Newton's Clock
Chaos in the Solar System

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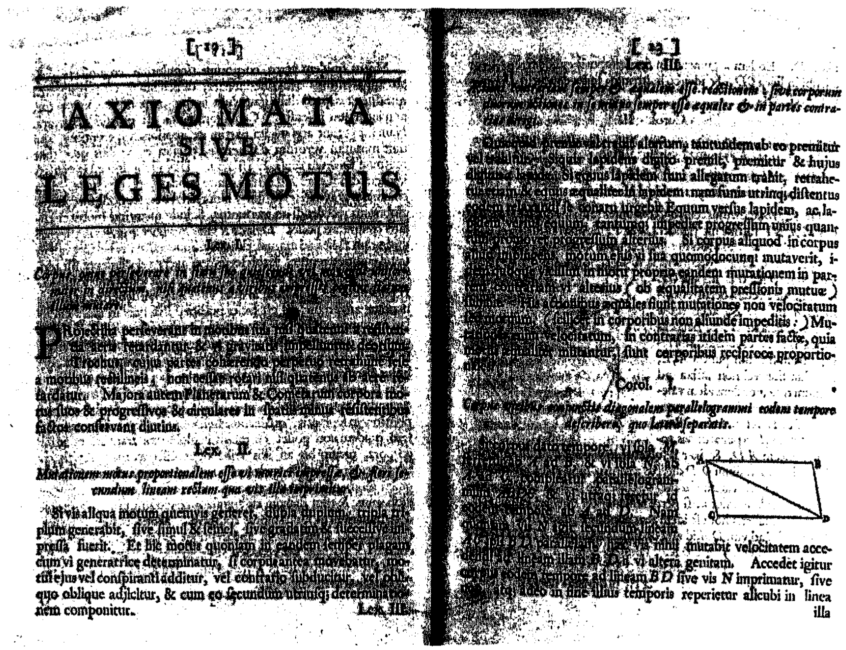
and his contemporaries, though lacking many of the advanced mathematical tools that mathematicians now take for granted, could solve in a few minutes numerous problems that a majority of present-day mathematicians would have to struggle with far longer to solve.

Despite the mathematical language that Newton employed, the publication of the *Principia*, fueled by Halley's promotion of an atmosphere of keen anticipation, created a tremendous stir. Whether or not the book was actually read, the philosophy it embodied had an immediate impact, and it colored philosophical discourse ever afterward. It also represented the true beginning of theoretical physics.

The book's introduction contains the famous three laws of Newton, the seeds of which had appeared in many earlier writings, especially those of Galileo Galilei, René Descartes, and Pierre Gassendi. Newton's contribution was the expression of these three laws of motion in a terse, quantitative form, gathered together to form the foundation of theoretical mechanics. As part of this formulation, he presented a novel and invaluable definition encapsulating the concept of "quantity of motion." He stated it (in Latin) as follows: "The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly." In modern terms, this means that every object has an associated quantity, called momentum, obtained by multiplying its mass and its velocity.

The first law, derived from the investigations of Galileo, Descartes, and others, specifies that "every body continues in its state of rest, or of uniform motion in a [straight] line, unless it is compelled to change that state by forces impressed upon it." Often called the law of inertia, this statement expresses the counterintuitive fact that, contrary to the evidence of everyday experience that all things—from a stone rolling down a hillside to a spinning top—will eventually stop moving, no force at all is required to sustain motion, and that a moving body naturally travels at a constant speed in an unchanging direction. This uniform motion persists in the absence of such forces as air resistance and friction.

Newton and his contemporaries understood the importance of the law of inertia in explaining the motion of a planet around the sun. In the simplest possible case—that of a circular orbit—the planet at each instant moves uniformly along a tangent to its orbit; simultaneously, it is drawn by the ever-present attraction of the sun. These two motions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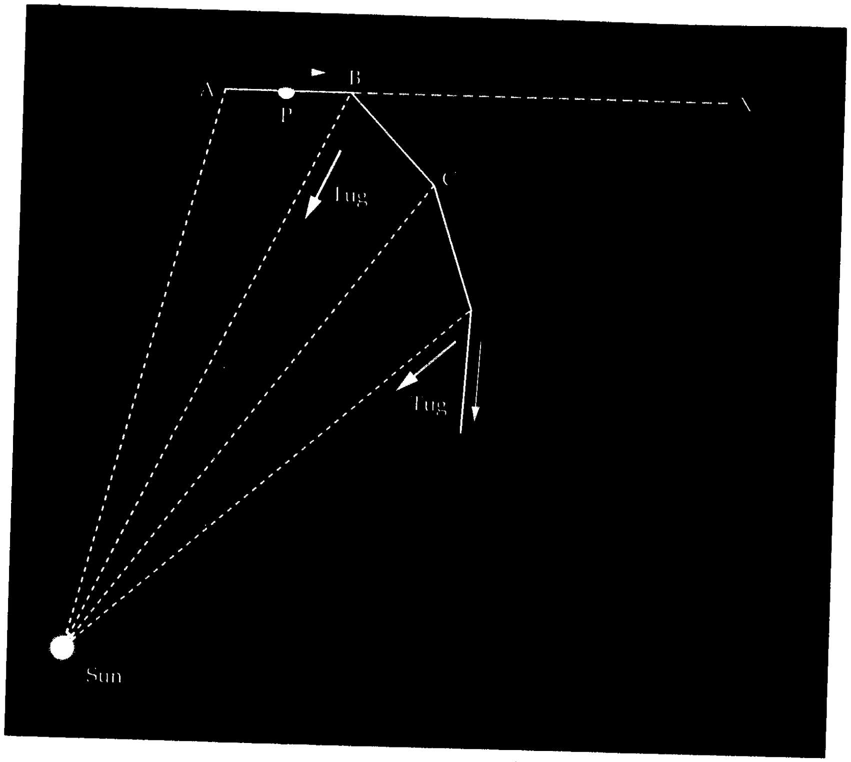
Newton's three laws of motion, as stated in Latin in his *Philosophiae naturalis principia mathematica.* (Library of Congress.)

combine in such a way that the planet traces out a circular path. Planets and comets, meeting less resistance in emptier, wider spaces, preserve their motions for much longer than ordinary terrestrial objects.

The second law states that "the change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which the force is impressed." In other words, Newton related force to a change in momentum (not to acceleration, as most contemporary physics textbooks state). If the mass remains constant, a change in motion is then equivalent to a change in velocity; that is, to the acceleration of a material object. Thus, all forces cause acceleration (a change in the speed or direction of an object), and doubling the force means doubling the acceleration. Interestingly, Newton's second law also describes a situation in which the mass of an object changes. This type of motion didn't become important until the advent of rockets, which propel themselves by spewing out hot gases and thus continuously change their mass.



How a sequence of discrete tugs of attraction influences a planet's motion. Without a tug at B, the planet P would move off in the direction of X. When the gravitational force exerted by the sun acts continuously, the planet takes an elliptical path.

The importance of Newton's choice of momentum as a key concept in dynamics lies in the notion that momentum is one of two quantities that, taken together, yield everything there is to know about a dynamical system at a given instant. The second quantity is simply position, which determines the force's strength and direction. Newton's insight concerning the pairing of momentum and position was placed on firmer ground more than a century and a half later by the mathematicians William Rowan Hamilton and Karl Gustav Jacob Jacobi. It is a duality that lies at the heart of modern dynamical theory.

The third law says that "to every action there is always opposed an equal rection: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts." Often misunderstood,

this law probably arose from Newton's studies of colliding bodies. Imagine two identical balls suspended side by side so that they touch each other. A remarkable exchange is produced if one ball is pulled aside and then released so that it strikes the other, stationary ball. The impact sends the stationary ball into motion, and the initially moving ball comes to a standstill. Out of such experiments, Newton drew the generalization that came to be known as the law of conservation of momentum. In other words, in the absence of an external force acting on a given system, the total momentum of the system remains constant. The third law is simply an alternative way of stating this fundamental principle.

Newton's axiomatic framework allowed him to pursue a strategy in which he could construct a simplified, idealized mathematical model of the physical system he wanted to probe—in this case, the solar system. Using mathematics, Newton could work out the consequences of certain actions and compare them with measurements and empirical observations. That comparison, in turn, would suggest ways in which the model could be adjusted and refined to achieve even greater realism. In essence, this strategy of maintaining a tight interplay between mathematical analysis and physical experience afforded a marvelously productive way of using mathematics to explain the workings of nature. Revolutionary in Newton's time, this kind of approach is taken for granted in modern research.

Newton's theory of universal gravitation formed the core of Book III of the *Principia*, which appeared at long last after two books devoted to abstract arguments that set the mathematical stage. By the end of this concluding book, Newton could contend that gravity acts on all bodies in the universe. In other words, every particle of matter in the universe attracts every other particle with a force that depends simply but precisely on the masses of the particles and the distances between them. Moreover, that universal relationship could be condensed into a brief algebraic formula expressing the fact that the gravitational force acting between two bodies is proportional to the product of the individual masses divided by the square of the distance between them.

But Newton didn't really answer the question of what makes planets move. He freely admitted that his law of gravitation specifies that the sun acts on the planets and produces orbits of the type mandated by Kepler but says nothing about exactly *how* the sun acts