Chapter 2

Fundamental Theoretical Formulation: The Microscopic Kinetic Equation

A. Maxwell's and Newton's Equations

We start from the simplest and most basic description of a collection of charged particles: Maxwell's equation for the electromagnetic field and Newton's equation (or its quantum mechanical equivalent, Schrodinger's equation) for the particle motion. The sources of the electromagnetic field are any external charges and current plus those due to the plasma particles.

We shall adopt a classical point of view here, using Newton's equation for the mechanical motion and treating the electrons and ions as point particles. So long as the de Broglie wavelength, $\sqrt{h^2/2\pi mT}$, for each particle is smaller than any other length in the problem, this is a good approximation.[1] In the high density/low temperature regime where quantum mechanical effects become important, the description of the mechanical motion must be appropriately modified and, more importantly, other expansion parameters replace ε_p. This regime is outside the scope of the text.

Our basic equations are then

$$m_i\dot{\mathbf{v}}_i = m_i\ddot{\mathbf{x}}_i = q_i\left(\tilde{\mathbf{E}} + \mathbf{v}_i \times \tilde{\mathbf{B}}/c\right) \quad (2.1)$$

$$i = 1, 2, ..., N_0 \quad (2.2)$$

and

$$\nabla \cdot \tilde{\mathbf{E}} = 4\pi\left(\rho_p + \rho_e\right) \quad (2.3)$$

$$\nabla \times \tilde{\mathbf{E}} + \dot{\tilde{\mathbf{B}}}/c = 0 \quad (2.4)$$

$$\nabla \times \tilde{\mathbf{B}} = \frac{4\pi}{c}\left(\mathbf{j}_p + \mathbf{j}_e + \dot{\tilde{\mathbf{E}}}/4\pi\right) \quad (2.5)$$

$$\nabla \cdot \tilde{\mathbf{B}} = 0 \quad (2.6)$$

where the subscripts p, e denote plasma particle and external source terms, respectively, and N_0 is the total number of particles. Since this is a microscopic theory, we need only one

[1] For electrons, this length is $\lambda_e = 3 \times 10^{-8}\text{cm}/\sqrt{T_{\text{eV}}}$.

electric field vector, $\tilde{\mathbf{E}}$, and one magnetic field vector, $\tilde{\mathbf{B}}$. All "material" effects are contained in the charge and current densities,

$$\rho_p(\mathbf{x},t) = \sum_{i=1}^{N_0} q_i \; \delta[\mathbf{x} - \mathbf{x}_i(t)] \tag{2.7}$$

and

$$\mathbf{j}_p(\mathbf{x},t) = \sum_{i=1}^{N_0} q_i \; \mathbf{v}_i(t) \; \delta[\mathbf{x} - \mathbf{x}_i(t)], \tag{2.8}$$

and since we shall be examining these in great detail, there is no particular advantage in introducing the auxiliary fields **D** and **H**.

B. The Microscopic Distribution Function and Kinetic Equation

Our study of plasma physics is based entirely on the set of equations (2.1), (2.3), and (2.7). They are exact, but, of course not soluble in closed form, since the total number of particles, N_0, is typically of order 10^6 or larger. It is convenient to reformulate (2.1) and (2.7) by introducing the concept of the microscopic distribution function of Klimontovich. For each species, σ, we define

$$\tilde{f}_\sigma(\mathbf{x},\mathbf{v},t) = \frac{1}{\bar{n}_\sigma} \sum_{i=1}^{N_\sigma} \delta[\mathbf{x} - \mathbf{x}_i(t)] \; \delta[\mathbf{v} - \mathbf{v}_i(t)] \tag{2.9}$$

where N_σ is the total number of particles of this species and $\bar{n}_\sigma$ is their average density, $\bar{n}_\sigma = N_\sigma/V$, V being the volume of the system. In terms of $\tilde{f}_\sigma$, we have from (2.7)

$$\rho_p(\mathbf{x},t) = ⨍ d^3v \;\; \bar{n} q \tilde{f}(\mathbf{x},\mathbf{v},t) \tag{2.10}$$

$$\mathbf{j}_p(\mathbf{x},t) = ⨍ d^3v \;\; \bar{n} q \mathbf{v} \tilde{f}(\mathbf{x},\mathbf{v},t) \tag{2.11}$$

where the symbol ⨍ means integration over $\mathbf{v}$ *and* summation over the species index σ. (Note that in (2.10) we have suppressed the species index, as we shall generally do in order to minimize notational clutter.) Our choice of normalization,[2] whose advantages will be apparent later, gives

$$\int d^3x d^3v \tilde{f} = V. \tag{2.12}$$

It also follows that the integral of $\tilde{f}_\sigma/V$ over any region R of the $(\mathbf{x},\mathbf{v})$ phase space is equal to the fraction of particles of species σ contained in R. Thus, $\tilde{f}_\sigma$ has the character of a distribution function in the $(\mathbf{x},\mathbf{v})$ phase space. We call it the microscopic distribution

[2] Another choice, made by Lenard, is to normalize to the total number of particles N, $\int d^3x d^3v \tilde{f} = N$. The difference between the two is a factor of $\bar{n}$, which appears in our equations, but not Lenard's. A third choice, made by Ichimaru, is to use box normalization, which essentially means making the replacement $\int d^3k \to \sum_k (2\pi)^3/V$.

function to distinguish it from the function we will obtain presently by taking a suitable average of $\tilde{f}_\sigma$.

Using the equations of motion (2.1) and the definition (2.9) of $\tilde{f}_\sigma$, we have

$$\frac{\partial}{\partial t}\tilde{f}(\mathbf{x},\mathbf{v},t) = -\frac{1}{\bar{n}}\sum_{i=1}^{N}\left[\mathbf{v}_i\cdot\nabla\delta\left(\mathbf{x}-\mathbf{x}_i\right)\delta\left(\mathbf{v}-\mathbf{v}_i\right)+\mathbf{a}_i\cdot\nabla_{\mathbf{v}}\delta\left(\mathbf{v}-\mathbf{v}_i\right)\delta\left(\mathbf{x}-\mathbf{x}_i\right)\right] \tag{2.13}$$

where $\mathbf{a}_i = \dot{\mathbf{v}}_i$ is the acceleration of particle i. We now take advantage of the delta functions to replace $\mathbf{v}_i$ by $\mathbf{v}$ in the first term and $\mathbf{a}_i$ by $(q/m)(\tilde{\mathbf{E}}+\mathbf{v}\times\tilde{\mathbf{B}}/c)$ in the second, $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ being the fields at $\mathbf{x}$ (we have again suppressed the species indices). Thus, we have replaced the equations of motion (2.1) by the **microscopic kinetic equations** (one for each species)

$$\frac{\partial\tilde{f}}{\partial t}+\mathbf{v}\cdot\frac{\partial\tilde{f}}{\partial\mathbf{x}}+\frac{q}{m}\left(\tilde{\mathbf{E}}+\frac{1}{c}\mathbf{v}\times\tilde{\mathbf{B}}\right)\cdot\frac{\partial\tilde{f}}{\partial\mathbf{v}}=0. \tag{2.14}$$

While (2.14) (which is also referred to as the Klimontovich equation[3]) is deceptively simple in appearance, involving only 6 independent phase space variables plus the time, it is nonlinear. Even more important, $\tilde{f}$ is stochastic, i.e., a wildly varying function of $\mathbf{x}$, $\mathbf{v}$, and t, due to the delta functions involved in its definition. Naturally, none of the many-body complexity of (2.1) has been eliminated. Since we have simply reformulated the problem without introducing any approximations, (2.14) is exactly equivalent to (2.1).

C. Ensemble Averages and Fluctuations

As in all statistical mechanical problems, and exact formulation is of little practical value. The quantities of principle physical interest are just the *average* properties and the fluctuations about these. We therefore introduce the usual notion of *statistical ensemble*, i.e., a set of very many (conceptual) copies of the physical system, each having the same observable, macroscopic properties (density, mean velocity, temperature, etc.) but differing in the microscopic variables, e.g., the initial particle positions and velocities. We denote the ensemble averages by

$$f \equiv \langle\tilde{f}\rangle \tag{2.15}$$

$$\mathbf{E} \equiv \langle\tilde{\mathbf{E}}\rangle \tag{2.16}$$

$$\mathbf{B} \equiv \langle\tilde{\mathbf{B}}\rangle \tag{2.17}$$

No special notation is required for the external sources, since these will be the same for each member of the ensemble. Thus

$$\langle\rho\rangle = \rho_e + ⨍ d\mathbf{v}\,\bar{n}qf, \tag{2.18}$$

$$\langle\mathbf{j}\rangle = \mathbf{j}_e + ⨍ d\mathbf{v}\,\bar{n}q\mathbf{v}f. \tag{2.19}$$

In accordance with the usual ideas of statistical mechanics, the ensemble averages $\mathbf{E}$ and $\mathbf{B}$ are to be identified with the physical electromagnetic fields in an experiment. The

[3]reference

averaged distribution function, f, has just the significance of the single particle distribution function of classical kinetic theory: $\bar{n}f(\mathbf{x},\mathbf{v},t)$ is the number of particles per unit phase space volume that would be found at time t in the vicinity of the phase space point $(\mathbf{x},\mathbf{v})$. [We shall sometimes use ξ to denote the set $(\mathbf{x},\mathbf{v})$.] Thus, f, $\mathbf{E}$, and $\mathbf{B}$ are the principal physically significant, macroscopic dynamical variables. Next most important are fluctuations about these averages:

$$\delta\tilde{f} \equiv \tilde{f} - f \tag{2.20}$$
$$\delta\tilde{\mathbf{E}} \equiv \tilde{\mathbf{E}} - \mathbf{E} \tag{2.21}$$
$$\delta\tilde{\mathbf{B}} \equiv \tilde{\mathbf{B}} - \mathbf{B} \tag{2.22}$$

where, by definition,

$$\langle\delta\tilde{f}\rangle = \langle\delta\tilde{\mathbf{E}}\rangle = \langle\delta\tilde{\mathbf{B}}\rangle = 0 \tag{2.23}$$

It will often simplify the notation to dentoe the total Lorentz force per unit charge by

$$\tilde{\mathbf{F}} = \tilde{\mathbf{E}} + \mathbf{v}\times\tilde{\mathbf{B}}/c \tag{2.24}$$

with

$$\mathbf{F} = \langle\tilde{\mathbf{F}}\rangle, \tag{2.25}$$
$$\delta\tilde{\mathbf{F}} = \tilde{\mathbf{F}} - \mathbf{F}. \tag{2.26}$$

Taking the ensemble average of (2.14) gives an equation for the single particle distribution function, f,[4]

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{F}\cdot\frac{\partial f}{\partial \mathbf{v}} = -\frac{q}{m}\left\langle \delta\tilde{\mathbf{F}}\cdot\frac{\partial\delta\tilde{f}}{\partial\mathbf{v}}\right\rangle \tag{2.27}$$

and subtracting this from (2.14) gives an equation for the fluctuations,[5]

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{q}{m}\mathbf{F}\cdot\nabla_{\mathbf{v}}\right]\delta\tilde{f} + \frac{q}{m}\delta\tilde{\mathbf{F}}\cdot\nabla_{\mathbf{v}}f = -\frac{q}{m}\nabla_{\mathbf{v}}\cdot\left[\delta\tilde{\mathbf{F}}\delta\tilde{f} - \langle\delta\tilde{\mathbf{F}}\delta\tilde{f}\rangle\right]. \tag{2.28}$$

Since Maxwell's equations are linear, their partition into average and fluctuation parts is trivial

$$\nabla\cdot\mathbf{E} = 4\pi\left(\rho_e + \textstyle\fint d\mathbf{v}\bar{n}qf\right) \tag{2.29}$$
$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\left(\mathbf{j}_e + \textstyle\fint d\mathbf{v}\bar{n}q\mathbf{v}f\right) + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{2.30}$$
$$\nabla\times\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = 0 \tag{2.31}$$
$$\nabla\cdot\mathbf{B} = 0 \tag{2.32}$$

[4]The ensemble average of a product is given by $\langle\tilde{A}\tilde{B}\rangle = AB + \langle\delta\tilde{A}\delta\tilde{B}\rangle$.
[5]The velocity divergence symbol stands for $\nabla_{\mathbf{v}} = \frac{\partial}{\partial\mathbf{v}} = \hat{\mathbf{x}}\frac{\partial}{\partial v_x} + \hat{\mathbf{y}}\frac{\partial}{\partial v_y} + \hat{\mathbf{z}}\frac{\partial}{\partial v_z}$.

and

$$\nabla \cdot \delta\tilde{\mathbf{E}} = 4\pi \left(\rho_e + ⨍ d\mathbf{v}\bar{n}q\delta\tilde{f}\right) \tag{2.33}$$

$$\nabla \times \delta\tilde{\mathbf{B}} = \frac{4\pi}{c}\left(\mathbf{j}_e + ⨍ d\mathbf{v}\bar{n}q\mathbf{v}\delta\tilde{f}\right) + \frac{1}{c}\frac{\partial\delta\tilde{\mathbf{E}}}{\partial t} \tag{2.34}$$

$$\nabla \times \delta\tilde{\mathbf{E}} + \frac{1}{c}\frac{\partial\delta\tilde{\mathbf{B}}}{\partial t} = 0 \tag{2.35}$$

$$\nabla \cdot \delta\tilde{\mathbf{B}} = 0 \tag{2.36}$$

Again, we are simply making definitions, and the set (2.27) through (2.33) is *identical*, as regards both contents and difficulty, with the original set (2.1) through (2.7). However, the problem is now formulated in a way to facilitate the approximations necessary if we are to make any progress. We see that (2.27) and (2.29) would constitute a closed set of equations for the ensemble averages f, $\mathbf{E}$, and $\mathbf{B}$ if only we knew enough about the fluctuations to compute the average value $\langle\delta\tilde{\mathbf{F}}\delta\tilde{f}\rangle$ which occurs on the right side of (2.27). On the other hand, if we knew f, $\mathbf{E}$, and $\mathbf{B}$, we need only solve the equations (2.28) and (2.33) for the fluctuations (a task made formidable, of course, by their nonlinear character). At the very least, some approximation scheme to decouple the fluctuations from the ensemble averages would be helpful.

D. The Expansion in Fluctuations

The rationale of the method of approximation which we shall use is very simple: we suppose the fluctuations to be, in some sense, "small," and we therefore expand in the fluctuations. Specifically, to lowest order, we neglect terms of second order in the fluctuations, such as the right hand side of (2.27); this completely decouples (2.27) and (2.29) from (2.28) and (2.33). Thus, to lowest order (we shall call it first order, since the next order involves retention of terms quadratic in the fluctuations) we have just an ensemble-averaged kinetic equation, plus the ensemble-averaged Maxwell equations,

$$\mathcal{L}f \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{q}{m}\mathbf{F}\cdot\nabla_{\mathbf{v}}\right) f = 0 \tag{2.37}$$

$$\nabla \cdot \mathbf{E} = 4\pi \left(\rho_e + ⨍ d\mathbf{v}\bar{n}qf\right) \tag{2.38}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\left(\mathbf{j}_e + ⨍ d\mathbf{v}\bar{n}q\mathbf{v}f\right) + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{2.39}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = 0 \tag{2.40}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.41}$$

This set of equations was first written down, on phenomenological grounds, by A. Vlasov[6] and is generally referred to by his name, although the misnomer "collisionless Boltzmann

[6] reference

equation" is sometimes used. We shall refer to this lowest order of the expansion as the **Vlasov approximation**. The differential operator $\mathcal{L}$ is sometimes called the Vlasov operator.

In the second order we retain the quadratic terms, $\langle\delta\tilde{\mathbf{F}}\delta\tilde{f}\rangle$ on the right hand side of (2.27) but neglect terms of third order in the fluctuations. When (2.28) and (2.33) are solved for $\delta\tilde{f}$ and $\delta\tilde{\mathbf{F}}$, the contributions of the right hand side of (2.28) will lead to terms of third order in (2.27), so in this order we can neglect the right hand side of (2.28) from the start. We then have

$$\mathcal{L}f = -\frac{q}{m}\nabla_{\mathbf{v}}\cdot\langle\delta\tilde{f}\delta\tilde{\mathbf{F}}\rangle \quad (2.42)$$

$$\mathcal{L}\delta\tilde{f} + \frac{q}{m}\delta\tilde{f}\cdot\nabla_{\mathbf{v}}f = 0 \quad (2.43)$$

plus the Maxwell equations (2.29) and (2.33) for the self-consistent determination of $\mathbf{F}$ and $\delta\tilde{\mathbf{F}}$. We shall designate this as the **quasilinear approximation** since (2.43) is linear in $\delta\tilde{f}$, albeit nonlinear terms are retained in equation (2.42) for f. The fluctuations modify the average distribution, f, but interactions among the fluctuations, such as mode coupling, are neglected.

Finally, in third order we retain the right hand side of (2.28), so that the third order equations are formally identical with the exact equations, (2.27) through (2.33). As we shall see later, the approximation consists in solving (2.27) and (2.33) by a perturbation expansion, keeping only terms of third order in the fluctuations. Only in this order do we have mode coupling of fluctuations, nonlinear wave-particle interactions, self-interaction of large-amplitude waves and similar exotic phenomena, so we may describe it as the **nonlinear wave approximation**.

For a plasma in equilibrium, one can prove that this "expansion in fluctuations" is tantamount to an expansion in the plasma parameter, ε_p, and hence well justified if $\varepsilon_p \ll 1$. However, the most interesting problems in plasma physics involve non-equilibrium phenomena, where this expansion procedure can really be justified only on an *a posteriori* basis, for each problem.

To avoid confusion with other treatments, we should emphasize that even within the Vlasov approximation one may make an expansion in the fields $\mathbf{E}$ and $\mathbf{B}$ (or in their deviation from the values characterizing some elementary solution), and hence encounter equations of "second order" or "third order" in the fields. These equations will be formally similar to those describing what we have called the quasilinear and nonlinear waves approximations, simply because of the obvious formal similarity between the Vlasov and Klimontovich equations. The possibility of confusion is compounded by the circumstance that in many problems the formal analysis (expansion in diagrams, etc.) may be quite similar. However, the physical interpretations are quite different, since within the Vlasov approximation we deal *only* with ensemble-averaged quantities, whereas the quasilinear and nonlinear wave approximations involve, in an essential way, stochastic variables.

E. An Overview

In subsequent chapters we shalll study systematically the consequences of these various orders of approximation. Before doing so, we make a few comments on their general properties:

1) The Vlasov approximation shows clearly the self-consistent aspect of plasma physics, with f determined by $\mathbf{E}$ and $\mathbf{B}$, whose sources are determined by f, along with ρ_e and $\mathbf{j}_e$.

2) Most of our understanding of the properties of the Vlasov system is based on a linearization of f about some time and space independent "equilibrium" or "unperturbed" function, $f_0(\mathbf{v})$. Since *any* $f_0(\mathbf{v})$ satisfies (2.37) when $\mathbf{F} = 0$, we must look elsewhere for guidance in making a sensible choice for f_0. For this, we need to consider the second order effects. The neglect of fluctuations at the Vlasov level means that of the total force on a given particle we are including only the average part, $\mathbf{F}$, and ignoring the rapidly fluctuating portions which arise from the discrete, particulate character of the plasma. However, it is just the latter which determines the equilibrium f_0. In fact, the $\langle \delta\tilde{f}\delta\tilde{\mathbf{F}}\rangle$ term in (2.27) corresponds to two physical effects:

1. "close" collisions (meaning those with impact parameter less than the Debye length, as we shall see later); and
2. "quasilinear" modifications of the average distribution function by the fluctuations.

For a plasma near equilibrium, where the fluctuations are, and remain, small, of order ε_p, only the first of these two effects is important and the right hand side of (2.27), which we write as

$$\frac{\delta f}{\delta t} \equiv -\frac{q}{m}\nabla_{\mathbf{v}} \cdot \langle \delta\tilde{\mathbf{F}}\delta\tilde{f}\rangle, \tag{2.44}$$

satisfies the H-theorem, i.e., tends to drive f_0 towards a Maxwellian distribution,

$$f_M(\mathbf{v}) = \frac{\exp\left(-v^2/a^2\right)}{a^3\pi^{3/2}}. \tag{2.45}$$

We shall often make this choice for f_0.

3) For many purposes, even the Vlasov description is too difficult to solve, and we deal instead with the velocity moments of f, i.e., density n, mean velocity $\mathbf{u}$, and pressure tensor p, as functions of $\mathbf{x}$ and t. It is easy to derive equations for these variables from (2.37), albeit the set does not close without further approximations. This leads to "two-fluid magnetohydrodynamics (MHD)," so-called because there are equations and dependent variables for each of the two (or more) species in the plasma. A further approximation, valid at low frequencies and large wavelengths, reduces this to "one fluid MHD." These fluid approximations are strictly justified only in the case of large collision frequencies, since the short mean free path tends to preserve the initial grouping of particles. However, they often give a good account of many phenomena, even when their use is not clearly justified, probably because they represent the basic conservation laws—mass, momentum, and energy.

4) It is often useful to make the "electrostatic approximation," neglecting the $\mathbf{v} \times \mathbf{B}$ part of the force. This represents an enormous simplification for the analysis, but it must be justified in each particular context.

5) So far as an external magnetic field, $\mathbf{B}_0$, is concerned, the simplest case is, of course, $\mathbf{B}_0 = 0$, and we shall consider that first in discussing the Vlasov equation. Next simplest is the case of very strong $\mathbf{B}_0$ (i.e., the cyclotron frequency is much larger than all other significant frequencies and the cyclotron radius is much smaller than all other significant lengths) when the Alfvén guiding center approximation and related techniques, which we shall discuss later, are applicable.

6) The relation amongst the various approximations or "models" of the plasma can be summarized in a block diagram (Fig 2.1).

7) Many of the approximations depicted here can be derived directly, on phenomenological grounds. For example, if we simply introduce a density function, $f(\mathbf{x}, \mathbf{v}, t)$, in six-dimensional phase space, then in the absence of "collisions" between particles, conservation of particles gives a six dimensional continuity equation,

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{v} f) + \nabla_{\mathbf{v}} \cdot \left(\frac{d\mathbf{v}}{dt} f \right) = 0. \tag{2.46}$$

With external fields $\mathbf{E}_e$ and $\mathbf{B}_e$ given by

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m} (\mathbf{E}_e + \mathbf{v} \times \mathbf{B}_e / c) \tag{2.47}$$

$$\nabla_{\mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \tag{2.48}$$

so that

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q}{m} (\mathbf{E}_e + \mathbf{v} \times \mathbf{B}_e / c) \cdot \nabla_{\mathbf{v}} \right) f = 0. \tag{2.49}$$

If we allow the "external" fields to have as sources also the plasma charge and current densities described by f, i.e., replace $\mathbf{E}_e$ and $\mathbf{B}_e$ with $\mathbf{E}$ and $\mathbf{B}$ satisfying

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q}{m} \mathbf{F} \cdot \nabla_{\mathbf{v}} \right) f = 0 \tag{2.50}$$

$$\nabla \cdot \mathbf{E} = 4\pi \left(\rho_e + \textstyle\sum\!\!\!\!\int d\mathbf{v}\, \bar{n} q f \right) \tag{2.51}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \left(\mathbf{j}_e + \textstyle\sum\!\!\!\!\int d\mathbf{v}\, \bar{n} q \mathbf{v} f \right) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \tag{2.52}$$

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2.53}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.54}$$

then we have just the Vlasov formulation. As we will see later, this includes, in the "self-consistent" fields, $\mathbf{E} - \mathbf{E}_e$ and $\mathbf{B} - \mathbf{B}_e$, the particle interactions associated with impact parameter, b, greater than L_D and neglects the "close" collisions, $b < L_D$. It is the latter which are described by the $\langle \delta \tilde{f} \delta \tilde{\mathbf{F}} \rangle$ terms neglected in the Vlasov approximation.

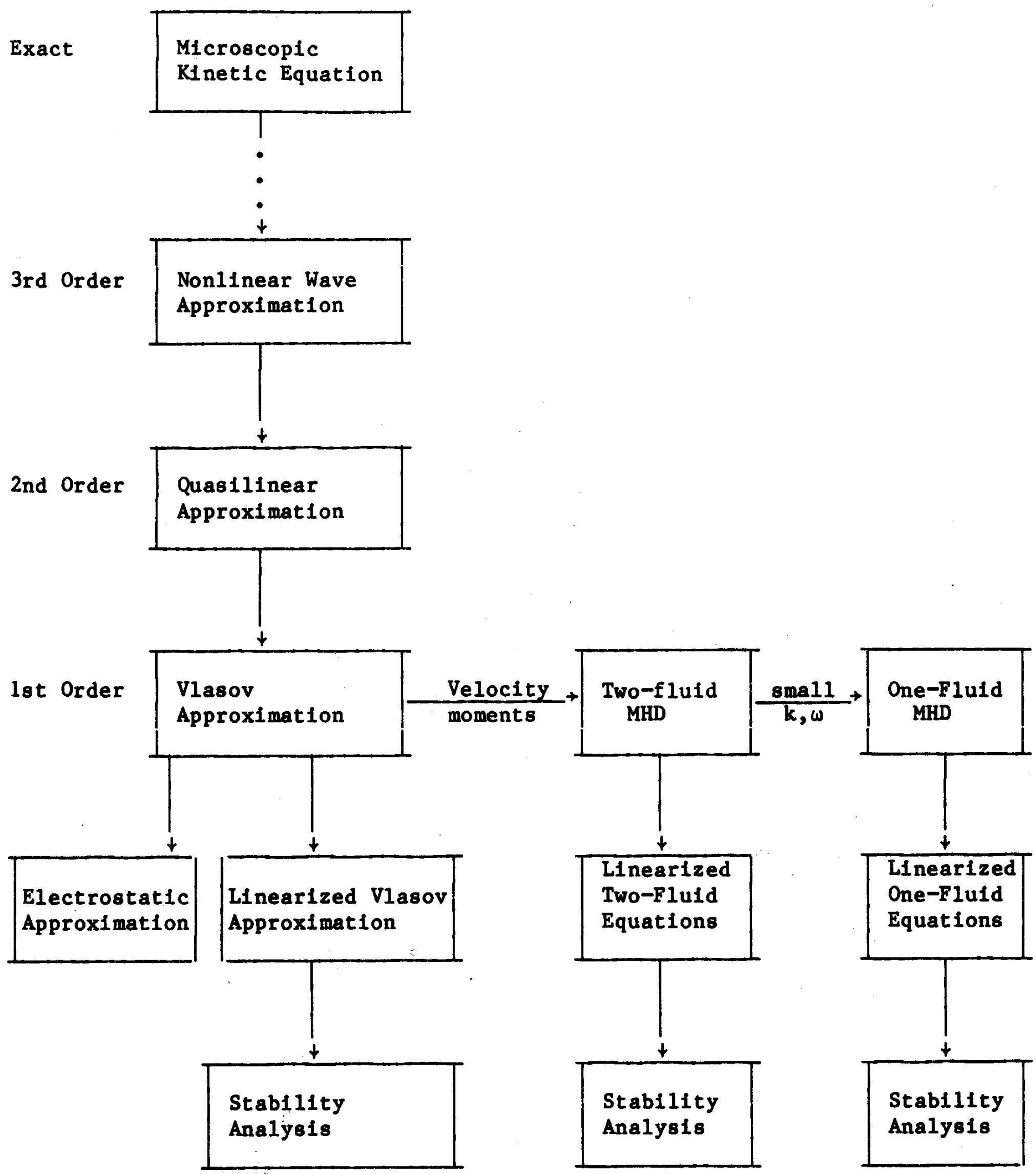

Figure 2.1: Ordering by expansion in fluctuations.