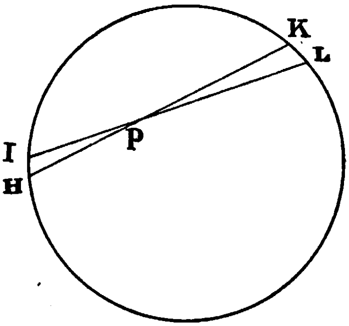
SECTION XII

The attractive forces of spherical bodies.

PROPOSITION LXX. THEOREM XXX

If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from those points, I say, that a corpuscle placed within that surface will not be attracted by those forces any way.

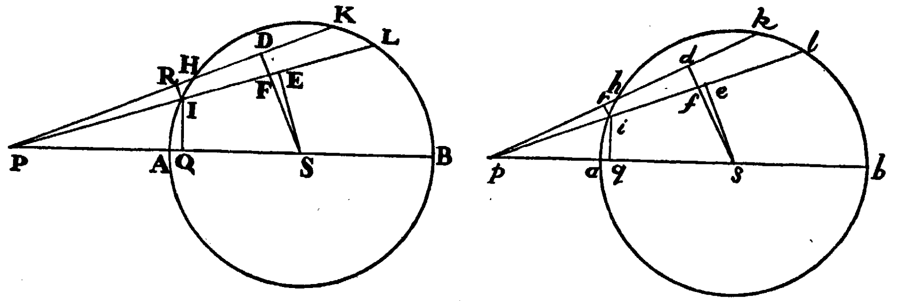
Let HIKL be that spherical surface, and P a corpuscle placed within. Through P let there be drawn to this surface two lines HK, IL, intercepting very small arcs HI, KL; and because (by Cor. III, Lem. VII) the triangles HPI, LPK are alike, those arcs will be proportional to the distances HP, LP; and any particles at HI and KL of the spherical surface, terminated by right lines passing through P, will be as the square of those distances. Therefore the forces of these particles exerted upon the body P are equal between themselves. For the forces are directly as the particles, and inversely as the square of the distances. And these two ratios compose the ratio of equality, 1 : 1. The attractions therefore, being equal, but exerted in opposite directions, destroy each other. And by a like reasoning all the attractions through the whole spherical surface are destroyed by contrary attractions. Therefore the body P will not be any way impelled by those attractions. Q.E.D.



PROPOSITION LXXI. THEOREM XXXI

The same things supposed as above, I say, that a corpuscle placed without the spherical surface is attracted towards the centre of the sphere with a force inversely proportional to the square of its distance from that centre.

Let AHKB, *ahkb* be two equal spherical surfaces described about the centres S, *s;* their diameters AB, *ab;* and let P and *p* be two corpuscles situate without the spheres in those diameters produced. Let there be drawn from the corpuscles the lines PHK, PIL, *phk*, *pil*, cutting off from the great circles AHB, *ahb*, the equal arcs HK, *hk*, IL, *il*; and to those lines let fall the perpendiculars SD, *sd*, SE, *se*, IR, *ir*; of which let SD, *sd*, cut PL, *pl*, in F and *f*.



Let fall also to the diameters the perpendiculars IQ, *iq*. Let now the angles DPE, *dpe* vanish; and because DS and *ds*, ES and *es* are equal, the lines PE, PF, and *pe*, *pf*, and the short lines DF, *df* may be taken for equal; because their last ratio, when the angles DPE, *dpe* vanish together, is the ratio of equality. These things being thus determined, it follows that

$$\mathrm{PI} : \mathrm{PF} = \mathrm{RI} : \mathrm{DF}$$

and $\quad pf : pi = df$ or $\mathrm{DF} : ri$.

Multiplying corresponding terms,

$$\mathrm{PI} \cdot pf : \mathrm{PF} \cdot pi = \mathrm{RI} : ri = \text{arc IH} : \text{arc } ih \text{ (by Cor. III, Lem. VII)}.$$

Again, $\quad \mathrm{PI} : \mathrm{PS} = \mathrm{IQ} : \mathrm{SE}$

and $\quad ps : pi = se$ or $\mathrm{SE} : iq$.

Hence, $\quad \mathrm{PI} \cdot ps : \mathrm{PS} \cdot pi = \mathrm{IQ} : iq$.

Multiplying together corresponding terms of this and the similarly derived preceding proportion,

$$\mathrm{PI}^2 \cdot pf \cdot ps : pi^2 \cdot \mathrm{PF} \cdot \mathrm{PS} = \mathrm{HI} \cdot \mathrm{IQ} : ih \cdot iq,$$

that is, as the circular surface which is described by the arc IH, as the semicircle AKB revolves about the diameter AB, is to the circular surface described by the arc *ih* as the semicircle *akb* revolves about the diameter *ab*. And the forces with which these surfaces attract the corpuscles P and *p* in the direction of lines tending to those surfaces are directly, by the hypothesis, as the surfaces themselves, and inversely as the squares of the distances of the surfaces from those corpuscles; that is, as $pf \cdot ps$ to $\mathrm{PF} \cdot \mathrm{PS}$. And these forces again are to the oblique parts of them which (by the resolution of forces as in Cor. II of the Laws) tend to the centres in the directions of the lines PS, *ps*, as PI to PQ, and *pi* to *pq*; that is (because of the like triangles PIQ and PSF, *piq* and *psf*), as PS to PF and *ps* to *pf*. Thence, the attraction of the corpuscle P towards S is to the attraction of the corpuscle *p* towards *s* as $\frac{\text{PF} \cdot pf \cdot ps}{\text{PS}}$ is to $\frac{pf \cdot \text{PF} \cdot \text{PS}}{ps}$, that is, as ps^2 to PS^2. And, by a like reasoning, the forces with which the surfaces described by the revolution of the arcs KL, *kl* attract those corpuscles, will be as ps^2 to PS^2. And in the same ratio will be the forces of all the circular surfaces into which each of the spherical surfaces may be divided by taking *sd* always equal to SD, and *se* equal to SE. And therefore, by composition, the forces of the entire spherical surfaces exerted upon those corpuscles will be in the same ratio. Q.E.D.

PROPOSITION LXXII. THEOREM XXXII

If to the several points of a sphere there tend equal centripetal forces decreasing as the square of the distances from those points; and there be given both the density of the sphere and the ratio of the diameter of the sphere to the distance of the corpuscle from its centre: I say, that the force with which the corpuscle is attracted is proportional to the semidiameter of the sphere.

For conceive two corpuscles to be severally attracted by two spheres, one by one, the other by the other, and their distances from the centres of the spheres to be proportional to the diameters of the spheres respectively; and the spheres to be resolved into like particles, disposed in a like situation to the corpuscles. Then the attractions of one corpuscle towards the several particles of one sphere will be to the attractions of the other towards as many analogous particles of the other sphere in a ratio compounded of the ratio of the particles directly, and the square of the distances inversely. But the particles are as the spheres, that is, as the cubes of the diameters, and the distances are as the diameters; and the first ratio directly with the last ratio taken twice inversely, becomes the ratio of diameter to diameter. Q.E.D.

COR. I. Hence if corpuscles revolve in circles about spheres composed of matter equally attracting, and the distances from the centres of the spheres be proportional to their diameters, the periodic times will be equal.