then it is she who will be older. The relativity of simultaneity is evident in Fig. 1, in which the worldlines of Dick and Jane are illustrated in the frame of Mom and Dad. Events 1 and $\bar{2}$ are events, like the twins' birthday, which are simultaneous to Mom and Dad. Events 1 and 2 are events which are simultaneous to the twins. It is clear that in S', the final frame of the twins, Jane's birthday is earlier than Dick's.

All this is a straightforward application of the formalism of special relativity. What is not straightforward, and what often confuses students, is the meaning of the result. First, what does the "age" or "aging" of one of the twins mean? The answer, of course, is that each twin can be considered to be a clock. Biological aging is no different in principle from the ticking of the counter in an atomic clock. The age of Jane at event 2, then, can equally well be considered to be the age inferred from her biological aging, or the reading of the atomic clock that was strapped to her wrist and started from zero at the moment Jane was born. It is a well-defined unambiguous, observer-independent value. When we say that the age of Jane at event 2 is greater than the age of Dick at event 1, we are comparing measurements of time about which there is no disagreement.

The issue that is much more slippery is the inference from the ages at events 1 and 2 that Jane has somehow gotten older than Dick. The skeptical student can argue that the age difference between events 1 and 2 is a purely formal, abstract, idea with no physical immediacy. Since events 1 and 2 are in different locations, no direct comparison of the ages of Dick and Jane is possible. For this student it is important to point out that Dick and Jane can simply *visit* one another. Dick can walk over to Jane's position; Jane can do the walking; they can meet somewhere in the middle. It does not matter *as long as they walk slowly.* By comparing their atomic clocks (or biological ages) they will find that indeed Jane in every sense of the word really *is* older by $\gamma v L$. It follows that the age difference has absolute meaning. After Dick and Jane have (*slowly!*) reunited and are at the same location, any observer will see Jane as older by $\gamma v L$.

This point will be important to the argument below, so we give here a demonstration that certain relativistic time differences can be arbitrarily small. To separate this demonstration from the context of Fig. 1, we will consider motion in a frame with coordinates X,T, as depicted in Fig. 2. Two observers, $\mathcal{O}_1$ and $\mathcal{O}_2$, initially are separated by D. For definiteness we will let the observers each move at speed v_s, one moving to the right, one to the left. The time T for them to meet (the time measured by an observer stationary in the frame) is $D/2v_s$. The "proper time" measured by the observers themselves is $D/2v_s\gamma_s = \sqrt{1-v_s^2}\ D/2v_s$. The difference between this proper time and the time measured by the stationary observers is

$$\frac{D}{2v_s}(1-\sqrt{1-v_s^2}). \tag{1}$$

If v_s is much less than the speed of light, this can be approximated as

$$\frac{D}{2v_s}(1-\sqrt{1-v_s^2}) \approx \frac{Dv_s}{4c^2}, \tag{2}$$

where, in the last expression, we have explicitly exhibited the factors of c. For any D, this time difference can be made arbitrarily small, by choosing v_s/c sufficiently small. The time duration observed by moving and stationary observers can therefore be in arbitrarily good agreement. This argument is easily extended to nonsymmetric cases, in which the two observers move through different distances. As long as

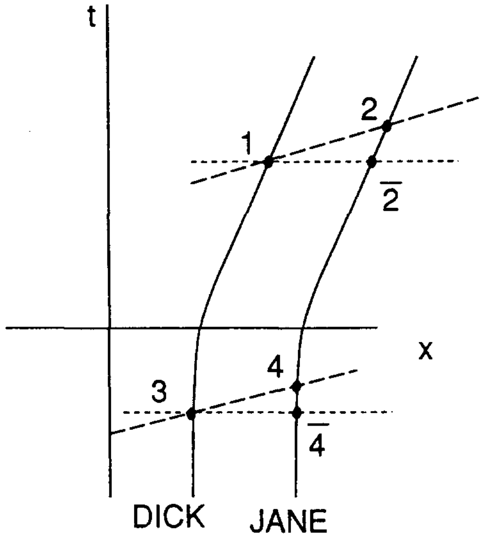

Fig. 1. Worldlines of Dick and Jane in the coordinate system of Mom and Dad. Events 1 and 2 are simultaneous to Dick and Jane; events 1 and $\bar{2}$ are simultaneous to Mom and Dad.

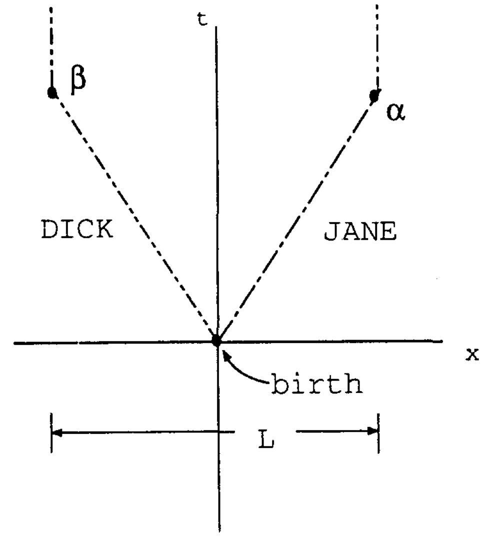

Fig. 3. Dick and Jane, the early years. The twins are born at the same point, then move apart slowly to a separation L.

the observer speeds are arbitrarily slow, all time differences will be arbitrarily small.

Without inducing any new differential aging, Dick and Jane can indeed simply "stroll" to some common meeting place and discover that Jane is now older than Dick. Slow motion also can answer student questions about early family history. Dick and Jane started out spatially very close at birth. How did they get separated by distance L without destroying the simultaneity of their aging? A simple answer is that they slowly drifted apart.

The stage is now set for our extension of the Boughn fable.

II. THE PARABLE OF THE UNCLE

Imagine that a family disagreement with Mom and Dad led to an uncle setting out, before the birth of the twins, and—after some acceleration—coming to rest in a reference frame that happens to be S', the frame in which the twins

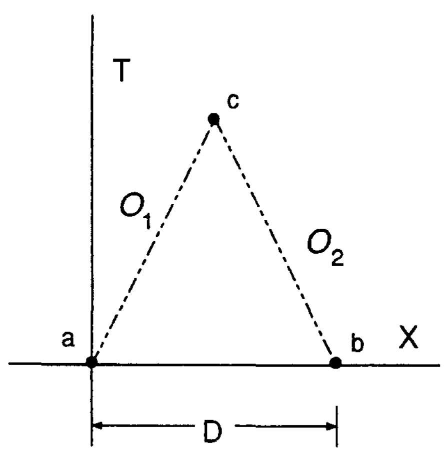

Fig. 2. Worldlines of two observers $\mathcal{O}_1$ and $\mathcal{O}_2$, who meet at event c to compare clock readings.

will eventually come to rest. There is a disturbing paradox in how the uncle will view the voyage of the twins. The uncle *will* see the twins born at the same place, then drift very slowly apart, and then begin their rocket voyage. He will not see them ignite their rocket engines simultaneously (their actions are simultaneous in the frame of Mom and Dad, not that of the uncle) so it is not surprising that the uncle will see the twins, when they finally come to rest in his frame, to have different ages.

But there is something quite strange here. To see this most clearly, we can consider the view of things in the frame of the uncle before either of the twins started off in a rocket. In Fig. 1, events 3 and $\bar{4}$, are simultaneous to Mom and Dad and hence cannot be simultaneous in the frame of the uncle. (For the uncle, events 3 and 4 are simultaneous.) If events 3 and $\bar{4}$ represent the twins' birthdays (an event that is simultaneous to the twins and to Mom and Dad) then in the frame of the uncle, Jane's birthday (event $\bar{4}$) came at an earlier time; Jane must be older. With a simple application of the Lorentz transformations we can show that in the frame of the uncle the time difference $\Delta t'$ between events 3 and $\bar{4}$ is $\Delta t' = \gamma v L$. Although this is an indication of an age difference between Dick and Jane, it is not the best measure of that age difference. We carefully defined the "age" of Jane, at any point on her world line, as the time she herself measures since birth. According to this definition the uncle-frame difference between the ages of Dick and Jane should be Jane's age at event 4 minus Dick's age at event 3. It is straightforward to show that this age difference is vL.

By either measure the actuaries in the uncle frame would conclude that aging has taken place *before* the twins enter their rockets. This leads to a disturbing question: How could Dick and Jane possibly have gotten "out of synchronization" to the uncle? He observed them to be born at the same place at the same time, and in subsequently separating they need not have made any fast motions.

The resolution is somewhat surprising: Dick and Jane lose simultaneity during the *arbitrarily slow* motion of their separation! To see this we look back in the family album to find Fig. 3, a depiction, in the Mom and Dad frame, of the early motions of the twins. The early, slow, motions from the origin (the maternity ward) to events α and β separated the twins by L; each twin was moving at speed v_s which we shall, in a moment, assume to be very small. The twins subsequent stationary "motion" is represented by the vertical segments, which—far in the future—will lead them to rocket ships.

The times t'_α and t'_β, of events α and β, as observed by the uncle, are easily found from the Lorentz transformations, and from the known coordinates of the events in the Mom and Dad frame:

$$t'_\alpha = \gamma[t_\alpha - v x_\alpha] = \gamma[(L/2v_s) - vL/2]. \quad (3)$$

For event β the x coordinate is $x_\beta = -L/2$ so the t' coordinate for the event is

$$t'_\beta = \gamma[(L/2v_s) + vL/2]. \quad (4)$$

The time difference, as measured by the uncle, will therefore be

$$\Delta t' \equiv t'_\beta - t'_\alpha = \gamma v L. \quad (5)$$

This result is independent of v_s; the time difference observed by the uncle remains, no matter how slowly Dick and Jane move. The result is, of course, the result we have already noted as the uncle-measured time difference between the birthdays of Dick and Jane.

We have given different answers to the question "where does the differential aging occur?": (i) It all occurs during the twins' rocket trip. (ii) Some occurs in the twins early (prerocket) years. The lack of a unique answer shows the lack of meaning of the question. Age differences in relativity have a well-defined meaning, but the origin of age differences cannot be assigned to any specific part of a worldline.

[1]S. P. Boughn, "The case of the identically accelerated twins," Am. J. Phys. **57**, 791–793 (1989). Other references to the twin paradox are listed in this paper.

[2]Here and elsewhere we use words like "see" and "view" somewhat inappropriately. We do not mean that the uncle makes observations of distant events by collecting light signals from those events. Rather, we mean that observations are made according to the usual procedure in relativity. Here, that would mean that an observer who is part of the uncle's special relativity reference system, but is located *right at* an event being observed (such as the igniting of a rocket engine), notes the position and time of that event.

Shielding of an oscillating electric field by a hollow conductor

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The electric and magnetic fields for a hollow conducting sphere located in a slowly varying uniform electric field background are computed to first-order in a power series expansion in the field frequency. These results are used to define an equivalent RC circuit and to test the circuit approach which is often used in electromagnetic compatibility (EMC). The case of an infinite cylindrical conducting tube under the influence of the same external field is also analyzed. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

The knowledge of the penetration of the electric and magnetic fields in electronic equipment is important to properly protect these ever increasingly sensitive devices from external influences. In fact, the shielding of a receptor set from a source of electrical disturbance is an interesting subject of research in electromagnetic compatibility (EMC), which is defined by IEEE as "the ability of a device, equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment."[1]

The physicist's approach to evaluating the electromagnetic shielding is based upon the solution of Maxwell's equations with appropriate boundary conditions on the shielding surfaces, but the mathematical machinery is so complex that, even when the calculations can be carried out, the physical insight is often missed.[2–4] As a consequence, from an engineering point of view, to estimate in practice the electromagnetic field inside the shielding enclosure, it is always necessary to use a simplified theory of electromagnetic shielding.

Among the techniques developed so far in EMC to deal with this kind of calculation we will consider here the so-called "circuit approach" in which the actual physical system is replaced by an equivalent RC circuit. This approach is based upon the fact that the external electromagnetic field will induce on the shielding enclosure a charge distribution which will vary in time because the external field is oscillating. This will produce a current flow in the conductor and it seems rather natural to substitute for the conductor an equivalent electric circuit whose characteristics are defined on heuristic grounds because in general they cannot be computed accurately, not even by numerical simulation.[4]

The main goal of this paper is to analyze a couple of simple but interesting examples in which explicit (although approximate) expressions for these phenomena may be easily computed. In this way we can illustrate and compare the

