Challenge Problems - Physics III

1. Two particles of equal mass m and equal charge q are suspended from a common point in a uniform gravitational field by light strings of equal length. Find the equilibrium angle of separation of the two strings.

2. Consider an ellipse centered at the origin with a semi-major axis a and a semi-minor axis b. The equation of the ellipse is $(x/a)^2 + (y/b)^2 = 1$. Explicitly calculate the area enclosed by the ellipse using Cartesian coordinates. That is, use an infinitesimal element of area that is a rectangle of width dx.

3. Consider the ellipse in challenge problem 2, but this time calculate the area enclosed by the ellipse using polar coordinates. Use an infinitesimal element of area that is a triangle with one vertex at the origin with an angle $d\theta$.

4. Consider an electric dipole that consists of two point charges $\pm q$ a distance $2a$ apart, centered at the origin as in Example 19.3. (a) Derive the exact expression for the components of the electric field at an arbitrary point (x, y, $z = 0$). [Note that this result is completely general, because even though $z = 0$, the x and z directions are symmetric.] Be sure to simplify your answer; if you don't, the answer you obtain for part (b) will be incorrect. (b) Obtain an approximate expression for the electric field in the so-called "far field" regime, where both x and y are much larger than a. You will need to use the Taylor series expansion (or binomial expansion) of $(1 + x)^n$. I recommend understanding Example 19.3 thoroughly before you attempt this problem. NOTE: Another method of determining the dipole field is to take the limit as a becomes infinitesimally small and q becomes infinitely large simultaneously, with the requirement that the product, qa, remains constant. In this limit there is no bare charge (the point charges cancel each other out), called a "monopole," but there is a "dipole." Recall that the product $q(2a)$ is just the dipole moment.

5. Consider a line segment of total charge Q that has a length ℓ, centered at the origin as in Example 19.4, except let the segment be centered on the origin (i.e., $a = -\ell/2$). Calculate the components of the electric field at an arbitrary point (x, y, $z = 0$). [Note that this result is completely general, because even though $z = 0$, the x and z directions are symmetric.] In Example 19.4, they assumed that $y = 0$ also, so this result will be more general.

6. A uniformly charged thin ring has a radius r and a total charge Q. A charge $-q$ is constrained to move on the axis of the ring. Find the (angular) frequency ω of small oscillations of the negative charge about the center of the ring.

7. Why does excess charge on a conductor move to the surface? Intuitively we answer this by arguing that like charges repel and therefore they desire to get as far away from each other as possible. For a more quantitative statement, one method is to use the concept of energy. From mechanics, we know that objects like to have the lowest potential energy possible. Now that we know how to calculate the electric potential energy, we can solve the following problem: Put excess charge Q on a solid spherical conductor of radius b. Let's assume that the charge spreads itself uniformly throughout a spherical shell of inner radius a and outer radius b. In other words, it's close to the surface, but not all at the surface. From our calculation of energy stored in a capacitor, we found that one explanation is that the energy is stored in the electric field, with an energy density $u_E = \epsilon_0 E^2/2$, which has units of J m^{-3}. The current task is to integrate this density over all space (i.e., from the origin out to a distance that is infinitely far away). We know the electric field everywhere from Gauss's Law, so the calculation should be an exercise in integration. Once you have obtained the answer, vary a, i.e., make the charge Q spread closer to the surface. The result should be that the energy (which is really potential energy) is a minimum when $a = b$ and the charge is on the surface.

8. Calculate the magnitude of the magnetic field on the axis of a solenoid of length L and radius R as a function of distance z from one end of the solenoid. Assume that the solenoid is made up of N current loops stacked concentrically. Since you know the field on the axis of a single current loop, all you have to do is add (i.e., integrate) the contributions to the field from each loop.

9. Obtain an approximation for the radial (perpendicular to the axis) component of the magnetic field near the axis of a single circular loop of wire carrying a current I. Use Gauss's Law for magnetism, and choose as your Gaussian surface a cylinder of radius r, centered on the axis, with one end a distance z away from the loop, and the other end a distance $z + dz$. That is, both the length and radius of the cylinder are small. Since you know the z component of the magnetic field, you should be able to obtain the radial component.

10. The electric potential that an electron "sees" due to a proton (at the origin) is $e/(4\pi\epsilon_0 r)$. This is the potential that is used, in quantum mechanics, to solve the Schrodinger equation and calculate the wave function ψ for that electron. It is, of course, related to the electric field generated by the nucleus $E_r = e/4\pi\epsilon_0 r^2$. This is good for hydrogen, but what happens when we consider helium? A simple model, which is too simplistic but gives a feeling for the complexity inherent in a "three-body problem," is to assume that the first electron is in the ground state $n = 1$. Then the second electron experiences not only the electric field due to the nucleus (with charge $+2e$) but also the field due to the first electron. Of course, the first electron is affected by the second electron, and so the ground state is modified, but we'll ignore that for this problem. Here, I'm asking you to calculate the electric field $E_r(r)$ due to the first electron in the ground state. The charge density of the first electron in the ground state can be found from the Schrodinger equation is

$$\rho(r) = \frac{-e}{\pi}\left(\frac{Z}{a_0}\right)^3 e^{-2Zr/a_0},$$

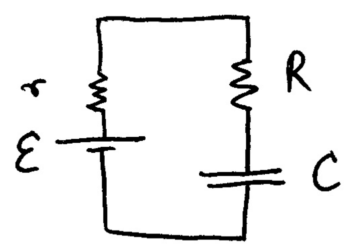
where $a_0 = 4\pi\epsilon_0\hbar^2/me^2$ is the Bohr radius. Your task in this problem is to calculate E_r due to this ρ. The simplest method, since the system is spherically symmetric, is to use Gauss's Law to obtain the radial component of the electric field. Of course, for use in the Schrodinger equation, the potential V must be obtained from an integration of $\vec{E}$.

NOTE: If you are curious, the wave function of the first electron is

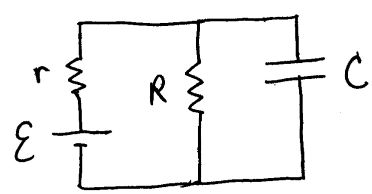
$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0},$$

and the charge density is given by $\rho = -e|\psi|^2$.

11. Charging a capacitor. Circuit theory tells us that the time constant for charging a capacitor in series with two resistors, R and r, is $\tau = (R+r)C$. See the first figure below. (Note: I've added an internal resistance to the battery.)



This problem challenges you to find the time constant for charging a capacitor in *parallel* with a resistor. See the figure below. (Note: here the internal resistance of the battery is essential, for the capacitor would charge "instantaneously" if r was zero.) HINT: You will have to use Kirchhoff's rules to determine the charge on the capacitor as a function of time.



12. Find the equivalent resistance between points a and b for the infinite series of resistors in the figure.