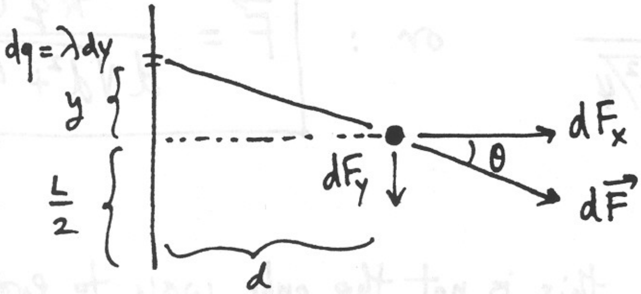
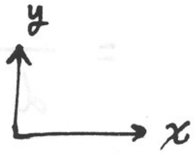
Find the force on a point charge q, a distance d away from the center of a rod of length L and total charge Q.

$\left(\lambda = \frac{Q}{L}\right)$





By symmetry, $\hat{j}$ components cancel, but $\hat{i}$ components add.

$$F_x = \int dF_x = \int dF \cos\theta \qquad \text{where}$$

$$\cos\theta = \frac{d}{\sqrt{d^2+y^2}} \quad \text{and} \quad dF = kq\,\frac{\lambda\,dy}{d^2+y^2}$$

$$F_x = kq\lambda d \int_{-L/2}^{+L/2} dy\,\frac{1}{(d^2+y^2)^{3/2}} \qquad \text{let } y = d\tan\alpha, \quad dy = d\sec^2\alpha\,d\alpha$$

$$= \frac{kq\lambda}{d}\int_{-\alpha_0}^{\alpha_0} \frac{\sec^2\alpha}{(1+\tan^2\alpha)^{3/2}}\,d\alpha \qquad 1+\tan^2\alpha = \sec^2\alpha, \quad \alpha_0 = \tan^{-1}(L/2d)$$

$$= \frac{kq\lambda}{d}\int_{-\alpha_0}^{\alpha_0} \cos\alpha\,d\alpha = \frac{2kq\lambda}{d}\sin\alpha_0$$

Use trigonometry to show that if $\tan\alpha_0 = L/2d$

then $\sin\alpha_0 = \dfrac{(L/2d)}{\sqrt{1+(L/2d)^2}}$

The final answer is

$$F_x = \frac{2kq\lambda}{d} \cdot \frac{L}{2d} \cdot \frac{1}{\sqrt{1+(L/2d)^2}}$$

$$= \frac{kqQ}{d\sqrt{d^2+L^2/4}} \qquad \text{or :} \quad \boxed{\vec{F} = \frac{kqQ}{d\sqrt{d^2+L^2/4}}\,\hat{\imath}}$$

It turns out that this is not the only way to evaluate this integral — although it may be the simplest, conceptually. Another method, , is to write all the quantities in terms of θ (rather than y), and then integrate over θ. This is essentially equivalent to what we have just done, because we have let

$$\frac{y}{d} = \tan\alpha$$

and looking at our diagram, we see that $\alpha = \theta$!

The moral is: do the integral whichever way you feel most comfortable.