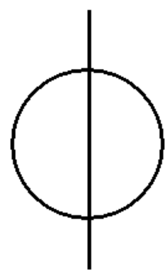
Challenge Problems - Physics II

1. Calculate, using integral calculus, the moment of inertia I of a hoop of mass M and radius R, through an axis that passes through the center of the hoop, but is in the plane of the hoop. NOTE: the moment of inertia that we know and love ($I = MR^2$) is through an axis that is perpendicular to the plane of the hoop.



2. Assuming the earth to be a homogeneous sphere of radius r and mass m, show that the period T of rotation about its axis is related to its radius by

$$T = \left(\frac{4\pi m}{5L}\right) r^2,$$

where L is the angular momentum of the earth due to its rotation. Suppose that the radius r changes by a very small amount Δr due to some internal effect, e.g., thermal expansion. (a) Show that the fractional change in the period T is given approximately by

$$\frac{\Delta T}{T} = \frac{2\Delta r}{r}.$$

HINT: Use differentials dr and dT to approximate the changes in these quantities. (b) By how many miles would the earth need to expand for the period to change by $\frac{1}{4}$ day/year, so that leap years would not be needed?

3. Given an object moving with simple harmonic motion, $x(t) = A\cos(\omega t + \phi)$, with ω known, determine the constants A and ϕ in terms of the initial conditions $x_0 = x(0)$, $v_0 = v(0)$.

4. Confirm, by direct substitution, that the solution to the equation of motion for a damped harmonic oscillator

$$-kx - b\,\frac{dx}{dt} = m\,\frac{d^2x}{dt^2},$$

is given by

$$x(t) = Ae^{-\alpha t}\cos(\omega' t + \phi),$$

where

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

and $\alpha = b/2m$. Note that when $b \to 0$, this reduces to the known solution of simple harmonic motion.