Math Review Name ______________________________

PS150 Practice Questions

Show your work in the space provided. Write your answer in the appropriate box.

1. Evaluate the following expression: $\sqrt{8^2 + 6^2}$

2. Equations like the one shown below often arise in the study of static equilibrium. Solve the equation for x, in terms of L, M, m and g. (L, M, m, and g are all *constants*, or *parameters* in the equation. The statement "in terms of" means that those parameters will appear in your final expression.) [Answer: $x = \frac{mL}{2(m+M)}$]

$$mg\left(\frac{L}{2} - x\right) = Mgx$$

3. Using the following system of equations, solve for y. Your answers should be real numbers, of magnitude less than 50.

$$
\begin{aligned}
120 &= yx - 5x^2 \\
0 &= y - 10x
\end{aligned}
$$

4. The equations shown below were obtained while analyzing a projectile motion problem. From this system of equations, solve for v_0 in terms of h and g. (v_0 and t are both variables; h and g are constants.)

$$
\begin{aligned}
h &= v_0 t - \frac{1}{2} g t^2 \\
0 &= v_0 - gt
\end{aligned}
$$

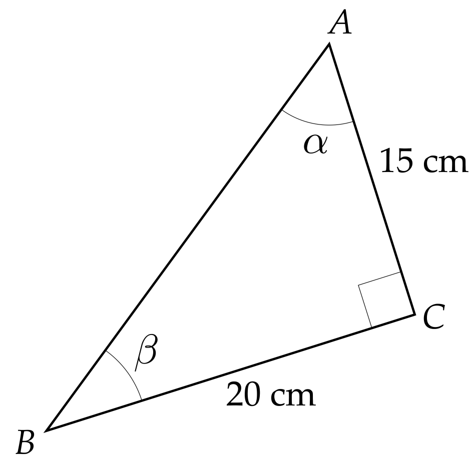
5. Newton's law of gravitation involves equations like the one shown below. Solve it for the variable r, in terms of the parameter d.

$$G\frac{2Mm}{r^2} = G\frac{Mm}{(d-r)^2}$$

6. Newton's second law of motion is often used to study the dynamics of an object in circular motion, and the equations shown below arise in that context. Solve this system of equations for v^2 in terms of R, θ, and g. [Answer: $v^2 = gR\tan\theta$]

$$N\cos\theta = mg$$
$$N\sin\theta = m\frac{v^2}{R}$$

7. Right triangles being ubiquitous in mechanics, their geometry and trigonometry is a daily exercise. Find the following from the right triangle ACB given below on the right;



a) $|AB| =$

b) $\tan\alpha =$

c) $\sin\alpha =$

d) $\cos\alpha =$

e) $\tan\beta =$

f) $\sin\beta =$

[Answer: 3/5]

g) $\cos\beta =$

h) How is $\cos\beta$ related to $\sin\alpha$?

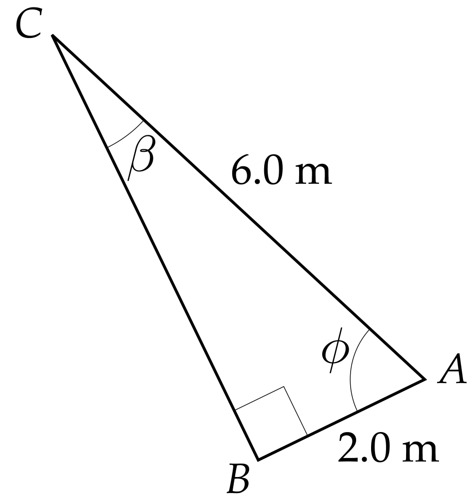
i) How is $\sin\beta$ related to $\cos\alpha$?

j) $2\,\sin\alpha\cos\alpha =$

k) $\sin(2\alpha) =$

l) How is $2\sin\alpha\cos\alpha$ related to $\sin(2\alpha)$?

8. Find the following from the right triangle CBA given below on the right;



a) $|CB| =$

b) $\tan\phi =$

[Answer: 2.83]

c) $\sin\phi =$

d) $\cos\phi =$

e) $\tan\beta =$

f) $\sin\beta =$

g) $\cos\beta =$

h) How is $\cos\beta$ related to $\sin\phi$?

i) How is $\sin\beta$ related to $\cos\phi$?

j) $2\,\sin\phi\cos\phi =$

k) $\sin(2\phi) =$

l) How is $2\sin\phi\cos\phi$ related to $\sin(2\alpha)$?

9. Newton's second law often leads to algebraic equations involving a variety of constants and variables. Solve the following equations for a in terms of each equation's other variables and constants.

   i) $v + w = a^2 yz$

   ii) $T = mg + ma$

   iii) $T = mg - Ma$

   iv) $\sin\alpha = \mu\cos\alpha + \frac{m}{m+M}\left(\frac{a}{g}\right)$

   [Answer: $a = g\left(\frac{m+M}{m}\right)(\sin\alpha - \mu\cos\alpha)$]

   v) $\mu\ \sin\alpha = \cos\alpha + \frac{m+M}{m}\left(\frac{g}{a}\right)$

   vi) $x + y = \frac{1}{a}$

   vii) $\frac{1}{x} + \frac{1}{y} = \frac{1}{a}$

10. The following system of equations arises from Newton's second law applied to block pressed into a rough wall. Solve the two simultaneous equations for P in terms of m, g, θ, and μ_s.

$$N = P\cos\theta$$

$$P\sin\theta + \mu_s N - mg = 0$$

11. Equations like the following often arise in the study of projectile motion. Eliminate t from the equations and find y as a function of x

    i) $x = 2t$, $y = 3t - 5t^2$

    ii) $x = vt$, $y = vt + at^2$ [Hint: consider v and a as constants] [Answer: $y(x) = x + \frac{a}{v^2}x^2$]

    iii) $x = v_i t$, $y = v_i t - \frac{1}{2}gt^2$ [Hint: consider v_i and g are constants]

    iv) $x = v_i\cos\theta_0 t$, $y = v_i\sin\theta_0 t - \frac{1}{2}gt^2$ [Hint: consider v_i, g, and θ_0 are constants]

12. Solve the following equations for the variable

1) $8 - 5x = 2x + 3$

2) $-27 = 7 - 2x$

3) $3\beta - 5 = -6$ [Answer: $\beta = -1/3$]

4) $3\gamma - 5 = -6$

5) $4\delta + 1 = 7 - 5\delta$

6) $17\theta + 34 = -17\theta - 34$

7) $-17\Lambda + 34 = +17\Lambda - 34$

8) $4x + 5 = 12$

9) $5x - 3x = -7$

10) $4(x - \frac{3}{4}) = \frac{9}{7}$ [Answer: $x = 15/14$]

11) $2(3a - 4) = a + 9$

12) $-3(2b + 5) = 15$

13) $\frac{1}{2} = \frac{2c - 3}{5}$

13. Consider the following functions $x(t)$ and $y(t)$.

$$\begin{aligned} x(t) &= 17t \\ y(t) &= 49t - 4.9t^2 \end{aligned}$$

1) Find the value of x and y at $t = 0, 1, 2, 3.5$, and 7.2.

2) What is the value of y when $x = 20$? [Answer: 50.9]

3) What is the value(s) of x when $y = -25$?

4) Find $y(x)$.

14. Equations like the one shown below arise when considering energy in mass-spring systems. Solve the following equation for the values of v_1 in-terms of k, M and A_1.

$$\frac{1}{2}kA_1^2 = \frac{1}{2}Mv_1^2$$

15. The equation shown below was obtained by considering momentum in a system of two colliding masses. Solve the equation for the value of v_2 in terms of m, M and v_1.

$$Mv_1 + m(0) = Mv_2 + mv_2$$

16. Solve for v in-terms of the rest of the variables,

$$\frac{1}{2}kA^2 + \frac{1}{2}m(0)^2 = \frac{1}{2}kx^2 + \frac{1}{2}Mv^2.$$

Notice that the equation is quadratic in v and hence may have up to two solutions.

17. The following equations arise in the study of rotational motion. Find the value of $I(x)$ from the relation when $x = R/2$,

$$I(x) = \frac{1}{2}MR^2 + Mx^2$$

18. Find the value of $I(x)$ from the following relation when $x = L/2$,

$$I(x) = \frac{1}{12}ML^2 + Mx^2$$

19. Suppose $t = 10^{-2}$. Without using a calculator, evaluate each of the following expressions.

1) $t^2 = \boxed{}$

2) $t^3 = \boxed{}$

3) $\sqrt{t^6} = \boxed{}$

4) $\sqrt[3]{t^6} = \boxed{}$ [Answer: 10^{-4}]

5) $t\left(t^2\right) = \boxed{}$

6) $t\left(t^2\right)^2 = \boxed{}$

7) $\left(t\left(t^2\right)\right)^2 = \boxed{}$

8) $\left(t\left(t^2\right)\right)^{-2} = \boxed{}$

9) $\dfrac{t^3}{t} = \boxed{}$

10) $\left(\dfrac{t^3}{t}\right) t^{-2} = \boxed{}$