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Newtonian mechanics

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In the Beginning was Mechanics.

MAX VON LAUE, *History of Physics* (1950)

I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena.

NEWTON, Preface to the *Principia* (1686)

Prologue

ONE OF THE MOST prominent features of the universe is motion. Galaxies have motions with respect to other galaxies, all stars have motions, the planets have distinctive motions against the background of the stars, the events that capture our attention most quickly in everyday life are those involving motion, and even the apparently inert book that you are now reading is made up of atoms in rapid motion about their equilibrium positions. "Give me matter and motion," said the seventeenth-century French philosopher René Descartes, "and I will construct the universe." There can be no doubt that motion is a phenomenon we must learn to deal with at all levels if we are to understand the world around us.

Isaac Newton developed a precise and powerful theory regarding motion, according to which the *changes* of motion of any object are the result of *forces* acting on it. In so doing he created the subject with which this book is concerned and which is called classical or Newtonian mechanics. It was a landmark in the history of science, because it replaced a merely descriptive account of phenomena with a rational and marvelously successful scheme of cause and effect. Indeed, the strict causal nature of Newtonian mechanics had an impressive influence in the development of Western thought and civilization generally, provoking fundamental questions about the interrelationships of science, philosophy, and religion, with repercussions in social ideas and other areas of human endeavor.

Classical mechanics is a subject with a fascinating dual character. For it starts out from the kinds of everyday experiences that are as old as mankind, yet it brings us face to face with some of the most profound questions about the universe in which we find ourselves. Is it not remarkable that the flight of a thrown pebble, or the fall of an apple, should contain the clue to the mechanics of the heavens and should ultimately involve some of the most basic questions that we are able to formulate about the nature of space and time? Sometimes mechanics is presented as though it consisted merely of the routine application of self-evident or revealed truths. Nothing could be further from the case; it is a superb example of a physical theory, slowly evolved and refined through the continuing interplay between observation and hypothesis.

The richness of our first-hand acquaintance with mechanics is impressive, and through the partnership of mind and eye and hand we solve, by direct action, innumerable dynamical problems without benefit of mathematical analysis. Like Molière's famous character, M. Jourdain, who learned that he had been speaking prose all his life without realizing it, every human being is an expert in the consequences of the laws of mechanics, whether or not he has ever seen these laws written down. The skilled sportsman or athlete has an almost incredible degree of judgment and control of the amount and direction of muscular effort needed to achieve a desired result. It has been estimated, for example, that the World Series baseball championship would have changed hands in 1962 if one crucial swing at the ball had been a mere millimeter lower.[1] But experiencing and controlling the motions of objects in this very personal sense is a far cry from analyzing them in terms of physical laws and equations. It is the task of classical mechanics to discover and formulate the essential principles, so that they can be applied to any situation, particularly to inanimate objects interacting with one another. Our intimate familiarity with our own muscular actions and their consequences, although it represents a kind of understanding (and an important kind, too), does not help us much here.

The greatest triumph of classical mechanics was Newton's own success in analyzing the workings of the solar system—a feat immortalized in the famous couplet of his contemporary and admirer, the poet Alexander Pope:

[1] P. Kirkpatrick, *Am. J. Phys.*, **31**, 606 (1963).

Nature and Nature's Laws lay hid in night
God said "Let Newton be," and all was light.[1]

Men had observed the motions of the heavenly bodies since time immemorial. They had noticed various regularities and had learned to predict such things as conjunctions of the planets and eclipses of the sun and moon. Then, in the sixteenth century, the Danish astronomer Tycho Brahe amassed meticulous records, of unprecedented accuracy, of the planetary motions. His assistant, Johannes Kepler, after wrestling with this enormous body of information for years, found that all the observations could be summarized as follows:

1. The planets move in ellipses having the sun at one focus.
2. The line joining the sun to a given planet sweeps out equal areas in equal times.
3. The square of a planet's year, divided by the cube of its mean distance from the sun, is the same for all planets.

This represented a magnificent advance in man's knowledge of the mechanics of the heavens, but it was still a description rather than a theory. Why? was the question that still looked for an answer. Then came Newton, with his concept of force as the cause of changes of motion, and with his postulate of a particular law of force—the inverse-square law of gravitation. Using these he demonstrated how Kepler's laws were just one consequence of a scheme of things that also included the falling apple and other terrestrial motions. (Later in this book we shall go into the details of this great achievement of Newton's.)

If universal gravitation had done no more than to relate planetary periods and distances, it would still have been a splendid theory. But, like any other good theory in physics, it had predictive value; that is, it could be applied to situations besides the ones from which it was deduced. Investigating the predictions of a theory may involve looking for hitherto unsuspected phenomena, or it may involve recognizing that an already familiar phenomenon must fit into the new framework. In either case the theory is subjected to searching tests, by which it must

[1] To which there is the almost equally famous, although facetious, riposte:

It did not last; the Devil, howling "Ho,
Let Einstein be!" restored the status quo.
(Sir John Squire)

stand or fall. With Newton's theory of gravitation, the initial tests resided almost entirely in the analysis of known effects—but what a list! Here are some of the phenomena for which Newton proceeded to give *quantitative* explanations:

1. The bulging of the earth and Jupiter because of their rotation.

2. The variation of the acceleration of gravity with latitude over the earth's surface.

3. The generation of the tides by the combined action of sun and moon.

4. The paths of the comets through the solar system.

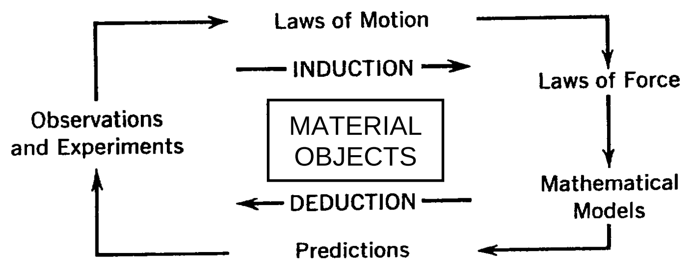
5. The slow steady change in direction of the earth's axis of rotation produced by gravitational torques from the sun and moon. (A complete cycle of this variation takes about 25,000 years, and the so-called "precession of the equinoxes" is a manifestation of it.)

This marvelous illumination of the workings of nature represented the last part of Newton's program, as he described it in our opening quotation "... and then from these forces to demonstrate the other phenomena." This modest phrase conceals not only the immensity of the achievement but also the magnitude of the role played by mathematics in this development. Newton had, in the theory of universal gravitation, created what would be called today a mathematical model of the solar system. And having once made the model, he followed out a host of its other implications. The working out was purely mathematical, but the final step—the test of the conclusions—involved a return to the world of physical experience, in the detailed checking of his predictions against the quantitative data of astronomy.

Although Newton's mechanical picture of the universe was amply confirmed in his own time, he did not live to see some of its greatest triumphs. Perhaps the most impressive of these was the use of his laws to identify previously unrecognized members of the solar system. By a painstaking and lengthy analysis of the motions of the known planets, it was inferred that disturbing influences due to other planets must be at work. Thus it was that Neptune was discovered in 1846, and Pluto in 1930. In each case it was a matter of deducing where a telescope should be pointed to reveal a new planet, identifiable through its changing position with respect to the general background of the stars.

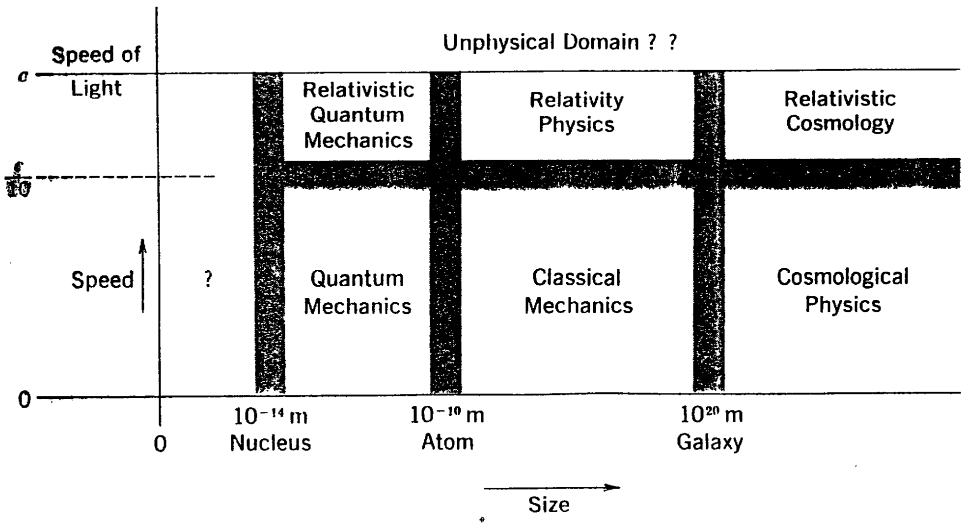
What more striking and convincing evidence could there be that the theory works?

Probably everyone who reads this book has some prior acquaintance with classical mechanics and with its expression in mathematically precise statements. And this may make it hard to realize that, as with any other physical theory, its development was not just a matter of mathematical logic applied undiscriminatingly to a mass of data. Was Newton inexorably driven to the inverse-square law? By no means. It was the result of guesswork, intuition, and imagination. In Newton's own words: "I began to think of gravity extending to the orbit of the Moon, and ... from Kepler's Rule of the periodic times of the Planets ... I deduced that the forces which keep the Planets in their orbits must be reciprocally as the squares of their distances from the centers about which they revolve; and thereby compared the force requisite to keep the Moon in her orbit with the force of gravity at the surface of the Earth, and found them to answer pretty nearly." An intellectual leap of this sort—although seldom as great as Newton's—is involved in the creation of any theory or model. It is a process of induction, and it goes beyond the facts immediately at hand. Some facts may even be temporarily brushed aside or ignored in the interests of pursuing the main idea, for a partially correct theory is often better than no theory at all. And at all stages there is a constant interplay between experiment and theory, in the process of which fresh observations are continually suggesting themselves and modification of the theory is an ever-present possibility. The following diagram, the relevance of which goes beyond the realm of classical mechanics, suggests this pattern of man's investigation of matter and motion.



The enormous success of classical mechanics made it seem, at one stage, that nothing more was needed to account for the whole world of physical phenomena. This belief reached a pinnacle toward the end of the nineteenth century, when some optimistic physicists felt that physics was, in principle, complete. They could hardly have chosen a more unfortunate time at which to form such a conclusion, for within the next few decades physics underwent its greatest upheaval since Newton. The discovery of radioactivity, of the electron and the nucleus, and the subtleties of electromagnetism, called for fundamentally new ideas. Thus we know today that Newtonian mechanics, like every physical theory, has its fundamental limitations. The analysis of motions at extremely high speeds requires the use of modified descriptions of space and time, as spelled out by Albert Einstein's special theory of relativity. In the analysis of phenomena on the atomic or subatomic scale, the still more drastic modifications described by quantum theory are required. And Newton's particular version of gravitational theory, for all its success, has had to admit modifications embodied in Einstein's general theory of relativity. But this does not alter the fact that, in an enormous range and variety of situations, Newtonian mechanics provides us with the means to analyze and predict the motions of physical objects, from electrons to galaxies. Its range of validity, and its limits, are indicated very qualitatively in the figure below.

In developing the subject of classical mechanics in this book, we shall try to indicate how the horizons of its application to the physical world, and the horizons of one's own view, can be gradually broadened. Mechanics, as we shall try to present it, is not at all a cut-and-dried subject that would justify its description



as "applied mathematics," in which the rules of the game are given at the outset and in which one's only concern is with applying the rules to a variety of situations. We wish to offer a different approach, in which at every stage one can be conscious of working with partial or limited data and of making use of assumptions that cannot be rigorously justified. But this is the essence of doing physics. Newton himself said as much. At the beginning of Book III of the *Principia* he propounds four "Rules of Reasoning in Philosophy," of which the last runs as follows:

"In experimental philosophy we are to look upon propositions inferred by general induction from phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions." The person who waits for complete information is on the way to dooming himself never to finish an experiment or to construct a useful theory. Lest this should be taken, however, as an encouragement to slipshod or superficial thinking, we shall end this introduction with a little fable due to George Polya.[1] He writes as a mathematician, but the moral for physicists (and others) is clear.

The Logician, the Mathematician, the Physicist, and the Engineer

"Look at this mathematician," said the logician. "He observes that the first 99 numbers are less than 100 and infers, hence, by what he calls induction, that all numbers are less than a hundred." "A physicist believes," said the mathematician, "that 60 is divisible by 1, 2, 3, 4, 5, and 6. He examines a few more cases, such as 10, 20, and 30, taken at random (as he says). Since 60 is also divisible by these, he considers the experimental evidence sufficient." "Yes, but look at the engineers," said the physicist. "An engineer suspected that all odd numbers are prime numbers. At any rate, 1 can be considered as a prime number, he argued. Then there come 3, 5, and 7, all indubitably primes. Then there comes 9; an awkward case; it does not seem to be a prime num-

[1] This cautionary tale is to be found in a book entitled *Induction and Analogy in Mathematics*, Princeton University Press, Princeton, N.J., 1954. This volume and its companion, *Patterns of Plausible Inference*, make delightful reading for any scientist.

ber. Yet 11 and 13 are certainly primes. 'Coming back to 9,' he said, 'I conclude that 9 must be an experimental error.'" But having done his teasing, Polya adds these remarks.

> It is only too obvious that induction can lead to error. Yet it is remarkable that induction sometimes leads to truth, since the chances of error seem so overwhelming. Should we begin with the study of the obvious cases in which induction fails, or with the study of those remarkable cases in which induction succeeds? The study of precious stones is understandably more attractive than that of ordinary pebbles and, moreover, it was much more the precious stones than the pebbles that led the mineralogists to the wonderful science of crystallography.

With that encouragement, we shall, in Chapter 1, begin our approach to the study of classical mechanics, which is one of the most perfect and polished gems in the physicist's treasury. We end this Prologue, however, with some preparatory exercises.

EXERCISES—HORS D'OEUVRES

The literal meaning of the phrase "hors d'oeuvre" is "outside the work." The exercises below correspond exactly to that definition, although it is hoped that they will also whet the appetite as hors d'oeuvres should. They deal mostly with order-of-magnitude estimates (i.e., estimates to the nearest power of 10) and judicious approximations—things that play an important role in a physicist's approach to problems but seldom get emphasized or systematically presented in textbooks. For example, everybody learns the binomial theorem, but how many students think of it as a useful tool for obtaining a quite good value for the hypotenuse of a right triangle, by the approximation

$$(a^2 + b^2)^{1/2} \approx a\left(1 + \frac{b^2}{2a^2}\right)$$

where we assume $b < a$? (Even in the worst possible case, with $b = a$, the result is wrong by only about 6 percent—1.5 instead of 1.414 . . . .) Moreover, it takes practice and some conscious effort to develop the habit of assessing, quite crudely, the magnitudes of quantities and the relative importance of various possible effects in a physical system. For example, in dealing with objects moving through liquids, can one quickly decide whether

viscosity or turbulence is going to be the chief source of resistance for an object of given speed and linear dimensions? An awareness of the effects of changes of scale can give valuable insights into the properties of systems. [A beautiful example of this is the well-known essay by J. B. S. Haldane, "On Being the Right Size," which is reprinted in *The World of Mathematics*, Vol. II (J. R. Newman, ed.), Simon and Schuster, New York, 1956.] By the use of such methods and ways of thought one can deepen one's appreciation of physical phenomena and can improve one's feeling for what the world is like and how it behaves.

It is surprising how much one can do with the help of a relatively small stock of primary information—which might include such items as the following:

Physical Magnitudes

Gravitational acceleration (g)	10 m/sec^2
Densities of solids and liquids	10^3–10^4 kg/m^3
Density of air at sea level	1 kg/m^3 (approx.)
Length of day	10^5 sec (approx.)
Length of year	3.16×10^7 sec $\approx 10^{7.5}$ sec
Earth's radius	6400 km
Angle subtended by finger thickness at arm's length	1° (approx.)
Thickness of paper	0.1 mm (approx.)
Mass of a paperclip	0.5 g (approx.)
Highest mountains, deepest oceans	10 km (approx.)
Earth–moon separation	3.8×10^5 km
Earth–sun separation	1.5×10^8 km
Atmospheric pressure	Equivalent to weight of 1 kg/cm^2 or a 10-m column of water
Avogadro's number	6.0×10^{23}
Atomic masses	1.6×10^{-27} kg to 4×10^{-25} kg
Linear dimensions of atoms	10^{-10} m (approx.)
Molecules/cm^3 in gas at STP	2.7×10^{19}
Atoms/cm^3 in solids	10^{23} (approx.)
Elementary charge (e)	1.6×10^{-19} C
Electron mass	10^{-30} kg (approx.)
Speed of light	3×10^8 m/sec
Wavelength of light	6×10^{-7} m (approx.)

Mathematical Magnitudes

$\pi^2 \approx 10$	$\log_{10} 4 \approx 0.60$
$e \approx 2.7$	$\log_{10} e \approx 0.43$
$\log_{10} 2 \approx 0.30$	$\log_{10} \pi \approx 0.50$
$\log_{10} 3 \approx 0.48$	$\log_e 10 \approx 2.3$

Angle (radians) = arc length/radius. Full circle = 2π rad.
1 rad $\approx 0.16 \times$ full circle $\approx 57°$.

Solid angle (steradians) = area/(radius)^2. Full sphere = 4π sr.
1 sr $\approx 0.08 \times$ full sphere.

Approximations

Binomial theorem:

For $x \ll 1$,
$$(1 + x)^n \approx 1 + nx$$
e.g.,
$$(1 + x)^3 \approx 1 + 3x$$
$$(1 - x)^{1/2} \approx 1 - \tfrac{1}{2}x \approx (1 + x)^{-1/2}$$

For $b \ll a$,
$$(a + b)^n = a^n\left(1 + \frac{b}{a}\right)^n \approx a^n\left(1 + n\frac{b}{a}\right)$$

Other expansions:

For $\theta \ll 1$ rad,
$$\sin\theta \approx \theta - \frac{\theta^3}{6} \to \theta$$
$$\cos\theta \approx 1 - \frac{\theta^2}{2} \to 1$$

For $x \ll 1$,
$$\log_e(1 + x) \approx x$$
$$\log_{10}(1 + x) \approx 0.43x$$

No answers are given to the problems that follow. For most of them, you yourself will be the best judge. You may want to turn to an encyclopedia or other reference book to check some of your assumptions or conclusions. If you are not prepared at this point to tackle them all, don't worry; you can always return to them later.

1 What is the order of magnitude of the number of times that the earth has rotated on its axis since the solar system was formed?

2 During the average lifetime of a human being, how many heartbeats are there? How many breaths?

3 Make reasoned estimates of (a) the total number of ancestors you would have (ignoring inbreeding) since the beginning of the human race, and (b) the number of hairs on your head.

4 The present world population (human) is about 3×10^9.

(a) How many square kilometers of land are there per person? How many feet long is the side of a square of that area?

(b) If one assumes that the population has been doubling every 50 years throughout the existence of the human race, when did Adam and Eve start it all? If the doubling every 50 years were to continue, how long would it be before people were standing shoulder to shoulder over all the land area of the world?

5 Estimate the order of magnitude of the mass of (a) a speck of dust; (b) a grain of salt (or sugar, or sand); (c) a mouse; (d) an elephant; (e) the water corresponding to 1 in. of rainfall over 1 square mile; (f) a small hill, 500 ft high; and (g) Mount Everest.

6 Estimate the order of magnitude of the number of atoms in (a) a pin's head, (b) a human being, (c) the earth's atmosphere, and (d) the whole earth.

7 Estimate the fraction of the total mass of the earth that is now in the form of living things.

8 Estimate (a) the total volume of ocean water on the earth, and (b) the total mass of salt in all the oceans.

9 It is estimated that there are about 10^{80} protons in the (known) universe. If all these were lumped into a sphere so that they were just touching, what would the radius of the sphere be? Ignore the spaces left when spherical objects are packed and take the radius of a proton to be about 10^{-15} m.

10 The sun is losing mass (in the form of radiant energy) at the rate of about 4 million tons per second. What fraction of its mass has it lost during the lifetime of the solar system?

11 Estimate the time in minutes that it would take for a theatre audience of about 1000 people to use up 10% of the available oxygen if the building were sealed. The average adult absorbs about one sixth of the oxygen that he or she inhales at each breath.

12 Solar energy falls on the earth at the rate of about 2 cal/cm^2/min. Estimate the amount of power, in megawatts or horsepower, represented by the solar energy falling on an area of 100 square miles—about the area of a good-sized city. How would this compare with the total power requirements of such a city? (1 cal = 4.2 J; 1 W = 1 J/sec; 1 hp = 746 W.)

13 Starting from an estimate of the total mileage that an automobile tire will give before wearing out, estimate what thickness of rubber is worn off during one revolution of the wheel. Consider the possible physical significance of the result. (With acknowledgment to E. M. Rogers, *Physics for the Inquiring Mind*, Princeton University Press, Princeton, N.J., 1960.)

14 An inexpensive wristwatch is found to lose 2 min/day.

(a) What is its fractional deviation from the correct rate?

(b) By how much could the length of a ruler (nominally 1 ft long) differ from exactly 12 in. and still be fractionally as accurate as the watch?

15 The astronomer Tycho Brahe made observations on the angular positions of stars and planets by using a quadrant, with one peephole at its center of curvature and another peephole mounted on the arc. One such quadrant had a radius of about 2 m, and Tycho's measurements could usually be trusted to 1 minute of arc ($\frac{1}{60}°$). What diameter of peepholes would have been needed for him to attain this accuracy?

16 Jupiter has a mass about 300 times that of the earth, but its mean density is only about one fifth that of the earth.

(a) What radius would a planet of Jupiter's mass and earth's density have?

(b) What radius would a planet of earth's mass and Jupiter's density have?

17 Identical spheres of material are tightly packed in a given volume of space.

(a) Consider why one does not need to know the radius of the spheres, but only the density of the material, in order to calculate the total mass contained in the volume, provided that the linear dimensions of the volume are large compared to the radius of the individual spheres.

(b) Consider the possibility of packing more material if two sizes of spheres may be chosen and used.

(c) Show that the total surface area of the spheres of part (a) *does* depend on the radius of the spheres (an important consideration in the design of such things as filters, which absorb in proportion to the total exposed surface area within a given volume).

18 Calculate the ratio of surface area to volume for (a) a sphere of radius r, (b) a cube of edge a, and (c) a right circular cylinder of diameter and height both equal to d. For a given value of the volume, which of these shapes has the greatest surface area? The least surface area?

19 How many seconds of arc does the diameter of the earth subtend at the sun? At what distance from an observer should a football be placed to subtend an equal angle?

20 From the time the lower limb of the sun touches the horizon it takes approximately 2 min for the sun to disappear beneath the horizon.

(a) Approximately what angle (expressed both in degrees and in radians) does the diameter of the sun subtend at the earth?

(b) At what distance from your eye does a coin of about $\frac{3}{4}$-in. diameter (e.g., a dime or a nickel) just block out the disk of the sun?

(c) What solid angle (in steradians) does the sun subtend at the earth?

21 How many inches per mile does a terrestrial great circle (e.g., a meridian of longitude) deviate from a straight line?

22 A crude measure of the roughness of a nearly spherical surface could be defined by $\Delta r/r$, where Δr is the height or depth of local irregularities. Estimate this ratio for an orange, a ping-pong ball, and the earth.

23 What is the probability (expressed as 1 chance in 10^n) that a good-sized meteorite falling to earth would strike a man-made structure? A human?

24 Two students want to measure the speed of sound by the following procedure. One of them, positioned some distance away from the other, sets off a firecracker. The second student starts a stopwatch when he sees the flash and stops it when he hears the bang. The speed of sound in air is roughly 300 m/sec, and the students must admit the possibility of an error (of undetermined sign) of perhaps 0.3 sec in the elapsed time recorded. If they wish to keep the error in the measured speed of sound to within 5%, what is the minimum distance over which they can perform the experiment?

25 A right triangle has sides of length 5 m and 1 m adjoining the right angle. Calculate the length of the hypotenuse from the binomial expansion to two terms only, and estimate the fractional error in this approximate result.

26 The radius of a sphere is measured with an uncertainty of 1%. What is the percentage uncertainty in the volume?

27 Construct a piece of semilogarithmic graph paper by using the graduations on your slide rule to mark off the ordinates and a normal ruler to mark off the abscissa. On this piece of paper draw a graph of the function $y = 2^x$.

28 The subjective sensations of loudness or brightness have been judged to be approximately proportional to the logarithm of the intensity, so that equal *multiples* of intensity are associated with equal arithmetic increases in sensation. (For example, intensities proportional to 2, 4, 8, and 16 would correspond to equal increases in sensation.) In acoustics, this has led to the measurement of sound intensities in *decibels*. Taking as a reference value the intensity I_0 of the faintest audible sound, the decibel level of a sound of intensity I is defined by the equation

$$\text{dB} = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

(a) An intolerable noise level is represented by about 120 dB. By what factor does the intensity of such a sound exceed the threshold intensity I_0?

(b) A similar logarithmic scale is used to describe the relative brightness of stars (as seen from the earth) in terms of *magnitudes.* Stars differing by "one magnitude" have a ratio of apparent brightness equal to about 2.5. Thus a "first-magnitude" (very bright) star is 2.5 times brighter than a second-magnitude star, $(2.5)^2$ times brighter than a third-magnitude star, and so on. (These differences are due largely to differences of distance.) The faintest stars detectable with the 200-in. Palomar telescope are of about the twenty-fourth magnitude. By what factor is the amount of light reaching us from such a star less than we receive from a first-magnitude star?

29 The universe appears to be undergoing a general expansion in which the galaxies are receding from us at speeds proportional to their distances. This is described by Hubble's law, $v = \alpha r$, where the constant α corresponds to v becoming equal to the speed of light, c ($= 3 \times 10^8$ m/sec), at $r \approx 10^{26}$ m. This would imply that the mean mass per unit volume in the universe is decreasing with time.

(a) Suppose that the universe is represented by a sphere of volume V at any instant. Show that the fractional increase of volume per unit time is given by

$$\frac{1}{V}\frac{dV}{dt} = 3\alpha$$

(b) Calculate the fractional decrease of mean density per second and per century.

30 The table lists the mean orbit radii of successive planets expressed in terms of the earth's orbit radius. The planets are numbered in order (n):

n	*Planet*	r/r_E
1	Mercury	0.39
2	Venus	0.72
3	Earth	1.00
4	Mars	1.52
5	Jupiter	5.20
6	Saturn	9.54
7	Uranus	19.2

(a) Make a graph in which $\log(r/r_E)$ is ordinate and the number n is abscissa. (Or, alternatively, plot values of r/r_E against n on semi-logarithmic paper.) On this same graph, replot the points for Jupiter, Saturn, and Uranus at values of n increased by unity (i.e., at $n = 6$, 7, and 8). The points representing the seven planets can then be reasonably well fitted by a straight line.

(b) If $n = 5$ in the revised plot is taken to represent the asteroid belt between the orbits of Mars and Jupiter, what value of r/r_E would your graph imply for this? Compare with the actual mean radius of the asteroid belt.

(c) If $n = 9$ is taken to suggest an orbit radius for the next planet (Neptune) beyond Uranus, what value of r/r_E would your graph imply? Compare with the observed value.

(d) Consider whether, in the light of (b) and (c), your graph can be regarded as the expression of a physical law with predictive value. (As a matter of history, it *was* so used. See the account of the discovery of Neptune near the end of Chapter 8.)