text. Zwiebach's book meets this need in a clear and accessible manner. It is well-grounded in familiar physical concepts, and proceeds through some of the most timely and exciting aspects of the subject.'

Professor Joseph Polchinski, University of California, Santa Barbara

'Zwiebach, a respected researcher in the field and a much beloved teacher at MIT, is truly faithful to his goal of making string theory accessible to advanced undergraduates – the test develops intuition before formalism, usually through simplified and illustrative examples ... Zwiebach avoids the temptation of including topics that would weigh the book down and make many students rush it back to the shelf and quit the course.'

Marcelo Gleiser, Physics Today

'... well-written ... takes us through the hottest topics in string theory research, requiring only a solid background in mechanics and some basic quantum mechanics ... This is not just one more text in the ever-growing canon of popular books on string theory ... '

Andreas Karch, Times Higher Education Supplement

'... the book provides an excellent basis for an introductory course on string theory and is well-suited for self-study by graduate students or any physicist who wants to learn the basics of string theory'.

Zentralblatt MATH

'... excellent introduction by Zwiebach ... aimed at advanced undergraduates who have some background in quantum mechanics and special relativity, but have not necessarily mastered quantum field theory and general relativity yet ... the book ... is a very thorough introduction to the subject ... Equipped with this background, the reader can safely start to tackle the books by Green, Schwarz and Witten and by Polchinski.'

Marcel L. Vonk, Mathematical Reviews Clippings

Cover illustration: a composite illustrating open string motion as we vary the strength of an electric field that points along the rotational axis of symmetry. There are three surfaces, each composed of two lobes joined at the origin and shown with the same color. Each surface is traced by a rotating open string that, at various times, appears as a line stretching from the boundary of a lobe down to the origin and then out to the boundary of the opposite lobe. The inner, middle, and elongated lobes arise as the magnitude of the electric field is increased. For further details, see Problem 19.2.

A First Course in String Theory

Second Edition

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1 A brief introduction

Here we meet string theory for the first time. We see how it fits into the historical development of physics, and how it aims to provide a unified description of all fundamental interactions.

1.1 The road to unification

Over the course of time, the development of physics has been marked by unifications: events when different phenomena were recognized to be related and theories were adjusted to reflect such recognition. One of the most significant of these unifications occurred in the nineteenth century.

For a while, electricity and magnetism had appeared to be unrelated physical phenomena. Electricity was studied first. The remarkable experiments of Henry Cavendish were performed in the period from 1771 to 1773. They were followed by the investigations of Charles Augustin de Coulomb, which were completed in 1785. These works provided a theory of static electricity, or electrostatics. Subsequent research into magnetism, however, began to reveal connections with electricity. In 1819 Hans Christian Oersted discovered that the electric current on a wire can deflect the needle of a compass placed nearby. Shortly thereafter, Jean-Baptiste Biot and Felix Savart (1820) and André-Marie Ampère (1820–1825) established the rules by which electric currents produce magnetic fields. A crucial step was taken by Michael Faraday (1831), who showed that changing magnetic fields generate electric fields. Equations that described all of these results became available, but they were, in fact, inconsistent. It was James Clerk Maxwell (1865) who constructed a consistent set of equations by adding a new term to one of the equations. Not only did this term remove the inconsistencies, but it also resulted in the prediction of electromagnetic waves. For this great insight, the equations of *electromagnetism* (or electrodynamics) are now called "Maxwell's equations." These equations unify electricity and magnetism into a consistent whole. This elegant and aesthetically pleasing unification was not optional. Separate theories of electricity and magnetism would be inconsistent.

Another fundamental unification of two types of phenomena occurred in the late 1960s, about one-hundred years after the work of Maxwell. This unification revealed the deep relationship between electromagnetic forces and the forces responsible for weak interactions. To appreciate the significance of this unification it is necessary first to review the main developments that occurred in physics since the time of Maxwell.

An important change of paradigm was triggered by Albert Einstein's special theory of relativity. In this theory one finds a striking conceptual unification of the separate notions of space and time. Different from a unification of forces, the merging of space and time into a spacetime continuum represented a new recognition of the nature of the *arena* where physical phenomena take place. Newtonian mechanics was replaced by relativistic mechanics, and older ideas of absolute time were abandoned. Mass and energy were shown to be interchangeable.

Another change of paradigm, perhaps an even more dramatic one, was brought forth by the discovery of quantum mechanics. Developed by Erwin Schrödinger, Werner Heisenberg, Paul Dirac and others, quantum theory was verified to be the correct framework to describe microscopic phenomena. In quantum mechanics classical observables become operators. If two operators fail to commute, the corresponding observables cannot be measured simultaneously. Quantum mechanics is a framework, more than a theory. It gives the rules by which theories must be used to extract physical predictions.

In addition to these developments, four fundamental forces had been recognized to exist in nature. Let us have a brief look at them.

One of them is the force of gravity. This force has been known since antiquity, but it was first described accurately by Isaac Newton. Gravity underwent a profound reformulation in Albert Einstein's theory of general relativity. In this theory, the spacetime arena of special relativity acquires a life of its own, and gravitational forces arise from the curvature of this dynamical spacetime. Einstein's general relativity is a classical theory of gravitation. It is not formulated as a quantum theory.

The second fundamental force is the electromagnetic force. As we discussed above, the electromagnetic force is well described by Maxwell's equations. Electromagnetism, or Maxwell theory, is formulated as a classical theory of electromagnetic fields. As opposed to Newtonian mechanics, which was modified by special relativity, Maxwell theory is fully consistent with special relativity.

The third fundamental force is the weak force. This force is responsible for the process of nuclear beta decay, in which a neutron decays into a proton, an electron, and an antineutrino. In general, processes that involve neutrinos are mediated by weak forces. While nuclear beta decay had been known since the end of the nineteenth century, the recognition that a new force was at play did not take hold until the middle of the twentieth century. The strength of this force is measured by the Fermi constant. Weak interactions are much weaker than electromagnetic interactions.

Finally, the fourth force is the strong force, nowadays called the color force. This force is at play in holding together the constituents of the neutron, the proton, the pions, and many other subnuclear particles. These constituents, called quarks, are held so tightly by the color force that they cannot be seen in isolation.

We are now in a position to return to the subject of unification. In the late 1960s the Weinberg–Salam model of *electroweak* interactions put together electromagnetism and the weak force into a unified framework. This unified model was neither dictated nor justified only by considerations of simplicity or elegance. It was necessary for a predictive and consistent theory of the weak interactions. The theory is initially formulated with four massless particles that carry the forces. A process of symmetry breaking gives mass to three of these particles: the W^+, the W^-, and the Z^0. These particles are the carriers of the weak force. The particle that remains massless is the photon, which is the carrier of the electromagnetic force.

Maxwell's equations, as we discussed before, are equations of classical electromagnetism. They do not provide a quantum theory. Physicists have discovered quantization methods, which can be used to turn a classical theory into a quantum theory – a theory that can be calculated using the principles of quantum mechanics. While classical electrodynamics can be used confidently to calculate the transmission of energy in power lines and the radiation patterns of radio antennas, it is neither an accurate nor a correct theory for microscopic phenomena. Quantum electrodynamics (QED), the quantum version of classical electrodynamics, is required for correct computations in this arena. In QED, the photon appears as the quantum of the electromagnetic field. The theory of weak interactions is also a quantum theory of particles, so the correct, unified theory is the quantum electroweak theory.

The quantization procedure is also successful in the case of the strong color force, and the resulting theory has been called quantum chromodynamics (QCD). The carriers of the color force are eight massless particles. These are colored gluons, and just like the quarks, they cannot be observed in isolation. The quarks respond to the gluons because they carry color. Quarks can come in three colors.

The electroweak theory together with QCD form the Standard Model of particle physics. In the Standard Model there is some interplay between the electroweak sector and the QCD sector because some particles feel both types of forces. But there is no real and deep unification of the weak force and the color force. The Standard Model summarizes completely the present knowledge of particle physics. So, in fact, we are not certain about any possible further unification.

In the Standard Model there are twelve force carriers: the eight gluons, the W^+, the W^-, the Z^0, and the photon. All of these are bosons. There are also many matter particles, all of which are fermions. The matter particles are of two types: leptons and quarks. The leptons include the electron e^-, the muon μ^-, the tau τ^-, and the associated neutrinos ν_e, ν_μ, and ν_τ. We can list them as

$$\text{leptons}: \quad e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau .$$

Since we must include their antiparticles, this adds up to a total of twelve leptons. The quarks carry color charge, electric charge, and can respond to the weak force as well. There are six different types of quarks. Poetically called flavors, these types are: up (u), down (d), charm (c), strange (s), top (t), and bottom (b). We can list them as

$$\text{quarks}: \quad u, d, c, s, t, b .$$

The u and d quarks, for example, carry different electric charges and respond differently to the weak force. Each of the six quark flavors listed above comes in three colors, so this gives $6 \times 3 = 18$ particles. Including the antiparticles, we get a total of 36 quarks. Adding leptons and quarks together we have a grand total of 48 matter particles. Adding matter particles and force carriers together we have a total of 60 particles in the Standard Model.

Despite the large number of particles it describes, the Standard Model is reasonably elegant and very powerful. As a complete theory of physics, however, it has two significant shortcomings. The first one is that it does not include gravity. The second one is that it has about twenty parameters that cannot be calculated within its framework. Perhaps the simplest example of such a parameter is the dimensionless (or unit-less) ratio of the mass of the muon to the mass of the electron. The value of this ratio is about 207, and it must be put into the model by hand.

Most physicists believe that the Standard Model is only a step towards the formulation of a complete theory of physics. A large number of physicists also suspect that some unification of the electroweak and strong forces into a Grand Unified Theory (GUT) will prove to be correct. At present, however, the unification of these two forces appears to be optional.

Another attractive possibility is that a more complete version of the Standard Model includes supersymmetry. Supersymmetry is a symmetry that relates bosons to fermions. Since all matter particles are fermions and all force carriers are bosons, this remarkable symmetry unifies matter and forces. In a theory with supersymmetry, bosons and fermions appear in pairs of equal mass. The particles of the Standard Model do not have this property, so supersymmetry, if it exists in nature, must be spontaneously broken. Supersymmetry is such an appealing symmetry that many physicists believe that it will eventually be discovered.

While the above extensions of the Standard Model may or may not occur, it is clear that the inclusion of gravity into the particle physics framework is not optional. Gravity must be included, with or without unification, if one is to have a complete theory. The effects of the gravitational force are presently quite negligible at the microscopic level, but they are crucial in studies of cosmology of the early universe.

There is, however, a major problem when one attempts to incorporate gravitational physics into the Standard Model. The Standard Model is a quantum theory, while Einstein's general relativity is a classical theory. It seems very difficult, if not altogether impossible, to have a consistent theory that is partly quantum and partly classical. Given the successes of quantum theory, it is widely believed that gravity must be turned into a quantum theory. The procedures of quantization, however, encounter profound difficulties in the case of gravity. The resulting theory of quantum gravity appears to be ill-defined. As a practical matter, in many circumstances one can work confidently with classical gravity coupled to the Standard Model. For example, this is done routinely in present-day descriptions of the universe. A theory of quantum gravity is necessary, however, to study physics at times very near to the Big Bang, and to study certain properties of black holes. Formulating a quantum theory that includes both gravity and the other forces seems fundamentally necessary. A *unification* of gravity with the other forces might be required to construct this complete theory.

1.2 String theory as a unified theory of physics

String theory is an excellent candidate for a unified theory of all forces in nature. It is also a rather impressive prototype of a complete theory of physics. In string theory all forces are truly unified in a deep and significant way. In fact, all the particles are unified. String theory is a quantum theory, and, because it includes gravitation, it is a quantum theory of gravity. Viewed from this perspective, and recalling the failure of Einstein's gravity to yield a quantum theory, one may conclude that in string theory all other interactions are necessary for the consistency of the quantum gravitational sector! While it may be difficult to measure the effects of quantum gravity directly, a theory of quantum gravity such as string theory may have testable predictions concerning the other interactions.

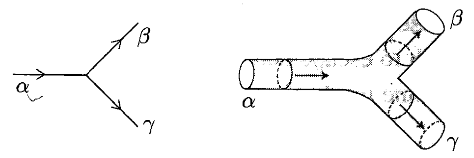

Fig. 1.1 The decay $\alpha \to \beta + \gamma$ as a particle process (left) and as a string process (right).

Why is string theory a truly unified theory? The reason is simple and goes to the heart of the theory. In string theory, each particle is identified as a particular vibrational mode of an elementary microscopic string. A musical analogy is very apt. Just as a violin string can vibrate in different modes and each mode corresponds to a different sound, the modes of vibration of a fundamental string can be recognized as the different particles we know. One of the vibrational states of strings is the graviton, the quantum of the gravitational field. Since there is just one type of string, and all particles arise from string vibrations, all particles are naturally incorporated into a single theory. When we think in string theory of a decay process $\alpha \to \beta + \gamma$, where an elementary particle α decays into particles β and γ, we imagine a single string vibrating in such a way that it is identified as particle α that breaks into two strings that vibrate in ways that identify them as particles β and γ (Figure 1.1). Since strings may turn out to be extremely tiny, it may be difficult to observe directly the string-like nature of particles.

Are we sure that string theory is a good quantum theory of gravity? There is no complete certainty yet, but the evidence is very good. Indeed, the problems that occur when one tries to quantize Einstein's theory do not seem to appear in string theory.

For a theory as ambitious as string theory, a certain degree of uniqueness is clearly desirable. It would be somewhat disappointing to have several consistent candidates for a theory of all interactions. The first sign that string theory is rather unique is that it does not have adjustable dimensionless parameters. As we mentioned before, the Standard Model of particle physics has about twenty parameters that must be adjusted to some precise values. A theory with adjustable dimensionless parameters is not really unique. When the parameters are set to different values one obtains different theories with potentially different predictions. String theory has one dimensionful parameter, the string length ℓ_s. Its value can be roughly imagined as the typical size of strings.

Another intriguing sign of the uniqueness of string theory is the fact that the dimensionality of spacetime is fixed. Our physical spacetime is four-dimensional, with one time dimension and three space dimensions. In the Standard Model this information is used to build the theory, it is not derived. In string theory, on the other hand, the number of spacetime dimensions emerges from a calculation. The answer is not four, but rather ten.

Some of these dimensions may hide from plain view if they curl up into a space that is small enough to escape detection in experiments done with low energies. If string theory is correct, some mechanism must ensure that the observable dimensionality of spacetime is four.

The lack of adjustable dimensionless parameters is a sign of the uniqueness of string theory: it means that the theory cannot be deformed or changed continuously by changing these parameters. But there could be other theories that cannot be reached by continuous deformations. So how many string theories are there?

Let us begin by noting two broad subdivisions. There are open strings and there are closed strings. Open strings have two endpoints, while closed strings have no endpoints. One can consider theories with only closed strings and theories with both open and closed strings. Since open strings generally can close to form closed strings, we do not consider theories with only open strings. The second subdivision is between bosonic string theories and superstring theories. Bosonic strings live in 26 dimensions, and all of their vibrations represent bosons. Since they lack fermions, bosonic string theories are not realistic. They are, however, much simpler than the superstrings, and most of the important concepts in string theory can be explained in the context of bosonic strings. The superstrings live in ten-dimensional spacetime, and their spectrum of states includes bosons *and* fermions. In fact, these two sets of particles are related by supersymmetry. Supersymmetry is therefore an important ingredient in string theory. All realistic models of string theory are built from superstrings. In all string theories the graviton appears as a vibrational mode of closed strings. In string theory gravity is unavoidable.

By the mid 1980s five ten-dimensional superstring theories were known to exist. In the years that followed, many interrelations between these theories were found. Moreover, another theory was discovered by taking a certain strong coupling limit of one of the superstrings. This theory is eleven-dimensional and has been dubbed M-theory, for lack of a better name. It has now become clear that the five superstrings and M-theory are only facets or different limits of a *single* unique theory! At present, this unique theory remains fairly mysterious. It is not yet clear whether or not the set of bosonic string theories is connected to the web of superstring theories.

All in all, we see that string theory is a truly unified and possibly unique theory. It is a candidate for a unified theory of physics, a theory Albert Einstein tried to find ever since his discovery of general relativity. Einstein would have been surprised, or perhaps disturbed, by the prominent role that quantum mechanics plays in string theory. But string theory appears to be a worthy successor of general relativity. It is almost certain that string theory will give rise to a new conception of spacetime. The prominence of quantum mechanics in string theory would not have surprised Paul Dirac. His writings on quantization suggest that he felt that deep quantum theories arise from the quantization of classical physics. This is precisely what happens in string theory. This book will explain in detail how string theory, at least in its simplest form, is nothing but the quantum mechanics of classical relativistic strings.

1.3 String theory and its verification

It should be said at the outset that, as of yet, there has been no experimental verification of string theory. In order to have experimental verification one needs a sharp prediction. It has been difficult to obtain such a prediction. String theory is still at an early stage of development, and it is not so easy to make predictions with a theory that is not well understood. Still, some interesting possibilities have emerged.

As we mentioned earlier, superstring theory requires a ten-dimensional spacetime: one dimension of time and nine of space. If string theory is correct, extra spatial dimensions must exist, even if we have not seen them yet. Can we test the existence of these extra dimensions? If the extra dimensions are the size of the Planck length ℓ_P (the length scale associated with four-dimensional gravity), they will remain beyond direct detection, perhaps forever. Indeed, $\ell_P \sim 10^{-33}$ cm, and this distance is many orders of magnitude smaller than 10^{-16} cm, which is roughly the smallest distance that has been explored with particle accelerators. This scenario was deemed to be most likely. It was assumed that in string theory the length scale ℓ_s coincides with the Planck length, in which case extra dimensions would be of Planck length, as well.

It turns out, however, that string theory allows extra dimensions that are as large as a tenth of a millimeter! Surprisingly, extra dimensions that large may have gone undetected. To make this work out, the string length ℓ_s is taken to be of the order of 10^{-18} cm. Moreover, our three-dimensional space emerges as a hypersurface embedded inside the nine-dimensional space. The hypersurface, or higher-dimensional membrane, is called a D-brane. D-branes are real, physical objects in string theory. In this setup, the presence of large extra dimensions is tested by gravitational experiments. Extra dimensions much larger than ℓ_P but still very small may be detected with particle accelerators. If extra dimensions are detected, this would be strong evidence for string theory. We discuss the subject of large extra dimensions in Chapter 3.

A striking confirmation of string theory may result from the discovery of a cosmic string. Left-over from early universe processes, a cosmic string can stretch across the observable universe and may be detected via gravitational lensing or, more indirectly, through the detection of gravitational waves. No cosmic strings have been detected to date, but the searches have not been exhaustive and they continue. If found, a cosmic string must be studied in detail to confirm that it is a string from string theory and not the kind of string that can arise from conventional theories of particle physics. We discuss the subject of cosmic strings in Chapter 7.

Another interesting possibility has to do with supersymmetry. If we start with a ten-dimensional superstring theory and compactify the six extra dimensions, the resulting four-dimensional theory is, in many cases, supersymmetric. No unique predictions have emerged for the specific details of the four-dimensional theory, but supersymmetry may be a rather generic feature. An experimental discovery of supersymmetry in future accelerators would suggest very strongly that string theory is on the right track.

Leaving aside predictions of new phenomena, we must ask whether the Standard Model emerges from string theory. It should, since string theory is supposed to be a unified theory of all interactions, and it must therefore reduce to the Standard Model for sufficiently low energies. While string theory certainly has room to include all known particles and interactions, and this is very good news indeed, no one has yet been able to show that they actually emerge in fine detail. In Chapter 21 we will study some models which use D-branes and have an uncanny resemblance to the world as we know it. In these models the particle content is in fact *precisely* that of the Standard Model (the particles are obtained with zero mass, however, and it is not clear whether the process that gives them mass can work out correctly). Our four-dimensional world is part of the D-branes, but these D-branes happen to have more than three spatial directions. The additional D-brane dimensions are wrapped on the compact space (we will learn how to imagine such configurations!). The gauge bosons and the matter particles in the model arise from vibrations of open strings that stretch between D-branes. As we will learn, the endpoints of open strings must remain attached to the D-branes. If you wish, the musical analogy for strings is improved. Just as the strings of a violin are held stretched by pegs, the D-branes hold fixed the endpoints of the open strings whose lowest vibrational modes could represent the particles of the Standard Model!

String theory shares with Einstein's gravity a problematic feature. Einstein's equations of gravitation admit many cosmological solutions. Each solution represents a consistent universe, but only one of them represents *our* observable universe. It is not easy to explain what selects the physical solution, but in cosmology this is done using arguments based on initial conditions, symmetry, and simplicity. The smaller the number of solutions a theory has, the more predictive it is. If the set of solutions is characterized by continuous parameters, selecting a solution is equivalent to adjusting the values of the parameters. In this way, a theory whose formulation requires no adjustable parameters may generate adjustable parameters through its solutions! It seems clear that in string theory the set of solutions (string models) is characterized by both discrete and continuous parameters.

In order to reproduce the Standard Model it seems clear that the string model must not have continuous parameters; such parameters imply the existence of massless fields that have not been observed. It was not easy to find models without continuous parameters, but that became possible recently in the context of flux compactifications; models in which the extra dimensions are threaded by analogs of electric and magnetic fields. There is an extraordinary large number of such models, certainly more than 10^{500} of them. There may be even more models that manage to avoid continuous parameters by other means. Physicists speak of a vast *landscape* of string solutions or models.

In this light we can wonder what are the possible outcomes of the search for a realistic string model. One possible outcome (the worst one) is that no string model in the landscape reproduces the Standard Model. This would rule out string theory. Another possible outcome (the best one) is that one string model reproduces the Standard Model. Moreover, the model represents a well-isolated point in the landscape. The parameters of the Standard Model are thus predicted. The landscape may be so large that a strange possibility emerges: many string models with almost identical properties all of which are consistent with the Standard Model to the accuracy that it is presently known. In this possibility there is a loss of predictive power. Other outcomes may be possible.

String theorists sometimes say that string theory has already made at least one successful *prediction*: it predicted gravity! (I heard this from John Schwarz.) There is a bit of jest in saying so – after all, gravity is the oldest known force in nature. I believe, however, that there is a very substantial point to be made here. String theory is the quantum mechanics of a relativistic string. In no sense whatsoever is gravity put into string theory by hand. It is a complete surprise that gravity emerges in string theory. Indeed, none of the vibrations of the *classical* relativistic string correspond to the particle of gravity. It is a truly remarkable fact that we find the particle of gravity among the *quantum* vibrations of the relativistic string. You will see in detail how this happens as you progress through this book. The striking quantum emergence of gravitation in string theory has the full flavor of a prediction.

1.4 Developments and outlook

String theory has been a very stimulating and active area of research ever since Michael Green and John Schwarz showed in 1984 that superstrings are not afflicted with fatal inconsistencies that threaten similar particle theories in ten dimensions. Much progress has been made since then.

String theory has provided new and powerful tools for the understanding of conventional particle physics theories, gauge theories in particular. These are the kinds of theories that are used to formulate the Standard Model. Close cousins of these gauge theories arise on string theory D-branes. We examine D-branes and the theories that arise on them in detail beginning in Chapter 15. A remarkable physical equivalence between a certain four-dimensional gauge theory and a closed superstring theory (the AdS/CFT correspondence) is discussed in Chapter 23. As we will explain, the correspondence has been used to understand hydrodynamical properties of the quark-gluon plasma created in the collision of gold nuclei at heavy ion colliders.

String theory has also made good strides towards a statistical mechanics interpretation of black hole entropy. We know from the pioneering work of Jacob Bekenstein and Stephen Hawking that black holes have both entropy and temperature. In statistical mechanics these properties arise if a system can be constructed in many degenerate ways using its basic constituents. Such an interpretation is not available in Einstein's gravitation, where black holes seem to have few, if any, constituents. In string theory, however, certain black holes can be built by assembling together various types of D-branes and strings in a controlled manner. For such black holes, the predicted Bekenstein entropy is obtained by counting the ways in which they can be built with their constituent D-branes and strings. In fact, the class of black holes amenable to string theory analysis continues to grow. We discuss this important development in Chapter 22.

String theory will be needed to study cosmology of the Very Early Universe. String theory may provide a concrete model for the realization of inflation – a period of dramatic exponential expansion that the universe is likely to have experienced at the earliest times. The theory of inflation suggests that our universe is a growing bubble or region inside a space that continues to inflate for eternity. Bubbles continue to emerge forever and some have remarked that every model in the landscape may be physically realized in some bubble. Inflation does not appear to be eternal in the past, so some kind of beginning seems necessary. The deepest mysteries of the universe seem to lie hidden in a regime where classical general relativity surely breaks down. String theory should allow us to peer into this unknown realm. Some day we may be able to understand how the universe comes into being, if it does, or how the universe could have existed forever in the past, if it did.

Most likely, answering such questions will require a mastery of string theory that goes beyond our present abilities. String theory is in fact an unfinished theory. Much has been learned about it, but in reality we have no complete formulation of the theory. A comparison with Einstein's theory is illuminating. Einstein's equations for general relativity are elegant and geometrical. They embody the conceptual foundation of the theory and feel completely up to the task of describing gravitation. No similar equations are known for string theory, and the conceptual foundation of the theory remains largely unknown. String theory is an exciting research area because the central ideas remain to be found.

Describing nature and formulating the theory – those remain the present-day challenges of string theory. If surmounted, we will have a theory of all interactions, allowing us to understand the fate of spacetime and the mysteries of a quantum mechanical universe. With such high stakes, physicists are likely to investigate string theory until definite answers are found.

2 Special relativity and extra dimensions

The word relativistic, as used in the term "relativistic strings," indicates consistency with Einstein's theory of special relativity. We review special relativity and introduce the light-cone frame, light-cone coordinates, and light-cone energy. We then turn to the idea of additional, compact space dimensions and show with an example from quantum mechanics that, if small, these dimensions have little effect at low energies.

2.1 Units and parameters

Units are nothing other than fixed quantities that we use for purposes of reference. A measurement involves finding the unit-free ratio of an observable quantity to the appropriate unit. Consider, for example, the definition of a second in the international system of units (SI system). The SI second (s) is defined to be the duration of 9 192 631 770 periods of the radiation emitted in the transition between the two hyperfine levels of the cesium-133 atom. When we measure the time elapsed between two events, we are really counting a unit-free, or dimensionless, number: the number that tells us how many seconds fit between the two events or, alternatively, how many periods of the cesium radiation fit between the two events. The same goes for length. The unit called the meter (m) is nowadays defined as the distance traveled by light in a certain fraction of a second (1/299 792 458 of a second, to be precise). Mass introduces a third unit, the prototype kilogram (kg), kept safely in Sèvres, France.

When doing dimensional analysis, we denote the units of length, time, and mass by L, T, and M, respectively. These are called the three basic units. A force, for example, has units

$$[F] = MLT^{-2}, \tag{2.1}$$

where $[X]$ denotes the units of the quantity X. Equation (2.1) follows from Newton's law that equates the force on an object to the product of its mass and its acceleration. The newton (N) is the SI unit of force, and it equals kg·m/s^2.

It is interesting that no additional basic units are needed to describe other quantities. Consider, for example, electric charge. Do we need a new unit to describe charge? Not