Problem The relativistic factor γ arises in nonrelativistic situations as well. For example, consider a river flowing with speed v, and a swimmer able to swim at speed c relative to the water. (a) Calculate the time t_u it takes the swimmer to swim a distance d upstream and back, where d is the distance measured relative to the stationary river bank. (b) Calculate the time t_a it takes the swimmer to swim a distance d directly across the river and back (perpendicular to the river bank). (c) Show that the ratio of the two times (t_u/t_a) is equal to γ.

Solution The Galilean transformation is needed here, which states that

$$\vec{v}_{SG} = \vec{v}_{SW} + \vec{v}_{WG},$$

or in words, "the velocity of the Swimmer relative to the Ground is equal to the velocity of the Swimmer relative to the Water plus the velocity of the Water relative to the Ground." Note that this is a vector equation, so the magnitudes don't necessarily add. In this problem, let's let $\vec{v}_{WG} = -v\hat{y}$ and $|\vec{v}_{SW}| = c$.

(a) While swimming upstream, the swimmer's speed relative to the ground is reduced $\vec{v}_{SG} = (+c - v)\hat{y}$ so that

$$\Delta t_u = \frac{d}{c - v}.$$

Similarly, swimming downstream the swimmer goes faster $\vec{v}_{SG} = (-c - v)\hat{y}$ and $\Delta t_d = d/(c + v)$. The total time taken is

$$\Delta t = \Delta t_u + \Delta t_d = \frac{d}{c - v} + \frac{d}{c + v} = \frac{2cd}{c^2 - v^2}.$$

(b) If the swimmer wants to move *directly* across the river, they must angle slightly upstream so they don't drift downstream. In this case, c is the hypotenuse of the right triangle, v is one side, and therefore $|\vec{v}_{SG}| = \sqrt{c^2 - v^2}$ is the speed of the swimmer relative to the ground. The time taken to swim across the river is the same as that to swim back (same speed), so that the total time taken is

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}}.$$

(c) The ratio shows that it is quicker to swim across and back

$$\frac{\Delta t_{up}}{\Delta t_{across}} = \frac{c}{c^2 - v^2}\sqrt{c^2 - v^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \geq 1.$$

This, in fact, is exactly the analysis needed to interpret the Michelson-Morley experiment. The "swimmer" in that case is light, and the "river" is the ether. Michelson and Morley measured the two travel times and tried to detect a difference, which would have allowed them to determine the speed of the Earth relative to the ether. However, since their result was that the two times were identical, Lorentz proposed that objects (i.e., their measuring apparatus) *contracted* in length by a factor γ in the direction of motion. This *ad hoc* proposal would result in the two travel times being identical. Of course, there is a "Lorentz contraction," but for reasons having to do with observers in different reference frames (i.e., special relativity), rather than an actual contraction of objects.