Physics I (Twelve Days of) Christmas Problems

Dec 26 You have a bag containing 340 coins and 160 bills totaling \$ 1809. There are pennies, nickels, fives, tens, and twenties. The total value of the tens is equal to the total value of the twenties. There are the same number of fives and twenties. What is the total value of the coins and bills bearing Lincoln's portrait?

Dec 27 The number e can be defined by the property

$$e^{\varepsilon} \approx 1 + \varepsilon \quad \text{for } \varepsilon \ll 1.$$

For any small value of ε, any number r raised to the power ε will differ from 1 by an amount proportional to ε. However, only for $r = e$ will the proportionality constant be 1. Taking the base 10 logarithm results in

$$\varepsilon \log_{10} e \approx \log_{10}(1 + \varepsilon). \tag{1}$$

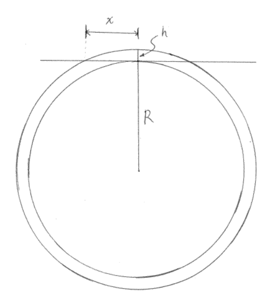
(a) Use this expression to calculate e by using small values for ε. Make a table showing that your value for e gets more and more precise as ε gets smaller and smaller.
(b) Show, by manipulating Eq. (1) above, that this way of calculating e is equivalent to

$$e = \lim_{N \to \infty} \left(1 + \frac{1}{N}\right)^N .$$

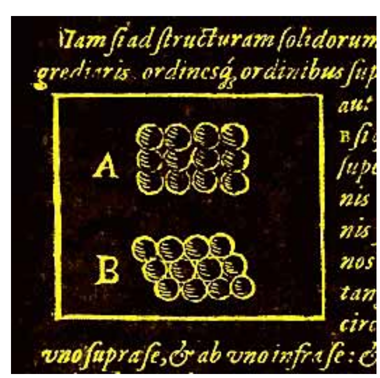
In addition, make a second table evaluating e for various values of N, showing that your estimate for e gets more precise as N increases.

Dec 28 One million barrels of oil is spilled into the ocean. (a) Assuming that the resulting oil slick covers all the oceans of the Earth, estimate the thickness (in meters) of the slick? (b) How many atoms (approximately) thick is this slick? (1 US barrel = 42 US gallons)

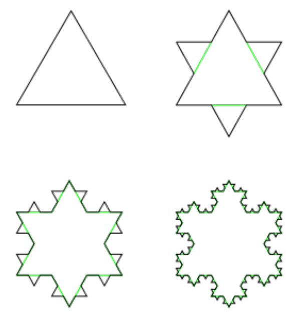
Dec 29 Consider a planet of radius R with an atmosphere of thickness h. How long is your line of sight through the atmosphere (i.e., x) when you look horizontally? Express x as a function of h and R, and then evaluate it for the Earth (take $h = 60$ km). NOTE: This also gives the distance x from your eyes to the horizon when your eyes are a height h above the Earth. (Partial answer: for $h = 2$ m, $x \approx 5$ km.)



Dec 30 How dense can you pack pennies in a plane? If you line them up in a rectangular array - as shown in the top figure A - you can show that the fraction of the area covered is $\pi/4 \approx 0.785$, which means that the pennies cover about 78.5% of the surface. If, however, you line them up hexagonally - as shown in the bottom figure B - you get a denser packing. Calculate the fractional area covered in this case. (The figure is from Kepler's 1611 book "On the six sided snowflake," where he investigated packing spheres in three dimensions.)

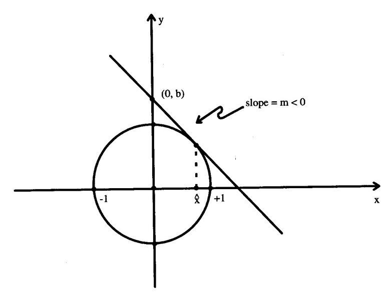


Dec 31 An interesting curve that is a "fractal" can be created as follows. Draw an equilateral triangle whose sides are length ℓ, then recursively alter each line segment as follows: 1) divide each line segment into three segments of equal length, 2) draw an equilateral triangle that has the middle segment from step 1 as its base and points outward, 3) remove the line segment that is the base of the triangle from step 2, 4) repeat. The first 4 iterations are shown in the figure. The perimeter diverges, i.e., is infinite, because at each step the perimeter increases by a factor of 4/3. However, the area enclosed by the curve as the number of iterations goes to infinity remains finite. Calculate this area.



Jan 1 Three blue socks and six red socks are in a drawer. If two socks are drawn at random from the drawer, what is the probability the two form a matching pair?

Jan 2 Consider a circle of unit radius centered on the origin (described by the equation $x^2 + y^2 = 1$). Consider a point on the y-axis at position $(0, b)$, where $b > 1$. Draw a line through $(0, b)$ that is tangent to the circle and also intersects the positive x-axis. Find (a) the slope m of the line, and (b) the x-coordinate (call it $\hat{x}$) of the tangent point.

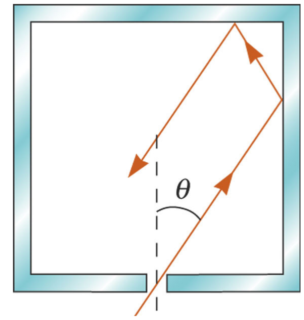


Jan 3 The five Platonic solids, cube, tetrahedron, dodecahedron, icosahedron, and octahedron, were the basis for Johannes Kepler's model of the solar system. He related the orbits of the six planets known at that time to the five Platonic solids. In *Mysterium Cosmographicum*, published in 1596, Kepler proposed a model of the solar system in which the five solids were set inside one another and separated by a series of inscribed and circumscribed spheres. The spheres represented the planetary orbits, and the solids acted as spacers.

Saturn's orbit, the outermost sphere, circumscribed a cube with a sphere (Jupiter's orbit) inscribed inside the cube. The order of the solids was that listed above: Saturn, cube, Jupiter, tetrahedron, Mars, dodecahedron, Earth, icosahedron, Venus, octahedron, and Mercury. This model was intended to predict the correct relative ratios of the planetary orbits, but not the absolute size of the solar system.

Calculate the relative sizes of the first two spheres - Saturn's and Jupiter's orbits - and compare your answer to the actual ratio of those planets' semi-major axes. What is the percentage error of Kepler's model?

Jan 4 A ray of light enters a square enclosure though a small hole in the center of one side. All the interior surfaces are mirrored. It is obvious that if $\theta = 45°$ the ray will exit the hole after being reflected once by each of the three mirrors. Determine a formula for θ that will give all the other angles that will result in the ray exiting through the hole after an arbitrary number of reflections from the interior mirrors.



Jan 5 In your sock drawer, you have some red socks and some blue socks. What is the smallest number of each type of sock that must be in the drawer so that if you remove two socks to wear, the probability that you get two red socks is 50%? What is the next smallest number?

Jan 6 Two boats leave from opposite banks of a river at the same time and travel at constant but different speeds. They pass each other 700 yards from one bank and continue to the other side of the river, where they turn around. On their return trip the boats pass again—this time 400 yards from the opposite bank. How wide is the river?