FOUR

Calendars and Time Reckoning

4.1 THE JULIAN AND GREGORIAN CALENDARS

One may regulate a calendar by means of the Sun alone, by means of the Moon alone, or by means of the Sun and Moon together. Thus, there are three principal types of calendar: solar, lunar, and luni-solar. At various times and in various cultures, all three types of calendars have been used. Indeed, all three types are still in use today. A good example of a lunar calendar is the Muslim calendar, which is still used in some countries of the Middle East, and which is used worldwide in Muslim religious practice. The most important luni-solar calendar still in use is the Jewish calendar. But the ancient Greek and Babylonian calendars were also of this type. The most familiar example of a solar calendar is the Gregorian calendar, which is used nearly worldwide today. However, to reckon time reliably in astronomical and historical work, one must also understand its relation to the Julian calendar that preceded it.

The Julian Calendar

Structure of the Julian Calendar The Julian calendar was instituted in Rome by Julius Caesar in the year we now call 45 B.C.. It reached its final form by A.D. 8 and continued in use without further change until A.D. 1582, when it was modified by the Gregorian reform. The Julian calendar adopts a mean length of 365 1/4 days for the year. This is in good agreement with the length of the tropical year, that is, the time from one spring equinox to the next. The Julian calendar is therefore a solar calendar and keeps good pace with the seasons. Two kinds of calendar year are distinguished: common years and leap years. Three years of every four are common years of 365 days each. One year of every four is a leap year of 366 days.

The months of the calendar year, and the number of days contained in them are

January	31	July	31
February	28 (29 in leap year)	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

The average length of the synodic month (the time from one new Moon to the next) is about 29 1/2 days. But, except for February, every month in the Julian calendar is longer than this. The calendar months therefore have no fixed relation to the Moon: the new Moon does not, for example, fall on a fixed day of the month.[1]

Years are customarily counted from the beginning of the Christian era. The first year of the Christian era is A.D. 1. The immediately preceding year is 1 B.C. There is no year 0. This arrangement is inconvenient for doing arithmetic. More convenient is the *astronomical way* of representing years before the beginning of the Christian era by negative numbers. In this system, the year immediately before A.D. 1 is called the year 0; the year before that, −1, and so on:

Historical way	Astronomical way
A.D. 2	+2
A.D. 1	+1
1 B.C.	0
2 B.C.	−1
3 B.C.	−2

The utility of the astronomical system can be made clear by an example. Let us compute the time elapsed between January 1, 23 B.C. and January 1, A.D. 47. The simplest approach is to express the B.C. date astronomically and then subtract:

$$23 \text{ B.C.} = -22.$$
$$47 - (-22) = 69.$$

Thus, 69 years elapsed between the two dates.

The leap years are those evenly divisible by four: A.D. 4, 8, 12, and so on. This rule may be extended to the years before the beginning of the Christian era, if the years are expressed astronomically: 0, −4, −8, −12 are all leap years. Note that if the years are expressed in the historical way the leap years are 1, 5, 9, 13 B.C.

The Roman manner of designating the days of the month was not the same as our own. The first day of the month was called *Kalendae*, or Kalends in English. The 5th of most months was called *Nonae* (Nones in English). The 13th day of most months was called *Idus* (Ides). However, four months had the Nones on the 7th and the Ides on the 15th (March, May, July, October). Other days of the month were specified in terms of the days remaining until the next of these three guideposts. For example,

Our way	*Roman way*
April 1	Kalends of April
2	4th day before the Nones of April
3	3rd day before the Nones of April
4	the day before the Nones of April
5	Nones of April
6	8th day before the Ides of April
7	7th day before the Ides of April
...	...
11	3rd day before the Ides of April
12	the day before the Ides of April
13	Ides of April
14	18th day before the Kalends of May
15	17th day before the Kalends of May
...	...
29	3rd day before the Kalends of May
30	the day before the Kalends of May
May 1	Kalends of May

The Roman way of counting the days continued in use to the end of the Middle Ages. In manuscripts of the fifteenth century, for example, one sees the Roman way and the modern way of counting used side by side. The fifteenth century was a period of transition. Note also the Roman manner of *inclusive counting*. We would say that April 11 is two days before the Ides. But the Romans called it the third day before the Ides—counting the 11th, 12th, and 13th. The Nones was so called because it came nine days before the Ides (counting inclusively). Time expressions based on inclusive counting survive in the Romance languages. For example, in French, an expression for a week is *huit jours*, literally eight days. Similarly, for two weeks, the French often say *quinze jours*, fifteen days.

The Julian calendar did not exist before 45 B.C., but that does not prevent us from using it as if it did. We say that Xerxes invaded Greece and fought the battle of Salamis in 480 B.C., or that Alexander died in 323 B.C. A Julian calendar date used in this way is always a translation into modern terms of a more ancient, and now defunct, system of chronology. An ancient Greek, for example, might have said that the battle of Salamis was fought in the year that Kalliades was archon of Athens[2] and that Alexander died during the archonship of Kephisodoros.[3]

History of the Julian Calendar The Roman calendar that Caesar eliminated was a luni-solar calendar, consisting of twelve months.[4,5] There were four months of 31 days, Martius, Maius, Quintilis (= July), and October; seven of 29 days, Ianuarius, Aprilis, Iunius, Sextilis (= August), September, November, and December; and one of 28 days (Februarius). The length of the year was therefore 355 days, in fair agreement with the length of twelve lunar months. But as this year was some ten days shorter than the tropical or solar year, its months would not maintain a fixed relation to the seasons. Consequently, roughly every other year an intercalary month, called *Intercalaris* or *Mercedonius*, consisting of 27 or 28 days, was inserted after February 23, and the five remaining days of February were dropped. Thus, the year with an intercalated month consisted of 377 or 378 days. Some scholars suggest that the intercalary month alternated regularly between its two possible lengths,[6] so that the calendar years went through the regular four-year cycle: 355, 377, 355, 378. Four successive calendar years therefore totaled 1,465 days, and the average year amounted to 366 days, about one day longer than the tropical year. The intercalation was in the charge of the pontifices (priests of the state religion). But, through neglect, incompetence, or corruption, the necessary intercalations had not been attended to, and by 50 B.C. the calendar was some two months out of step with the seasons.

Julius Caesar, who had been elected Pontifex Maximus in 63 B.C., abandoned the old luni-solar calendar entirely and adopted a purely solar calendar. In the technical details he followed the advice of Sosigenes, a Greek astronomer from Alexandria. To bring the calendar back into step with the seasons, it was decided to apply two intercalations to the year 46 B.C.. The first was the customary insertion of a month following February 23, which was scheduled to be done in that year anyway. The second was the insertion of two additional months totaling 67 days between the end of November and the beginning of December, to make up for previous intercalations that had been neglected. The effect of this was to bring the vernal equinox back to March. After this *annus confusionis* ("year of confusion"), as it was called by Macrobius, the new calendar began to operate in 45 B.C..

The year was to consist of 365 days, ten more than in the old calendar. To make up the new total, the ten days were distributed among the old 29-day months: January, Sextilis, and December received two days each, while April, June, September, and November each gained one day. The old 31-day months (March, May, Quintilis, and October) remained unchanged,[7] as did February. An intercalary day was to be added to the month of February one year out of every four. After Caesar's assassination in 44 B.C., the Senate decided to honor his memory by renaming his birth-month (Quintilis) Iulius.

Unfortunately, owing to a mistake by the pontifices, the intercalation was actually performed once every three years so that, by 9 B.C., 12 intercalary days had been inserted, while Caesar's formula had called for only 9. The pontifices, who were inclusive counters like all Romans, had misunderstood Sosigenes' prescription. To bring the calendar back into step with the original plan, Augustus decreed in 8 B.C. that all intercalations be omitted until A.D. 8. In that year, the Roman Senate honored Augustus by renaming for him the month of Sextilis, since it was in this month that Augustus was first admitted to the consulate and thrice entered the city in triumph. From A.D. 8 the Julian calendar operated without further change until the Gregorian reform of 1582.

The week was not originally a feature of the Julian calendar. There is some evidence for an eight-day cycle of market days in Rome. The seven-day week seems to have originated from the Jewish practice: six days of work and one day of rest. The Jews had no names for the days of the week, except the Sabbath, and simply numbered them. As the week penetrated to the western Mediterranean, the practice grew up of naming the days of the week after the planets.[8] Most of these planetary names are still apparent in the French:

Planet	*Latin*	*French*	*English*
Saturn	Dies Saturni	Samedi	Saturday
Sun	Dies Solis	Dimanche	Sunday
Moon	Dies Lunae	Lundi	Monday
Mars	Dies Martis	Mardi	Tuesday
Mercury	Dies Mercurii	Mercredi	Wednesday
Jupiter	Dies Jovis	Jeudi	Thursday
Venus	Dies Veneris	Vendredi	Friday

In the Teutonic languages, the names of the Roman deities Mars, Mercury, Jupiter, and Venus were replaced by their counterparts Tiu, Woden, Thor, and Frigga. The seven-day planetary week was made official by the emperor Constantine in 321.

The practice of reckoning years from the beginning of the Christian era was introduced in the sixth century A.D. by the Roman abbot Dionysius Exiguus. Before this time, a year was commonly specified by the names of the consuls for that year or, later, in terms of the number of years elapsed since the beginning of the reign of some emperor, for example, Diocletian. In his tables for computing the date of Easter, Dionysius Exiguus identified A.D. 532 with year 248 of the Diocletian era. This fixed once and for all the relation of the Christian era to the Julian calendar—but not quite correctly. Modern scholarship has placed the actual year of Jesus's birth between 8 and 4 B.C.

The Gregorian Reform

The Error in the Julian Year The *Julian year* (the average length of the Julian calendar year) is 365.25 days. But the time required for the Sun to travel from one tropic, all the way around the ecliptic, and return to the same tropic is about 365.2422 days. This is called the *tropical year*. Obviously, the tropical year can only be measured with such precision over an interval of many years. The Julian year exceeds the tropical year by 0.0078 day:

$$1 \text{ Julian year} = 1 \text{ tropical year} + 0.0078 \text{ day.}$$

In any one year, or even over a period of several years, this discrepancy would not be noticed. But over the centuries, it mounts up. In A.D. 300, to take a definite example, the vernal equinox fell on March 20. For the next several decades the equinox continued to fall on March 20 or 21. (The date of the equinox oscillated between the 20th and 21st, because of the leap day system.) But gradually, over a longer period of time, a systematic shift in the date of the equinox occurred. Consider an interval of 400 years. If we multiply the relation above by 400 we obtain

$$\begin{aligned} 400 \text{ Julian years} &= 400 \text{ tropical years} + 400 \times 0.0078 \text{ day} \\ &= 400 \text{ tropical years} + 3.12 \text{ days.} \end{aligned}$$

Therefore, the spring equinox of the year 700 did not take place on March 20, but on March 17. *Because of the difference in length between the Julian and the tropical year, the date of the equinox retrogresses through the Julian calendar by about 3 days every 400 years.* By the sixteenth century, the equinox had worked its way back to the 11th of March.

The Easter Problem The principal motive for reform was the desire to correct the ecclesiastical calendar of the Catholic church, particularly the placement

of Easter. As Easter is the festival of the resurrection, its celebration depended on the proper dating of the crucifixion and the events around it. According to the Gospels, the last supper occurred on a Thursday evening; the trial, crucifixion, and burial of Christ on Friday. On the evening of the same Friday, the Passover was celebrated by the Jews.[9] Finally, the resurrection occurred on the following Sunday.[10] The Passover, around which all these events center, is celebrated for the week beginning in the evening of the 14th day of Nisan in the Jewish calendar. Now, the Jewish calendar is of the luni-solar type, and the beginning of each month corresponds closely to a new Moon. It follows, then, that the 14th day of Nisan was the date of a full Moon. Moreover, the month of Nisan was traditionally connected with the spring equinox: a month was intercalated before Nisan whenever necessary to ensure that Passover week did not begin before the Jewish calendrical equinox. The proper time to celebrate Easter was therefore shortly after the first full Moon of spring.

In the early church, this general principle was interpreted in a number of different ways. Some Christians celebrated Easter on the third day after the full Moon, regardless of whether this was a Sunday or not. Most, however, celebrated Easter on a Sunday, although there was disagreement over which Sunday was proper. An attempt to regularize practice was made by the Council of Nicaea in 325. The rule adopted by the Council, expressed somewhat inexactly, was this: Easter is the Sunday following the full Moon that occurs on or just after the vernal equinox. The Council also decreed that if the date of Easter, so calculated, coincided with the Jewish Passover, then Easter should be celebrated one week later. This description of the Council's rule is the one commonly encountered today in nontechnical books on the subject, but it is inexact for the following reason: neither the true Sun nor the true Moon was used in the determination of Easter. For example, the Council fixed the date of the equinox at March 21. (This was correct for A.D. 325, as we have seen.) Moreover, the determination of the Easter Moon was not carried out through observation of the real Moon, but through calculation based on lunar cycles.

The Council of Nicaea does not seem to have regularized practice regarding the Moon, for different lunar cycles continued to be used in the East and the West. Thus, Easter was sometimes celebrated on different Sundays by different sects. For example, in A.D. 501, Pope Symmachus, following the cycle then used at Rome, celebrated Easter on March 25. But his political and religious opponents at Rome, the Laurentians, followed the Greek cycle and celebrated Easter that year on April 22. Moreover, they sent a delegation to the emperor at Constantinople to accuse Symmachus of anticipating the Easter festival.[11] Uniform practice between East and West was not achieved until 525, when the nineteen-year Metonic cycle was introduced at Rome. It had long been used in the East, where Greek influence predominated. Tables were prepared, based on this cycle, by means of which the date of Easter in any year could readily be determined. Again, the date of the full Moon on or next after March 21 was determined from these tables, not from astronomical observation; the Sunday following was Easter. Even after 525, other cycles continued to be used in Gaul and Britain. Feeling often ran high. The celebration of Easter on the wrong day was often deemed sufficient grounds for excommunication.[12] Completely uniform practice across Europe was not achieved until about A.D. 800.[13]

The Reform In practice, then, Easter was celebrated on a Sunday in March or April following March 21. But by the sixteenth century the date of the equinox had retrogressed to March 11, so that Easter was steadily moving toward the summer. The need for reform had long been felt, but the state of astronomy in Europe had been inadequate for the task.[14] In 1545, the Council of Trent authorized Pope Paul III to act, but neither Paul nor his successors were able to arrive at a solution. Work by the astronomers continued, however, and when Gregory XIII was elected to the papacy in 1572 he found several proposals awaiting him and agreed to act on them. The plan finally adopted had been proposed by Aloysius Lilius (the Latinized name of Luigi Giglio, an Italian physician and astronomer, d. 1576). The final arrangement was worked out by Christopher Clavius, Jesuit astronomer and tireless explainer and defender of the new system.[15] The reformed calendar was promulgated by Gregory in a papal bull issued in February, 1582.

The most difficult part of the reform involved adjustments to the luni-solar ecclesiastical calendar used for calculating Easter. The details of this part of the reform need not concern us. New lunar tables were constructed to restore the ecclesiastical Moon to agreement with the true Moon. This reformed luni-solar calendar has never been accepted by the Orthodox churches, which still reckon Easter according to the tables that the Roman Church abandoned in 1582. As a result, the Orthodox Easter may coincide with the Roman Easter, or it may lag behind it by one, four, or five weeks.[16]

By contrast, the reform of the solar, or Julian, calendar was simple. First, to bring the vernal equinox back to the 21st of March, the day following October 4, 1582, was called October 15. That is, ten days were omitted. However, there was no break in the sequence of the days of the week: this sequence has therefore continued uninterrupted since its inception. Second, to correct the discrepancy between the lengths of the calendar year and the tropical year, it was decided that three leap days every 400 years were to be omitted. These were to be centennial years not evenly divisible by 400. Thus, in the old Julian calendar the years 1600, 1700, 1800, 1900, 2000, 2100, and so on, were all leap years. But under the new Gregorian calendar, 1700, 1800, 1900, and 2100 are not leap years.

The new calendar was immediately adopted in the Catholic countries of southern Europe, but in the Protestant north, most refused to go along. Denmark did not change over until 1700; Great Britain, not until 1752. In a few countries that had been dominated by the Eastern church, the change was not made until the twentieth century. Thus, Russia did not adopt the Gregorian calendar until 1918, after the revolution.

Using the Julian and Gregorian Calendars

In historical writing, the common practice is to use the Julian calendar for dates before 1582 and the Gregorian for dates after 1582. Consistent practice therefore requires translating many Julian calendar dates—for example, from seventeenth-century England—into their Gregorian equivalents. However, in astronomical discussion it is sometimes preferable to use the Gregorian calendar even for the remote past, since the dates of the equinoxes and solstices are nearly fixed in that calendar. *The only safe practice is to clearly specify which calendar is being used whenever there is any possibility of confusion.* Sometimes, in older writing, one comes across references to the "old style" and "new style," which refer to the Julian and the Gregorian calendar, respectively.

Table 4.1 may be used to make conversions. For example, Russia changed from the old to the new calendar on February 1, 1918 (Julian calendar). Let us express this date in terms of the Gregorian calendar. From table 4.1, we find that in 1918 there was a 13-day difference between the two calendars. The corresponding Gregorian date is therefore February 14, 1918. To put things as clearly as possible, "February 1, 1918 (Julian calendar)" and "February 14, 1918 (Gregorian calendar)" are two different names for the same day: it was a Thursday. Note that when the Gregorian calendar was promulgated in 1582 the difference between the two calendars was 10 days. But 1700, 1800, and 1900 were leap years in the Julian calendar, and not in the Gregorian; thus, by 1918 the difference had grown to 13 days. The Russian Orthodox Church uses the Julian calendar to this day. They celebrate Christmas on December

TABLE 4.1. Equivalent Dates in the Julian and Gregorian Calendars

	Time Interval	Difference
From	−500 Mar 6 Julian (= Mar 1 Gregorian)	−5 days
Through	−300 Mar 4 Julian (= Feb 28 Gregorian)	
From	−300 Mar 5 Julian (= Mar 1 Gregorian)	−4 days
Through	−200 Mar 3 Julian (= Feb 28 Gregorian)	
From	−200 Mar 4 Julian (= Mar 1 Gregorian)	−3 days
Through	−100 Mar 2 Julian (= Feb 28 Gregorian)	
From	−100 Mar 3 Julian (= Mar 1 Gregorian)	−2 days
Through	100 Mar 1 Julian (= Feb 28 Gregorian)	
From	100 Mar 2 Julian (= Mar 1 Gregorian)	−1 day
Through	200 Feb 29 Julian (= Feb 28 Gregorian)	
From	200 Mar 1 Julian (= Mar 1 Gregorian)	+0 days
Through	300 Feb 28 Julian (= Feb 28 Gregorian)	
From	300 Feb 29 Julian (= Mar 1 Gregorian)	+1 day
Through	500 Feb 28 Julian (= Mar 1 Gregorian)	
From	500 Feb 29 Julian (= Mar 2 Gregorian)	+2 days
Through	600 Feb 28 Julian (= Mar 2 Gregorian)	
From	600 Feb 29 Julian (= Mar 3 Gregorian)	+3 days
Through	700 Feb 28 Julian (=Mar 3 Gregorian)	
From	700 Feb 29 Julian (= Mar 4 Gregorian)	+4 days
Through	900 Feb 28 Julian (= Mar 4 Gregorian)	
From	900 Feb 29 Julian (= Mar 5 Gregorian)	+5 days
Through	1000 Feb 28 Julian (= Mar 5 Gregorian)	
From	1000 Feb 29 Julian (= Mar 6 Gregorian)	+6 days
Through	1100 Feb 28 Julian (= Mar 6 Gregorian)	
From	1100 Feb 29 Julian (= Mar 7 Gregorian)	+7 days
Through	1300 Feb 28 Julian (= Mar 7 Gregorian)	
From	1300 Feb 29 Julian (= Mar 8 Gregorian)	+8 days
Through	1400 Feb 28 Julian (= Mar 8 Gregorian)	
From	1400 Feb 29 Julian (= Mar 9 Gregorian)	+9 days
Through	1500 Feb 28 Julian (= Mar 9 Gregorian)	
From	1500 Feb 29 Julian (=Mar 10 Gregorian)	+10 days
Through	1700 Feb 28 Julian (= Mar 10 Gregorian)	
From	1700 Feb 29 Julian (= Mar 11 Gregorian)	+11 days
Through	1800 Feb 28 Julian (= Mar 11 Gregorian)	
From	1800 Feb 29 Julian (= Mar 12 Gregorian)	+12 days
Through	1900 Feb 28 Julian (= Mar 12 Gregorian)	
From	1900 Feb 29 Julian (= Mar 13 Gregorian)	+13 days
Through	2100 Feb 28 Julian (= Mar 13 Gregorian)	

25 of the Julian calendar, which is January 7 in the Gregorian—13 days after the Christmas of the Roman Church.

As a second example of the relation between the two calendars, consider the birth date of George Washington. In encyclopedias, this date is given as February 22, 1732. However, an entry in the Washington family Bible preserved at Mt. Vernon reads

> George Washington Son to Augustine & Mary his Wife was Born ye 11th Day of February 1731/2 about 10 in the Morning & was Baptiz'd on the 30th of April following.[17]

Two features of this entry require comment. First, the date of birth recorded by the family was the 11th of February (Julian calendar). Virginia in 1732 was an English colony and therefore used the same calendar as did the English. The colonies changed with England to the Gregorian calendar in 1752. The date that eventually became a national holiday, February 22, is the Gregorian equivalent of the date recorded in the family Bible. In 1732 there was an 11-day difference between the two calendars.

The second feature that requires comment is the designation of the year as 1731/2. There were several different practices regarding the beginning of the year. The most common initial dates were December 25, January 1, March 1, and March 25. These different reckonings of the year were known as *styles*—not to be confused with the usage *old style, new style* for designating the Julian and Gregorian calendars. In England, the Nativity style (December 25) was used until the fourteenth century, when it was superseded by the Annunciation style (March 25). This was the style still in use in the first half of the eighteenth century, when Washington was born. That is, in England and the English colonies the year officially began on March 25. However, by this time most of Europe was using the January 1 style. Therefore, to avoid ambiguity, it was common to specify both years in cases where the date fell between January 1 and March 24. The designation 1731/2 therefore means "1731 in the March 25 style, but 1732 in the January 1 style." The January 1 style was adopted in England in 1752 in connection with the change to the Gregorian calendar. The January 1 style is always used in modern historical writing.

4.2 EXERCISE: USING THE JULIAN AND GREGORIAN CALENDARS

1. Octavian assumed imperial powers and took the name Augustus in January, 27 B.C. He died in August, A.D. 14. How long did he reign?
2. The following list gives the Julian calendar dates of the vernal equinox over an interval of 3,000 years.

Year	*Date of vernal equinox (Julian calendar)*
A.D. 1500	11 March
1000	14 March
500	18 March
0	22 March
−500	26 March
−1000	30 March
−1500	3 April

Express these dates in terms of the Gregorian calendar. For year −500 and later, use table 4.1. For the earlier dates you will have to apply the rule for the leap years governing the centurial years in the Gregorian calendar.

3. Consider the following common remark: Isaac Newton was born in 1642, the year of Galileo's death. The popularity of this remark stems from its symbolic value. It seems to signify a passing of the torch of intellect. And it even seems to be true. Galileo died on January 9, 1642.[18] Newton was born on December 25, 1642.[19]

However, as Galileo lived in Italy, where the Gregorian reform was immediately accepted, the date of his death is naturally expressed in terms of the Gregorian calendar. Newton was born in England when that nation still used the Julian calendar. (Both dates have been expressed in the January 1 style.)

Express both dates in terms of the same calendar—first try the Julian, then the Gregorian. Do both fall in the same calendar year in one system or the other?

4. Compute the length of the *Gregorian year*, that is, the average length of the calendar year according to the Gregorian calendar. (Hint: begin by counting the number of common years and the number of leap years in the 400-year cycle.) Is the Gregorian year too long or too short in comparison with the tropical year? How much time will elapse before the Gregorian calendar loses step with the Sun by one day? The tropical year is 365.2422 days long.

4.3 JULIAN DAY NUMBER

The *Julian day number* is a count of days, widely used by modern astronomers. The day January 1, 4713 B.C. is called day zero, and for each successive day the count increases by 1.

For example, the Julian day number of December 31, A.D. 1899, is 2,415,020. The Julian day number of September 15, A.D. 1948, is 2,432,810. Knowledge of the Julian day numbers makes the calculation of time intervals simple:

September 15, 1948 =	J.D. 2,432,810
December 31, 1899 =	J.D. 2,415,020
Difference	17,790

Thus, 17,790 days elapsed between the two dates. The calculation of this time interval by some other method would be much more complicated, for it would involve the reckoning of months of different lengths and the careful counting of leap days.

When the Julian day number is a whole number, as in the examples quoted so far, it signifies Greenwich mean noon of the calendar day:

September 15, 1948, noon (at Greenwich) = J.D. 2,432,810

If the time of day falls after noon, the appropriate number of hours may be added to the Julian day number:

September 15, 1948, 6 p.m. (Greenwich) = J.D. $2{,}432{,}810^d6^h$,

where ^{d} and ^{h} stand for days and hours. If the time falls before noon, the appropriate number of hours must be subtracted from the Julian day number:

September 15, 1948, 9 A.M. (Greenwich) = J.D. $2{,}432{,}809^d21^h$.

The Julian day number, although used now as a continuous count, originally specified the location of the day within a repeating period, called the *Julian period*. The length of the Julian period is 7,980 years. In principle, after 7,980 years have elapsed the Julian day numbers are supposed to start over again. (Whether the astronomers will actually consent to begin the count of days afresh at the start of the second Julian period in A.D. 3268, we shall have to wait and see!) In publications from the early part of the twentieth century, one often sees the expression "day of the Julian period," where we would now say, "Julian day number." The two expressions mean the same thing.

The Julian period and the practice of numbering the days within this period were introduced in 1583 by Joseph Justus Scaliger, the founder of modern chronology.[20] The period was formed by combination of three shorter periods. The first of these is the 19-year luni-solar (or Metonic) period, discussed in section 4.7. The second is a 28-year calendrical period: for any two

TABLE 4.2 Julian Day Number: Century Years. Days Elapsed at Greenwich Mean Noon of January 0

Julian Calendar						Gregorian Calendar	
A.D. 0	172 1057	A.D. 600	194 0207	A.D. 1200	215 9357	A.D. 1500†	226 8923
100	117 7582	700	197 6732	1300	219 5882	1600	230 5447
200	179 4107	800	201 3257	1400	223 2407	1700†	234 1972
300	183 0632	900	204 9782	1500	226 8932	1800†	237 8496
400	186 7157	1000	208 6307	1600	230 5457	1900†	241 5020
500	190 3682	1100	212 2832	1700	234 1982	2000	245 1544

†Common years.

years in the Julian calendar that are 28 years apart, all the days of the year will fall on the same days of the week. Thus, the calendars for the years 1901, 1929, 1957, 1985, and so on, are exactly the same. (Note that in the Gregorian calendar, this pattern is broken by the three century years in four that are not leap years.) The third period, called *indiction*, was a 15-year taxation period introduced in the Roman empire in the third century A.D. The Julian period is simply the product of these three: 19 × 28 × 15 = 7980 years. Scaliger's starting year for the Julian period, 4713 B.C., is the most recent year in which all three periods were simultaneously at their beginnings.

Tables 4.2, 4.3, and 4.4 provide a convenient way of obtaining the Julian day number for any date.

Precepts for Use of the Tables for Julian Day Number

Dates after the Beginning of the Christian Era For years before 1500, the date must be expressed in terms of the Julian calendar. For the year 1800 and thereafter, the date must be expressed in terms of the Gregorian calendar. Between the dates 1500 and 1800, either calendar may be used. In any case, the date must be expressed in terms of Greenwich mean time.

1. Enter the table of century years (table 4.2) with the century year immediately preceding the desired date and take out the tabular value. If the Gregorian calendar is being used and if the century year is marked with a dagger †, note this fact for use in step 2.
2. Enter the table of the years of the century (table 4.3), with the last two digits of the year in question and take out the tabular value. If the century year used in step 1 was marked with a dagger †, diminish the tabular value by one day unless the tabular value is zero.
3. Enter the table of the days of the year (table 4.4) with the day in question, and take out the tabular value. If the year in question is a leap year, and the table entry falls after February 28, add one day to the tabular value. The sum of the values obtained in steps 1, 2, and 3 then gives the Julian day number of the date desired. This Julian day number applies to noon of the calendar date.

First Example: September 15, A.D. 1948, Greenwich mean noon:

1. Century year	1900†		241 5020
2. Year of the century	48	17 532 − 1 =	1 7531
3. Day of the year	September 15	258 + 1 =	259
Julian day number			243 2810

Note that in step 2 the tabular value has been diminished by 1 because 1900 is a common year (marked with † in table 4.2). In step 3, the tabular value

TABLE 4.3. Julian Day Number: Years of the Century. Days Elapsed at Greenwich Mean Noon of January 0

0§	0	20*	7 305	40*	14 610	60*	21 915	80*	29 220
1	366	21	7 671	41	14 976	61	22 281	81	29 586
2	731	22	8 036	42	15 341	62	22 646	82	29 951
3	1 096	23	8 401	43	15 706	63	23 011	83	30 316
4*	1 461	24*	8 766	44*	16 071	64*	23 376	84*	30 681
5	1 827	25	9 132	45	16 437	65	23 742	85	31 047
6	2 192	26	9 497	46	16 802	66	24 107	86	31 412
7	2 557	27	9 862	47	17 167	67	24 472	87	31 777
8*	2 922	28*	10 227	48*	17 532	68*	24 837	88*	32 142
9	3 288	29	10 593	49	17 898	69	25 203	89	32 508
10	3 653	30	10 958	50	18 263	70	25 568	90	32 873
11	4 018	31	11 323	51	18 628	71	25 933	91	33 238
12*	4 383	32*	11 688	52*	18 993	72*	26 298	92*	33 603
13	4 749	33	12 054	53	19 359	73	26 664	93	33 969
14	5 114	34	12 419	54	19 724	74	27 029	94	34 334
15	5 479	35	12 784	55	20 089	75	27 394	95	34 699
16*	5 844	36*	13 149	56*	20 454	76*	27 759	96*	35 064
17	6 210	37	13 515	57	20 820	77	28 125	97	35 430
18	6 575	38	13 880	58	21 185	78	28 490	98	35 795
19	6 940	39	14 245	59	21 550	79	28 855	99	36 160

*Leap year.
§Leap year unless the century is marked †.
In Gregorian centuries marked †, subtract one day from the tabulated values for the years 1 through 99.

TABLE 4.4. Julian Day Number: Days of the Year

Day of Mo.	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	285	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	*	88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

*In leap years, after February 28, add 1 to the tabulated value.

has been increased by 1 because 1948 was a leap year and the date fell after February 28.

Second Example: February 9, A.D. 1584 (Gregorian calendar), 10:30 A.M. Greenwich mean time:

1. 1500† (Gregorian)		226 8923
2. 84	3 0681 − 1 =	3 0680
3. February 9		40
Julian day number		229 9643

1 1/2 hours before noon of the 9th: $2{,}299{,}642^{d}22^{h}30^{m}$

Note that although 1584 was a leap year, the tabular value in step 3 is not changed because the date fell before the end of February.

Dates before the Beginning of the Christian Era Express the date astronomically; add the smallest multiple (n) of 1,000 years that will convert the date into an A.D. date; determine the Julian day number of the A.D. date; then subtract the same multiple (n) of 365250. The result is the Julian day number desired.

Example: March 12, 3284 B.C. Greenwich mean noon:

March 12, B.C. 3284 = − 3283 March 12
4 × 1000 = 4000
sum = 717 March 12

1. 700	197 6732
2. 17	6210
3. March 12	71
Julian day number, March 12, A.D. 717 noon	198 3013
Less 4 × 365250	−146 1000
Julian day number, March 12, B.C. 3284, noon	52 2013

4.4 EXERCISE: USING JULIAN DAY NUMBERS

1. Work out the Julian day numbers for the following dates. The time is Greenwich noon unless otherwise noted.

   A. June 13, 1952 (answer: 243 4177).
   B. June 10, 323 B.C. (death of Alexander).
   C. November 12, 1594, 6 A.M. Greenwich (Gregorian calendar).

2. Days of the week: The Julian day number provides a handy method of determining the day of the week on which any calendar date falls. Divide the Julian day number by 7, discard the quotient, but retain the remainder. The remainder determines the day of the week:

Remainder	*Day of week*
0	Monday
1	Tuesday
2	Wednesday
3	Thursday
4	Friday
5	Saturday
6	Sunday

   A. Columbus, on his first voyage of discovery, first sighted land on October 12, 1492. What day of the week was this? (Answer: Friday.)
   B. July 4, 1776 (Gregorian) fell on what day of the week?

3. Length of the tropical year: The vernal equinox of 1973 fell on March 20 at 6 P.M. Greenwich time. Copernicus observed the vernal equinox of the year 1516, "4 1/3 hours after midnight on the 5th day before the Ides of March"[21] That is, the vernal equinox fell at 4:20 A.M. March 11, A.D. 1516. (Is this the Julian or the Gregorian calendar?) Copernicus's time of day is referred to his own locality, that is, to the meridian through Frauenberg, on the Baltic coast of Poland. Frauenberg lies about 19° east of Greenwich, which amounts to about 1 1/4 hour of time. Expressed in terms of Greenwich time, then, Copernicus's vernal equinox fell at about 3 A.M. (We ignore the small fraction of an hour.)

   Use these two equinoxes (1516 and 1973) to determine the length of the tropical year. To do this, compute the Julian day number of each observation, subtract to find the time elapsed, then divide by the number of years that passed. Compare your result with the modern figure for the tropical year, 365.2422 days.

4.5 THE EGYPTIAN CALENDAR

An understanding of the ancient Egyptian calendar is essential for every student of the history of astronomy. Because of its great regularity, the Egyptian calendar was adopted by Ptolemy as the most convenient for astronomical work, and it continued to be used by astronomers of all nations down to the beginning of the modern age. In the sixteenth century, Copernicus, for example, constructed his tables for the motion of the planets, not on the basis of the Julian year, but on the basis of the Egyptian year. When Copernicus wanted to calculate the time elapsed between one of Ptolemy's observations and one of his own, *he converted his own Julian calendar date into a date in the Egyptian calendar.*[22]

Structure

The Egyptian calendar from a very early date consisted of a year of twelve months, of thirty days each, followed by five additional days. The length of the year was therefore 365 days. Every year was the same: there were no leap years or intercalations. The names of the months are

1. Thoth	7. Phamenoth
2. Phaophi	8. Pharmuthi
3. Athyr	9. Pachon
4. Choiak	10. Payni
5. Tybi	11. Epiphi
6. Mecheir	12. Mesore
	Plus 5 additional days.

The names transcribed here, as commonly written by scholars today, represent their Greek forms. (Greeks of the Hellenistic period, living in Egypt, spelled the old Egyptian month names as well as they could in the Greek alphabet.) The additional days at the end of the year are sometimes called "epagomenal": the Greeks called them *epagomenai*, "added on."

The Egyptian year, being only 365 days, will after an interval of four years begin about one day too early with respect to the solar year. As a result, the Egyptian months retrogress through the seasons, making a complete cycle in about 1460 years (1461 Egyptian years = 1460 Julian years).[23] It therefore came

Virginis). The recent observation, made by Ptolemy himself, involved an opposition of Saturn to the mean Sun. The date (as given in *Almagest* XI, 5) was Mesore 24, in the 20th year of Hadrian. Compute the number of days between these two observations.

3. Conversion of dates, Egyptian to Julian: Ptolemy records the time of the middle of a partial lunar eclipse, which he observed at Alexandria, as follows: in year 20 of Hadrian, four equinoctial hours after midnight on the night between the 19th and 20th of Pharmouthi (Egyptian calendar). Convert this date into its equivalent in the Julian calendar. (Answer: March 6, A.D. 136, 4 A.M., Alexandria local time.)
4. Another conversion problem: In *Almagest* IV, 9, Ptolemy reports the beginning of a partial eclipse observed by him: in the 9th year of Hadrian, in the evening between the 17th and 18th of Pachon, 3 3/5 equinoctial hours before midnight. Express this date in terms of its Julian equivalent.

4.7 LUNI-SOLAR CALENDARS AND CYCLES

All luni-solar calendars contain two features. First, the months alternate between 29 and 30 days long. In this way, the calendar months closely match the synodic month (the time from new Moon to new Moon). (But because the synodic month is a little longer than 29 1/2 days, there must be a few more 30-day months than 29-day months.) Second, the calendar year contains sometimes twelve months and sometimes thirteen. Twelve synodic months amount to 354 days, which is shorter than the tropical year (365 1/4 days). Thus, if every calendar year had only twelve months, the calendar would progressively get out of step with the seasons. The occasional insertion of a thirteenth month restores the calendar to its desired relation to the seasons. In a well-regulated luni-solar calendar, the calendar months slosh back and forth a bit with respect to the seasons, but they do not continually gain or lose ground. For example, in the Jewish calendar, the month of Nisan comes always in the spring, but it does not always begin on the same date of the Gregorian calendar.

The Greek Civil Calendars

The Months of four Greek calendars[28]

Athens	Delos	Thessaly	Boeotia
1. Hekatombaion	Hekatombaion	Phyllikos	Hippodromios
Metageitnion	Metageitnion	1. Itonios	Panamos
Boedromion	Bouphonion	Panemos	Pamboiotios
Pyanepsion	Apatourion	Themistios	Damatrios
Maimakterion	Aresion	Agagylios	Alalkomenios*
Poseideon*	Poseideon	Hermaios	1. Boukatios
Gamelion	1. Lenaion	Apollonios*	Hermaios
Anthesterion	Hieros	Leschanopios	Prostaterios
Elaphebolion	Galaxion	Aphrios	Agrionios
Mounychion	Artemision	Thuios	Thiouios
Thargelion	Thargelion	Homoloios	Homoloios
Skirophorion	Panamos*	Hippodromios	Theilouthios

In the calendars of ancient Greece, the month began with the new Moon. Generally, months of 30 days, called "full" (*pleres*), alternated with months of 29 days, called "hollow" (*koiloi*). Ordinarily, the civil year consisted of twelve months, but occasionally a thirteenth month was intercalated.

Despite the simplicity of the basic calendrical scheme, Greek chronology is a difficult, even obscure, field. Most cities had their own calendars, which differed in the names of the months, the starting point of the year, and the place in the calendar where intercalary months were inserted. The list above gives the names of the months in four Greek calendars, starting from summer solstice. The first month of the year is marked 1. In the Athenian calendar, the year began with Hekatombaion, around the time of summer solstice. But in Delos the year began with Lenaion, around winter solstice. The months that were customarily doubled in leap years are marked with asterisks. For many cities, for example, for Argos and Sparta, the complete list of month names is not even known.

The most vexing complication is not, however, that each city followed its own practice, but that even in a single city the practice was not uniform. No regular pattern determined the intercalation of months. Moreover, individual days were sometimes intercalated or suppressed at will. For example, the Athenians held a theatrical presentation in connection with the cult of Dionysos on Elaphebolion 10. In 270 B.C., for some reason, the performance was postponed. Accordingly, the day following Elaphebolion 9 was counted as Elaphebolion 9 *embolimos* ("inserted"), and the next three days were counted as the second, third, and fourth "inserted" Elaphebolian 9.[29] Religious practice did not permit tampering with the *names* of days on which feasts were held, but the archons were free to intercalate days as needed, to place the feasts at a more convenient time. In a famous passage of *The Clouds* (lines 615–626), Aristophanes ridicules Athenian calendrical practice. The Moon complains that although she renders the Athenians many benefits—saving them a drachma each month in lighting costs through moonlight—nevertheless they do not reckon the days correctly, but jumble them all around. Consequently, the gods threaten her whenever they are cheated of their dinner because the sacrifices have not been held on the right days. As Samuel[30] points out, this illustrates that the festival calendar was out of step with the Moon, *and that the Athenians were aware of it.* Consequently, it is not surprising to see Athenian writers distinguish between "the new Moon according to the goddess" (Selene, the Moon) and "the new Moon according to the archon" (the head magistrate of the city).[31] We might call these the actual new Moon and the calendrical new Moon.

Because no fixed system was used to regulate the intercalation of either months or days, it is usually impossible to convert a date given in terms of the Athenian calendar into its exact Julian equivalent. Such a conversion would be possible only if we had a more or less complete record of the intercalations actually ordered by the authorities at Athens. No such record has come down to us. The same uncertainty attaches to most ancient calendars, with the notable exceptions of the Egyptian calendar and the Roman calendar after the Julian and Augustan reforms. The superiority of these two calendars derives from their regularity. In the Egyptian calendar there were no intercalations at all, while in the Julian calendar the only intercalation is the regular insertion of one day every four years.

Years were designated by the Greeks in several different manners. One practice involved the counting of Olympiads and the years (numbered one through four) within the Olympiad. The particular Olympiad was singled out both by number and by the name of the athlete who had won one of the important competitions, usually the foot race called the *stadion.*

More common was the use of the eponymous year, that is, the year named after a ruler then in power. The expression of eponymous years in terms of equivalent years of the Christian era requires a list of the rulers, and the lengths of their reigns, for the city or nation in question. We have fairly good king lists for Babylon, Persia, Egypt, Sparta, and so on, and lists of the archons of Athens and the consuls of Rome, so it usually is possible to determine at least the year to which an ancient writer refers.[32] As a rule, the farther back we go into the past, the less reliable the lists become.

A good example of these ancient manners of designating the year is provided by Diodorus of Sicily (Diodorus Siculus), a Greek historical writer who lived in Rome during the reigns of Caesar and Augustus. Diodorus completed an enormous work, of which less than half has come down to us, that treated the history of the whole known world from the time before the Trojan War down to Caesar's conquest of Gaul. Diodorus's arrangement is chronological. Each year's events are introduced by two or three equivalent designations of the year in question. For example, Diodorus begins his account of the year corresponding to 420/419 B.C. in the following way:

> When Astyphilos was archon at Athens, the Romans designated as consuls Lucius Quinctius and Aulus Sempronius, and the Greeks celebrated the 90th Olympiad, in which Hyperbios of Syracuse won the stadion. In this year, the Athenians, to abide by an oracle, restored to the Delians their island; and the Delians, who had been living at Adramyttium, returned to their homeland. . . .[33]

Luni-Solar Cycles

All ancient luni-solar calendars were originally regulated by observation, without the aid of any astronomical system. In most cultures, the month began with the first visibility of the crescent Moon—in the west, just after sunset. For this reason, in Babylonian as well as Jewish practice, the day began at sunset. A few generations of experience would suffice to show that the month varied between 29 and 30 days. Therefore, if because of unfavorable weather the new crescent could not be sighted on the 31st evening, a new month could be declared anyway.

The intercalation of months arose as a method of maintaining a roughly fixed relation between the seasons of the year and the months of the calendars. The ancient Jews inserted a thirteenth month to delay the beginning of the spring month if the lambs were still young and weak, if the winter rains had not stopped, if the roads for Passover pilgrims had not dried, if the barley had not yet ripened, and so on. Similar considerations must have governed the intercalation of months in all cultures that used a luni-solar calendar. Only later did observations of the heliacal risings and settings of the fixed stars play any part. It was much later still before any use was made of observations of solstices and equinoxes.

The Eight-Year Cycle The lengths of the two fundamental periods are

Synodic month: 29.5306 days,
Tropical year: 365.242 days.

Their ratio is 365.242/29.5306 = 12.3683. Thus, on the average, a calendar year ought to contain 12.3683 months.

A real calendar year, however, contains a whole number of months. Suppose we let every year contain twelve months. After the first year, the calendar will be deficient by 0.3683 months. The calendrical deficit after n years will be $n \times 0.3683$ months. We simply wait until this deficit amounts to a whole month; then it will be time to intercalate a month. For example, after three years the deficit will be 3×0.3683 months = 1.1 months. If we insert a thirteenth month in the third calendar year, then at the end of that year the deficit will be nearly (although not exactly) eliminated. Unfortunately, 3×0.3683 is not very near a whole number. The central problem, then, is to find an integer n such that $n \times 0.3683$ is as close to a whole number as possible. One possible solution is $n = 8$, for then we have

$$8 \times 0.3683 = 2.946, \text{ which is pretty nearly } 3.$$

This near-equality allows us to construct a luni-solar cycle. In eight calendar years, we insert three additional months. Of the eight calendar years, five will consist of twelve months, and three will consist of thirteen months:

Eight-year cycle

	5 years of 12 months =	60 months
	3 years of 13 months =	39 months
So,	8 calendar years =	99 months

The average length of the calendar year in this system is 99 months/8 = 12.3750 months, which is close to the figure we were trying to match (12.3683 months per year). The correspondence is not perfect, however. Indeed, the calendar year is 0.0067 months too long (12.3750 − 12.3683 = 0.0067). In about 149 years, this surplus will amount to a whole month. Thus, the eight-year cycle will operate satisfactorily for about 149 years, but then one month will have to be omitted to restore the balance.

The Nineteen-Year Cycle The eight-year cycle is tolerably accurate, but let us search for a better one. Again, the tropical year is longer than twelve synodic months by 0.3683 month. We search for an integer n such that $n \times 0.3683$ is a whole number. A very satisfactory solution is $n = 19$:

$$19 \times 0.3683 = 6.9977, \text{ which is very nearly } 7.$$

Thus, we may construct a nineteen-year luni-solar cycle. In nineteen calendar years, we insert seven additional months. Of the nineteen years, then, twelve will consist of twelve months and seven will consist of thirteen months:

Nineteen-year cycle

	12 years of 12 months =	144 months
	7 years of 13 months =	91 months
So,	19 calendar years =	235 months

The average length of the calendar year in this system is 235 months/19 = 12.3684 months, which agrees very well with the length of the solar year (12.3683 months).

Nineteen tropical years therefore contain 235 synodic months, almost exactly. The astronomical meaning of this statement is that after nineteen tropical years, both the Sun and the Moon return to the same positions on the ecliptic. The Sun returns to the same longitude after any interval containing a whole number of tropical years. The special feature of the nineteen-year period is that it also contains a whole number of synodic months. Thus, the Moon will be in the same phase on two dates that are nineteen years apart.

The explanation of the eight- and nineteen-year cycles given above is not meant to reflect the actual process of discovery: the ancient Greeks and Babylonians did not begin with a knowledge of the lengths of the year and the month. Rather, a knowledge of these cycles emerged after several generations of keeping track of the Moon.

The nineteen-year cycle was introduced at Athens in 432 B.C. by the astronomer Meton, for which reason it is also known as the *Metonic cycle.* The Greeks simply called it the *nineteen-year period.* Unfortunately, the Athenians never adopted it as the regulatory device of their calendar, although the archons may have taken it into account while pondering the need for an intercalation. Whether the Greeks discovered this cycle independently or learned it from the Babylonians, it is not possible to say. Borrowing may be considered likely in view of other demonstrated debts of Greek astronomy to Babylonian practice. On the other hand, the fundamental relation (235 months = 19 years) is very simple, and independent discovery cannot be ruled out.

Geminus on the Structure of the Nineteen-Year Cycle In chapter VIII of the *Introduction to the Phenomena*, Geminus gives a detailed account of the nineteen-year cycle as used by the Greeks. According to Geminus, this cycle was based on the identity

$$19 \text{ years} = 235 \text{ months} = 6{,}940 \text{ days}.$$

In one nineteen-year cycle there were, of course, twelve years of twelve months and seven years of thirteen months. Geminus adds that there were 125 full months (30 days each) and 110 hollow months (29 days). Thus, $125 \times 30 + 110 \times 29 = 6{,}940$ days.

Geminus asserts that the arrangement of full and hollow months should be as uniform as possible. There are 6,940 days in the nineteen-year period and 110 hollow months. If all the months were temporarily considered full, it would therefore be necessary to remove a day after every run of 63 days ($6{,}940/110 \simeq 63$). That is, every 64th day number would be removed. According to Geminus, the thirtieth day of the month is not always the one scheduled for removal. Rather, the hollow month is produced by removing whichever day falls after the running 63-day count. Such a procedure would have enormously complicated the construction of a calendar. Neugebauer[34] therefore doubts that this rule was ever followed. However, Geminus is unambiguous on this point, and both recent efforts at a reconstruction of the Metonic cycle have taken him at his word.[35]

The Callippic Cycle and the Callippic Calendar The length of the year implied by Meton's nineteen-year cycle is

$$6{,}940 \text{ days}/19 = 365\,\frac{5}{19} \text{ days}.$$

As Geminus points out, this is too long by

$$365\,\frac{5}{19} - 365\,\frac{1}{4} = \frac{1}{76} \text{ day}.$$

Therefore, after 76 years (which is four consecutive Metonic cycles), we will have counted one day too many in comparison with the solar year.

In the late fourth century B.C., Callippus proposed a new luni-solar cycle, the seventy-six-year or *Callippic cycle*, as it is often called. The Callippic cycle is formed from four consecutive nineteen-year cycles, but one day is dropped. The average length of the year in Callippus's cycle is therefore exactly 365 1/4 days. The cycle also preserves the good agreement with the length of the month that had already been achieved in Meton's nineteen-year cycle.

Callippus's seventy-six year cycle served as the basis of an artificial calendar used by some of the Greek astronomers. The best evidence for this comes from Ptolemy's citations of older observations in the *Almagest.* For example, Ptolemy cites an occultation of the Pleiades observed by Timocharis in the third century B.C.:

> Timocharis, who observed at Alexandria, records the following. In the 47th year of the First Callippic 76-year period, on the eighth of Anthesterion, . . . towards the end of the third hour [of the night], the southern half of the Moon was seen to cover exactly the rearmost third or half of the Pleiades.[36]

In this artificial calendar, the years were counted by their place in the seventy-six-year cycle. The month names were borrowed from the Athenian calendar. But it is important to stress that Callippus's calendar had no relation to the calendar of Athens. It was a scientific calendar used by astronomers for their own purposes. This extreme step was taken because the civil calendars of the Greeks were completely unsuitable for accurate counting of the days—for all the reasons mentioned above. Year one of the first Callippic cycle began with the summer solstice of 330 B.C. Timocharis's occultation of the Pleiades, quoted above, was observed in 283 B.C.

Ptolemy provides Egyptian calendar equivalents for the Callippic dates he cites. Thus, Ptolemy says that Anthesterion 8, year 47 of the first Callippic cycle, was equivalent to Athyr 29, year 465 of Nabonassar. All attempts to reconstruct Callippus's calendar have been based on the handful of equivalences provided by Ptolemy and the short description of the Metonic-Callippic cycle by Geminus. However, Geminus's discussion should be viewed as a pedagogical effort to explain the luni-solar cycle, rather than a serious historical account, and Ptolemy provides us very few hard facts. Thus, we cannot reconstruct the Callippic calendar with any certainty.

In the second century B.C., Hipparchus used the Callippic cycle only for specifying the year and preferred to name the day in terms of the Egyptian calendar. For example, in *Almagest* III, 1, Ptolemy cites a list of equinoxes observed by Hipparchus. According to Hipparchus, the autumnal equinox of 162 B.C. occurred in the 17th year of the third Callippic cycle, on Mesore 30, about sunset. This mixed reckoning, involving the use of a solar (Egyptian) calendar for the month and day, and a luni-solar (Callippic) calendar for the year, did not last long. In his own work, Ptolemy used the Egyptian calendar, which was the simplest, most rational option of all.

The Babylonian Calendar

The Babylonian year began with the new Moon of the spring month. Years contained either twelve or thirteen months. The thirteenth month was intercalated either by adding a second month VI or a second month XII.

Babylonian month names[37]

I	BAR	Nisannu	VII	DU_6	Tešrítu
II	GU_4	Ajjaru	VIII	APIN	Araẖsamnu
III	SIG	Simānu	IX	GAN	Kislímu
IV	ŠU	Du'ūzu	X	AB	Ṭebētu
V	IZI	Abu	XI	ZÍZ	Šabāṭu
VI	KIN	Ulūlu	XII	ŠE	Addaru
VI_2	KIN.A		XII_2	DIRIG, A	

In this list, the Babylonian month name is preceded by the Sumerian ideogram often used in Babylonian astronomical texts. Thus, the name of the spring month, *Nisannu* (which would require several cuneiform signs), is usually replaced by a single ideogram, BAR. (Subscripts and accent marks on some ideograms are the Assyriologists' way of distinguishing among several cuneiform signs with the same sound.)

Originally, the intercalations were performed irregularly. Notices were sent in the king's name to the priestly officials at temples throughout Babylonia. This practice was still followed in the Chaldaean period. Later, during the Persian period, the announcements of intercalations came from the scribes at the temple Esangila, who sent notices to the officials at other temples throughout Babylonia.[38] Thus, it appears that the regulation of the calendar passed into the hands of the bureaucracy. This is what made possible the eventual adoption of a regular system of intercalation.

The few official announcements of intercalary months that have survived prove that no regular system of intercalation was in place at the beginning of the Persian period. As discussed in section 1.1, already in MUL.APIN (650 B.C.) there was an attempt at formulating some guidelines for intercalation of months, based on the heliacal risings of the stars. But the appearance of a fixed luni-solar cycle was a later development. There is some evidence that an eight-year cycle was used for the brief period from 529 to 503 B.C. (three intercalary months inserted every eight years). From 499, the nineteen-year cycle was probably in use (seven intercalary months inserted every nineteen years). However, there are some gaps in our knowledge, since we do not have records of some intercalations. Also, the scribes had not yet finalized the rules for deciding when the intercalary month should follow month VI and when it should follow month XII. A definite glitch in the pattern occurred in 385, when that year (rather than the following year) was made a leap year. But from 383 B.C. down to the first century A.D. (when the cuneiform texts cease), a regular pattern of intercalations was followed.[39]

After Alexander's conquest and the establishment of the Seleucid dynasty, the Babylonian texts use the Seleucid era, which we shall abbreviate SE. That is, the old luni-solar calendar based on the nineteen-year cycle continued to function without interruption. But the years were counted from the year that Seleukos I decided to count as the official beginning of his reign. (1 Nisannu, year 1 of Seleucid era = 3 April 311 B.C.)[40]

In terms of the Seleucid era, the leap years are those marked with asterisks in the following sequence:

1* 2 3 4* 5 6 7* 8 9* 10 11 12* 13 14 15* 16 17 18** 19

Thus, years 1, 4, and so on, of the Seleucid era were leap years. In years marked with a single asterisk, month XII was doubled. In years marked with a double asterisk (i.e., year 18), month VI was doubled. To determine whether any year of the Seleucid era was a leap year or not, divide the year number by 19, discard the quotient, but retain the remainder and compare it with the sequence above.

Features of the Babylonian calendar persist in two luni-solar calendars still in use today—the Jewish calendar and the Christian ecclesiastical calendar. After Israel and Judah were conquered by the Babylonians, in the sixth century B.C., the Babylonian calendar was adopted by the Jews. The nineteen-year cycle remains the basic operating principle of the modern Jewish calendar, which is the official calendar of Israel, and which is used worldwide for Jewish religious practice. The month names in the modern Jewish calendar clearly reflect their Babylonian origins: Nisan corresponds to Nisannu, Iyyar to Ajjaru, and so on. The Christian church, drawing on both the Jewish calendar and the Greek astronomical tradition, adopted the nineteen-year cycle as the basis of the ecclesiastical calendar that governs the date of Easter. Thus, in the twentieth century, the celebration of religious festivals such as Passover and Easter is in part regulated by decisions made by anonymous Babylonian scribes 2,500 years ago. This is another striking example of the continuity of the Western astronomical tradition.

4.8 EXERCISE: USING THE NINETEEN-YEAR CYCLE

As discussed in section 4.7, nineteen tropical years contain a whole number of synodic months. This is the basis of the Metonic cycle: 19 years = 235 months. It follows that the dates of the new Moons in the Gregorian calendar should repeat the same pattern, almost exactly, after an interval of nineteen

years. The list below gives the dates of the first new Moons for the years 1960–1979. (The dates refer to Greenwich time.)

Year	*Date of first new Moon*	*Year*	*Date of first new Moon*
1961	January 16	1971	January 26
62	January 6	72	January 16
63	January 25	73	January 4
64	January 14	74	January 23
65	January 2	75	January 12
66	January 21	76	January 1
67	January 10	77	January 19
68	January 29	78	January 9
69	January 18	79	January 28
70	January 7		

During this nineteen-year period, the date of the first new Moon moved back and forth all over the month of January. However, after nineteen years, we find the pattern repeating, very nearly. The dates of the first new Moons for the next few years are

Year	*Date of first new Moon*
1980	January 17
1981	January 6
1982	January 25

We can predict the dates of all the new Moons in any desired year, by use of this list of new Moons. Suppose we want the new Moons for a year that is contained in the list, say 1963. Then, beginning with the date of the first new Moon, we add increments of 30 days and 29 days alternately (i.e., we alternate full and hollow months):

1963	First new Moon	January 25	
			+30 days
	Second new Moon	February 24	
			+29
	Third new Moon	March 25	
			+30
	Fourth new Moon	April 24	
			+29
	Fifth new Moon	May 23, etc.	

The dates obtained by this approximate scheme will rarely differ from the date of true new Moon by more than a day.

Suppose we want the new Moons for a year not in the list, say 1948. We then simply determine which year of the list occupies the same position in the nineteen-year cycle as does 1948. The answer is 1967, since 1948 + 19 = 1967. The new Moons for 1948 may therefore be written out, exactly as explained above, by use of the date of the first new Moon of 1967 as starting point:

1948	First new Moon	January 10	
			+30 days
	Second new Moon	February 9	
			+29
	Third new Moon	March 9, etc.	

Problems

1. Use the nineteen-year cycle and a pattern of alternating full and hollow months to write out the dates of the new Moons for the current year. Compare your results with the dates given by a calendar or almanac.
2. Do you see any evidence for an eight-year cycle in the list of new Moons given above?
3. The technical name for the position of a year in the nineteen-year cycle is the *golden number*. (This term originated in the Middle Ages.) The golden number may be obtained by dividing the year by 19, discarding the quotient, and adding 1 to the remainder. Thus, the golden number of 1961 is 5. (1961/19 = 103, with remainder 4. Golden number = 4 + 1.) The golden number for 1962 is 6; for 1963 it is 7, and so on. Golden numbers are usually written as Roman numerals.

   Construct a table of two columns. The first column should contain the golden numbers I through XIX. The second column should contain the date of the first new Moon of the year corresponding to each golden number.

4.9 THE THEORY OF STAR PHASES

The cycle of appearances and disappearances of the fixed stars was an important part of both early Greek and early Babylonian astronomy. As an example, take the case of the Pleiades. During the spring, the Pleiades disappeared for a month and a half when the Sun moved near them on the ecliptic. Then (in late May), the Pleiades emerged from their period of invisibility. They could be seen, for the first time in the year, rising in the east, a few minutes before dawn. This event was the *morning rising* of the Pleiades. It signaled the wheat harvest and the beginning of summer weather. In the same way, the morning rising of Arcturus was recognized everywhere in the Greek world as the beginning of autumn. The risings and settings of stars that occur just before sunrise, or just after sunset, are called *heliacal risings and settings* (because they occur in connection with the Sun). They are also called *fixed star phases.*

By the fifth century B.C., this lore was systematized into the *parapegma*, or star calendar. (The star calendar was a bit older among the Babylonians. As we have seen, the seventh-century B.C. compilation, MUL.APIN, included a star calendar.) A parapegma listed the heliacal risings and settings of the stars in chronological order. The user of the parapegma could tell the time of year by noting which stars were rising in the early morning. The parapegma served as a supplement to the chaotic civil calendars of the Greeks. Usually, but not always, the star phases were accompanied in the parapegma by weather predictions.

One could compile a list of the heliacal risings and settings of the constellations, simply by observations made at dawn and dusk over the course of a year. There is no need for any sort of theory. In this sense, the parapegma may be considered prescientific. But understanding the annual cycle of star phases was an important early goal of Greek scientific astronomy. Indeed, one of the oldest surviving works of Greek mathematical astronomy is devoted to this subject. This is the book (or really two books) written by Autolycus of Pitane around 320 B.C. and called *On Risings and Settings*. Autolycus defines the various kinds of heliacal risings and settings, then states and proves theorems concerning their sequence in time and the way the sequence depends on the star's position with respect to the ecliptic. No individual star is mentioned by name. Autolycus's goal is to provide a theory for understanding the phenomena. His style is that of Euclid.

True Star Phases

Autolycus and all the Greek scientific writers who followed him distinguished between *true* and *visible* star phases. An example of a true star phase is the *true morning rising* (TMR), which occurs when the star rises at the same moment as the Sun. At such a time the star would be invisible, owing to the brightness of the sky. The *visible morning rising* (VMR) would occur some weeks later, after the Sun had moved away from the star. The visible phases are the observable events of interest to farmers, sailors, poets, and astrologers. However, the true phases are more easily analyzed. Accordingly, Autolycus begins his treatise with a discussion of the true risings and settings. There are four true phases:

TMR	True morning rising	(Star rises at sunrise.)
TMS	True morning setting	(Star sets at sunrise.)
TER	True evening rising	(Star rises at sunset.)
TES	True evening setting	(Star sets at sunset.)

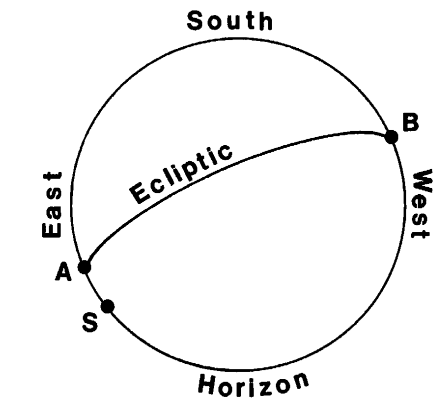

FIGURE 4.3.

Properties of True Star Phases For any star, the TMR and the TER occur half a year apart.

For any star, the TMS and the TES occur half a year apart.

These propositions are easily proved. Let star *S* be rising in the east, as in figure 4.3. Let the Sun be rising at *A*. The star is making its TMR. The TER will occur when the star is rising at *S* and the Sun is setting at *B*. The ecliptic is bisected by the horizon; thus there are six zodiac signs between *A* and *B*. If we suppose the Sun moves uniformly on the ecliptic, it will take the Sun half a year to go from *A* to *B*. Thus, the TMR and the TER occur six signs (about six months) apart in the year. The same sort of proof is easily made for the TMS and the TES.

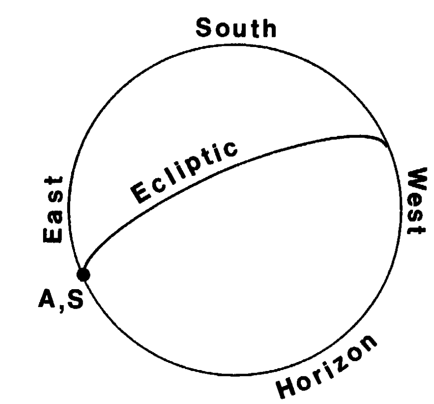

FIGURE 4.4.

The stars have their true phases in different orders according to whether they are south of the ecliptic, on the ecliptic, or north of the ecliptic.

Ecliptic Stars: If a star is exactly on the ecliptic, its TMR and TES will occur on the same day. Let star *S* be at ecliptic point *A*, as in figure 4.4. When the Sun is also at *A*, *S* and *A* rise together, thus producing the star's TMR. In the evening, *S* and *A* will set together in the west, thus producing the star's TES. (We assume that the Sun stays at the same ecliptic point for the whole day.) In the same way, one may show that for ecliptic stars, the TER and TMS occur on the same day.

Northern Stars: If a star is north of the ecliptic, the TMR will precede the TES. Let the northern star *S* be making its TMR, rising simultaneously with ecliptic point *A*, as in figure 4.3. Now, of any two points on the celestial sphere that rise simultaneously, the one that is farther north will stay up longer and set later. (We assume the observer is in the northern hemisphere.) *S* and *A* rise together. But *A* will set first. Thus, when *S* sets, the situation will resemble figure 4.5. *S* is on the western horizon. *A*, located farther south on the sphere, will already have set and will be below the horizon. The TES of star *S* occurs when the Sun is at *C*. Thus, we must wait a few weeks for the Sun to advance eastward on the ecliptic from *A* to *C*. The TES therefore follows the TMR.

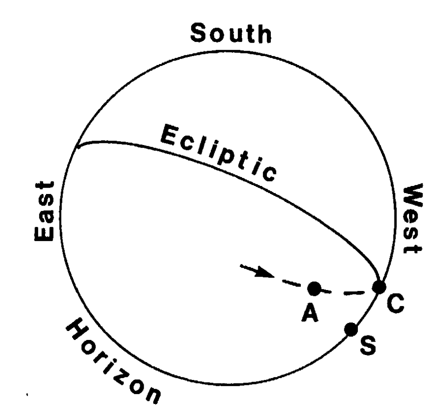

FIGURE 4.5.

Southern Stars: If a star is south of the ecliptic, the TMR will follow the TES. The proof may be made in the same way.

The proofs given above are more concise than Autolycus's proofs of the same propositions, but follow his basic method.

Example: Betelgeuse, a Southern Star Let us examine the annual cycle of a particular star, Betelgeuse, which lies in Orion's right shoulder. We will assume