Appendix G

Cosmic Rays and Muons

> *Coming out of space and incident on the high atmosphere, there is a thin rain of charged particles known as the primary cosmic radiation.* — Cecil Powell [Nobel Prize, Physics, 1950]

The charged particles that make up the "primary" cosmic rays are protons, α particles, heavier nuclei, and electrons, and they impact the Earth from all directions and with various energies. Most of these are protons (about 80%), second in abundance are α particles (about 14%), while electrons make up less than 1%. When they impact nuclei in the atmosphere — mostly oxygen and nitrogen nuclei — their energies are such that they create "showers" of hadrons, mostly pions, along with some kaons, and anti-protons, and anti-neutrons. These then decay into photons, electrons, positrons, neutrinos, and muons (which themselves decay into electrons and neutrinos). These are all called "secondary" cosmic rays.

Where do the primary cosmic rays come from? Some come from the sun (mostly due to solar flares), most come from galactic supernovae, and a few with the highest energy are suspected to originate from outside the Milky Way. You might suspect the solar wind—a neutral plasma that consists of low energy protons, electrons, and helium nuclei—as a source of cosmic rays. Due to their low energies, however, these particles are stopped from reaching the atmosphere by the Earth's magnetic field, except in the polar regions. While they have enough energy to cause aurora, they do not cause showers of secondary subatomic particles.

How many are there? About 1 charged particle per second per cm^2 impacts the Earth.[1] This is a far cry from the 6×10^{10} neutrinos s^{-1} cm^{-2} that come from the Sun.

What are their energies? The typical kinetic energy of these particles is about 10 MeV to 100 MeV, although there are some at higher energies. Figure G.1 shows the distribution of the measured energy per particle. In fact, the cosmic ray with the highest energy has been measured at 48 J! These ultra-high energy cosmic rays are suspected to be extra-galactic, as there is no plausible mechanism of acceleration to these energies by

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[1] Henley and Garcia, *Subatomic Physics*, page 597.

a supernova, for example. Again, compare these energies to those of solar neutrinos that have only 0.26 MeV.

What happens to the secondary cosmic rays? The pions decay via the following modes

$$\begin{aligned} \pi^0 &\rightarrow 2\gamma & \text{(G.1)} \\ \pi^\pm &\rightarrow \mu^\pm + \nu, & \text{(G.2)} \end{aligned}$$

where the neutral pions decay electromagnetically with an average lifetime of 8.4×10^{-17} s, and the photons subsequently create electron-positron pairs. Most of the energy of the original cosmic ray follows this path. Some of the energy goes into charged pions, which decay into muons with an average lifetime of 2.6×10^{-8} s. This long lifetime indicates that the decay is due to the weak interaction, and is therefore relatively unlikely. The muons then decay into electrons (or positrons) and neutrinos

$$\mu^\pm \rightarrow e^\pm + 2\nu, \qquad \text{(G.3)}$$

and their average lifetime is 2.2 μs, also a weak interaction.[2]

What happens to these secondary cosmic rays as they pass through the atmosphere? First of all, in addition to possible decay, the charged particles cause ionization of the atmospheric molecules and therefore lose energy. For example, a typical muon loses about 2 GeV of kinetic energy before it hits the ground (if it hasn't decayed yet), and by the time they do reach the ground, the average muon energy is about 4 GeV. Secondly, the showers spread out laterally from the direction of the primary cosmic ray. The main hadronic core (pions, etc.) covers a few meters by the time it hits the ground, and the electromagnetic particles (electrons, positrons, photons) have spread further, about 100 m. Finally, the muons have spread the furthest, almost 1 km.

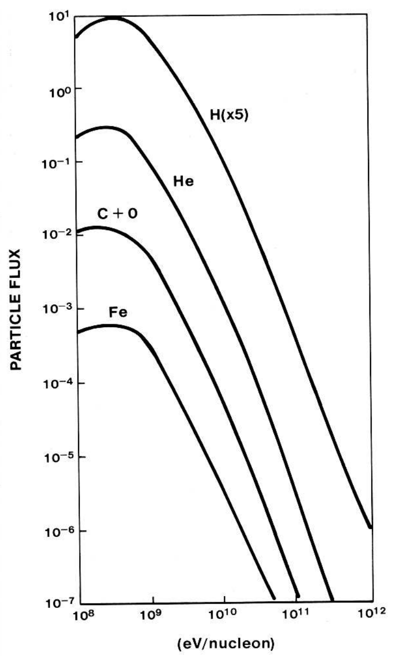

Figure G.1: The energy spectrum of the different nuclei that make up cosmic rays. Carbon and oxygen are lumped together. From Friedlander, *Cosmic Rays*, Figure 6.4.

Muons as clocks

This spreading means that muons are continually bombarding the Earth's surface and, since it is not clear what direction they came from, statistical methods must be used to interpret the muon flux. That is, the muons are all "born" at different altitudes, they travel downward with different speeds, and they "live" for different intervals of time. Therefore, you might expect that the muon flux would increase with increasing altitude, at least initially, reach a maximum at some altitude, and then finally decrease. This is precisely

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[2] Recall that the weak force is responsible for changing one family of quarks or leptons into another.

what is observed, but the exact shape of this curve is a convolution of a source function and a decay function, and therefore requires lots of modeling to interpret.

However, for our purpose — special relativity — we want to use the muons as a clock. In Chapter 5 we assume that our muons are all created at the same altitude, and all live for the same amount of time, 2.2 μs. You might think that we are not justified in doing this, because of the statistical spread of muon lifetimes, but that turns out not to be true. Scott and Burke state the case:

> It may seem at first glance that a real particle that is formed and later decays does not constitute an accurate clock, because of the uncertain nature of the decay process. Given a number of particles, some will decay at times less than the mean life, some will decay at times greater than the mean life, and in general it is impossible to predict exactly when any given particle will decay. However, it is possible to determine the *mean* lifetime of a number of particles to any desired accuracy simply by observing a sufficient number of such particles, and in this sense, decaying particles are just as good clocks as vibrating molecules. Indeed, for a vibrating molecule it is necessary to observe it for a large number of cycles in order to determine its frequency precisely; this is analogous to observing a large number of decays in an exponentially decaying system.[3]

Collateral Reading

- "The early history of cosmic ray research." by Q. Xu and L. M. Brown, *Am. J. Phys.*, **55** 23-33 (1987).
- Michael W. Friedlander, *Cosmic Rays*, Harvard University Press, 1989. (ERAU: `QC 485 .F75 1989`)

Problems

1. Calculate the energy in MeV of a 48-J proton. Also calculate γ and β for the same proton.
2. (a) Calculate the reaction energy for a pion decaying into a muon and a neutrino. (b) Using the conservation of momentum, calculate how much energy the muon has. HINT: You can assume the muon is non-relativistic (check this), but you must take relativistic effects into account for the neutrino. One approximation is to take the highly relativistic limit for the neutrino, where the relationship between its energy and momentum is $E_\nu = p_\nu c$. As usual, ignore the neutrino mass.
3. Why can't a π^0 decay into a μ^- and a μ^+?
4. If a muon μ^- is "born" due to a pion decay $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ at an altitude of 20 km, how fast must it be traveling to reach the ground before it decays 2.2 μs later? Express your answer in the form $\beta = 1 - \epsilon$, and calculate ϵ.

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[3] Scott and Burke, *Special Relativity Primer*, page 5.