QUESTIONS AND ANSWERS

Contributions to this section, both Questions and Answers, are welcomed. Please submit four copies to the editorial office. Please include a *title* for each submission, include name and address at the end, and put references in the standard format used in the American Journal of Physics. For further suggestions, sample Questions and Answers, and requested form for both Questions and Answers, see Robert H. Romer, "Editorial: 'Questions and Answers,' a new section of the American Journal of Physics," Am. J. Phys. **62** (6) 487–489 (1994).

Questions at any level and on any appropriate AJP topic, including the "quick and curious" question, are encouraged.

Question #61. Air bubble in an ultracentrifuge

High-speed ultracentrifuges are used to separate particles from the less dense medium in which they are suspended when sedimentation by ordinary gravitational forces fails. The effective gravitational force (centrifugal force) that such a particle experiences is generally large enough to overcome both the buoyant and viscous drag forces so that the particle migrates away from the axis of rotation and lodges at the bottom of the test tube.

What happens if an air bubble is trapped in a fluid-filled test tube when the tube is placed in an ultracentrifuge? Presumably, since it is less dense than the medium in which it finds itself, the bubble should, at least initially, move toward the axis of rotation of the centrifuge. Will such a bubble "maintain its integrity," or will it collapse before it reaches the top of the tube?

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Question #62. When did the indeterminacy principle become the uncertainty principle?

Old textbooks[1] refer to the indeterminacy principle of Heisenberg. Current textbooks refer to the uncertainty principle. But uncertainty is not synonymous with indeterminacy. When and why has this principle changed names? Is it a simple issue of translation from German to English? How is this principle called in different languages?

[1]P. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, New York, 1942), p. 272.

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Answer to Question #4. Is there a physics application that is best analyzed in terms of continued fractions?

Neuenschwander[1] asks for physics applications of continued fractions. Although not strictly physics, this application may be of interest to *AJP* readers, and also to our close cousins, the astronomers.

Every 4 years we are reminded that a year cannot be evenly divided into days, so that leap years and other irregularities must be introduced into the calendar to prevent January 1st from drifting through the seasons. With 5 h, 48 min, and 46 s "extra," the ratio of a year to a day is 365.242199. So the problem before us, as designers of a calendar, is to approximate this number as a ratio of integers both as accurately and as simply as possible. The theory of continued fractions provides the best solution (in a well-defined, technical sense) to this problem; historically, it is this property (of approximating numbers) that has been the main driving force behind the study of continued fractions.[2,3]

The continued fraction representation of a number is calculated using an iterative algorithm. Subtracting the integral part of the number (365) leaves a remainder which is necessarily less than 1 (0.242199). The reciprocal of the remainder must then be greater than one (4.1288...). Now the process is repeated on this number, subtracting the integer 4 and leaving a remainder of 0.1288... = 1/7.7617... Continuing in this way, we arrive at the continued fraction

$$365+\cfrac{1}{4+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{3+\dots}}}}\,.$$

If the original number is rational, then eventually a remainder of zero is reached and the fraction terminates; all irrational numbers are represented by infinite continued fractions which converge to them. Since the number 365.242199 is an observational datum, complete with uncertainty, carrying out the algorithm too far yields meaningless precision, as does any other method of representing numbers. The digits displayed above in the fraction are both meaningful and sufficiently precise to devise a calendar superior to the Gregorian calendar presently in use.

Truncating the continued fraction at successive levels produces the approximations 1/4, 7/29, 8/33, 31/128 (added to 365). The first of these is the Julian calendar, adding 1 leap day every 4 years. Higher accuracy is achieved at the expense of larger denominators, requiring more complicated rules for assigning leap years. None of the above fractions, however, is as complicated as the current Gregorian calendar: 1 leap year every 4th year, but not every 100th year, except every 400th year, which *is* a leap year. This gives 97 leap days in 400 years for a ratio of 365 97/400 = 365.2425, producing an average year 26 s longer than the true year. It may be quibbling to fuss over an error of 26 s out of 3×10^7, but note that the fourth fraction is both simpler and more accurate than the Gregorian ratio: By observing leap year every 4th year except the 128th (31 leap years out of 128), we get an error of only 1 s per year. This is, in my opinion, a remarkable result that should have been implemented but was not, due to the facts that 31/128 has no base ten mnemonic and that the Gregorian calendar was evidently devised by trial and error.

[1]Dwight E. Neuenschwander, "Question #4. Is there a physics application that is best analyzed in terms of continued fractions?," Am. J. Phys. **62**(10), 871 (1994).

[2]A. Khinchin, *Continued Fractions* (University of Chicago Press, Chicago, 1964).

[3]C. D. Olds, *Continued Fractions* (Mathematical Association of America, Washington, DC, 1963).

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Answer to Question #6. Faraday's law

There are so many cases in which it is unproblematic to regard induced emf as arising from magnetic field lines cutting across a circuit, and indeed that is how Faraday's law may be introduced in an elementary course. It would seem, however, that the external field strength near the midplane of a solenoid can be made arbitrarily small by making the solenoid arbitrarily long. Then it appears implausible that moving magnetic field lines can account for the induced emf, or, equivalently, for the electric field induced around the solenoid. French inquires whether this electric field may be related to the acceleration fields of the electrons in the solenoid.

As Mills[1] points out in his answer to A. P. French's question, and as several recent papers[2] in this journal have affirmed, there *is* a magnetic field outside a solenoid (or toroid, another conceptually difficult case) when the current is changing. This *quasistatic* field depends not, as a static field does, on I, but rather on dI/dt, and this field is so related to the induced emf as to vindicate the "cutting field lines" picture.

The following comments represent my own coming-to-terms with the solenoid emf paradox.

(1) Electric fields arise directly from changing currents in a way that is conceptually independent of magnetic fields or vector potentials. Solution of Maxwell's equations gives[3]

$$\mathbf{E}=-\int \frac{\left[\mu_0 \dfrac{\partial \mathbf{J}}{\partial t}+\dfrac{1}{\epsilon_0}\nabla \rho\right]_{\text{ret}}}{4\pi r}\, dV.$$

For a circular loop of area a, observed from a distance r (large compared with the size of the loop) in a direction at angle θ from the loop axis, the $\partial \mathbf{J}/\partial t$ term gives

$$E=\frac{\mu_0}{4\pi}\frac{dI/dt}{r^2}\, a \sin\theta,$$

where $\mathbf{E}$ is opposite to the sense of dI/dt in the near side of the loop.[4] Outside a long, thin solenoid in which the current is increasing at a uniform rate, the induced electric field can be obtained straightforwardly by summing up the contributions from the many current loops that comprise the solenoid, and the result agrees, as it must, with what we infer from Faraday's law.

The above result is congenial with French's surmise, although there appears little more to be gained by pushing the analysis all the way to the particle level. Both the quasistatic electric field and the radiation field of a current loop have their origins in the $\partial \mathbf{J}/\partial t$ term (which is proportional to the linear acceleration of the current-carrying electrons) in the solution to Maxwell's equations. The quasistatic field arises from the difference in $1/r$ from the near and far sides of the loop to the observer, while the radiation field arises from the difference in the retarded value of $\partial \mathbf{J}/\partial t$ and is in fact proportional to d^2I/dt^2.[5]

(2) On quite general grounds, emf induced by changing currents must be *associated with* cutting magnetic field lines. Since the electric fields induced by changing currents propagate only at the speed of light, a changing magnetic flux in a solenoid cannot give rise instantly to emf in an encircling wire of large radius.[6] Yet Faraday's law, obtained by integrating $\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$, admits rigorously of no time delay between the change in magnetic flux through a circuit and the emf induced in it. This paradox is resolved by fully exploiting the fact that, since magnetic field lines must (by $\nabla\cdot\mathbf{B}=0$) be continuous, flux can only be created in closed loops.

The field lines threading a solenoid do, after all, emerge from the ends and wrap around outside as the "return field." The flux returning through the midplane of the solenoid is equal to the interior flux, but for a very long solenoid the flux density there is low because the field lines are spread far apart. If the current is increasing, new field lines shed by the windings crowd inward within the solenoid while their exterior continuations propagate rapidly outward to take their places far away in the external static field. The outward-propagating field strength can be much stronger than the ultimate static field in the midplane near the solenoid. New magnetic field lines are also shed by a *decreasing* current, but these are opposite in direction to the static field. Propagating inward on the inside and outward on the outside, they neutralize pre-existing field lines and reduce the static field strength.

In the case of a *toroid* in which the current is steadily increasing, for each current loop that is created inside, a reverse-directed loop is shed outward and collapses on the axis at some distance from the toroid. Such transient field loops, as counterintuitive as they may be, must exist because in their absence Faraday's law would imply that the effect of changing magnetic flux within the toroid could be felt instantaneously at some distance.

(3) For the case of a small current loop, one can easily and explicitly demonstrate that the magnetic flux through the plane of the loop, at large distances from the loop, consists of field lines which have in fact traveled outward to their present positions as the current increased to its steady value; and, that the emf induced around a circular path in that plane, concentric with the loop, is calculable in terms of outward-moving field lines cutting the loop.

The key is that in applying the Biot–Savart law to sum up the magnetic field from different parts of the loop, we must allow for the propagation of information at the speed of light, which, for an increasing current, means that the current in the near side of the loop appears larger to an observer than the current in the far side. This leads to the following expression[7] for the quasistatic field in the plane of the loop (of area a), at large distance r: