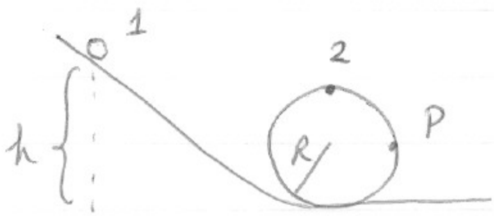
#76 Since the sphere rolls without slipping, there is no energy lost to kinetic friction, and hence mechanical energy is conserved, $\Delta E = 0$. I must remember to include rotational kinetic energy, though.



(a) At point 1, the sphere is released from rest so it has zero kinetic energy, but mgh of gravitational potential energy.

At point 2, it has $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ of kinetic energy and $mg(2R)$ of potential. Equating these gives the speed at point 2:

$$mgh = mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

NOTE: I'm assuming that the sphere is small $r \ll h$, $r \ll R$

but $\omega = \frac{v}{r}$ (rolling without slipping)

and $I = \frac{2}{5}mr^2$ (solid sphere)

so $$\boxed{mg(h-2R) = \frac{1}{2}mv^2\left(1 + \frac{2}{5}\right) = \frac{7}{10}mv^2} \quad (1)$$

The other condition that must be satisfied for the sphere to complete the loop, is that the normal force must be greater than zero at point 2: $N > 0$. Hence,

$\frac{v^2}{R} > g$ where v^2 is obtained from (1) above

#76 $v^2 = \frac{10}{7} g (h - 2R)$ which must be greater than Rg

So $\frac{10}{7} g (h-2R) > Rg$

OR $h - 2R > \frac{7}{10} R \Rightarrow h > \frac{27}{10} R = \boxed{2.7R}$

— Recall that if the object is not rolling (just sliding on a frictionless track), there is no rotational kinetic energy, and the $\frac{2}{5} \to 0$, and we get $h > 2.5R$. So a rolling object must be released from a larger height.

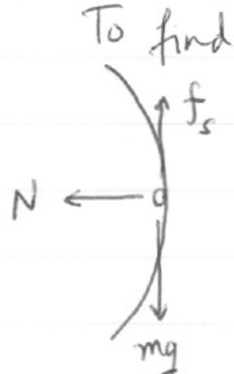
(b) If $h = 3R$, then at point P the speed is obtained in the same fashion, equating total energy

$$mgh = mgR + \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = mgR + \frac{7}{10} mv^2$$

OR

$$v^2 = \frac{10}{7} g (h - R) = \frac{10}{7} g (3R - R) = \frac{20}{7} gR \quad (2)$$

To find the forces at P, we need a force diagram,

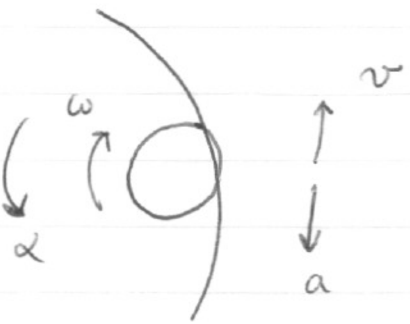


where I've drawn a static friction force upward, because the sphere is rolling but slowing down.

The Normal force must cause the centripetal acceleration

$$N = m\frac{v^2}{R} = \frac{m}{R} \frac{20}{7} gR = \boxed{\frac{20}{7} mg}$$

#76: To find the vertical forces, especially f_s I need to draw a close-up of the sphere and indicate the directions of v, a, ω, α. Note, this is NOT a force diagram



The only force that can cause a torque (and hence α) in the proper direction is a static friction force upward.

Newton's second law gives $\Sigma F_y = \underbrace{mg - f_s = ma}_{(3)}$

and the rotational version $\Sigma \tau = \underbrace{f_s r = I\alpha}_{(4)}$

In Eq (4) $I = \frac{2}{5} m r^2$

and $\alpha = \frac{a}{r}$

Solving (3) and (4) for the two unknowns gives $a = \frac{5}{7} g$ and $f_s = \frac{2}{7} mg$

which means that the net vertical force is $\boxed{\frac{5}{7} g \text{ down}}$