On the Gregorian Revision of the Julian Calendar

Jacques Dutka

1. Introduction

After a history of centuries of attempts within the Roman Catholic Church to reform the yearly calendar established by Julius Caesar, the decisive step was taken when Pope Gregory XIII replaced the Julian calendar by a revision. The first part of the revision consisted of omitting 10 days from the year 1582 so that Thursday, October 4, was followed by Friday, October 15. The second part of the revision, of far more lasting significance, was to modify the Julian scheme by which every numbered year, not divisible by four, consisted of 365 days, while every year divisible by four was a leap year, consisting of 366 days, so that the mean Julian year is 365¼ days. The modification retained the Julian scheme except for centurial years—those ending with two zeros. Those centurial years that are not divisible by 400 are ordinary (or common) years of 365 days, while only those centurial years divisible by 400 are leap years of 366 days. Thus in the Gregorian calendar, in every four centuries following a centurial year there are $(365.25)\ 400 - 3 = 146097$ days, so that the mean Gregorian year is $146097/400 = 365\ 97/400$ days.

As might have been expected, the Gregorian revision soon aroused vigorous opposition in Protestant countries at the time, and even disquiet among many Catholics. But eventually (in some cases after several centuries), it came to be accepted by virtually all Christian churches—a notable exception to this day being the Greek Orthodox Church.

Despite sporadic attempts to reform it that are still periodically advanced, the Gregorian calendar is now accepted, at least for civil matters, in virtually every country.

Many attempts since 1582 have been made to explain the curious 365 97/400 days for the mean Gregorian year. Eminent writers have offered explanations of varying degrees of ingenuity that are simply wrong. The primary purpose of this article is to present the available evidence for this number and to permit the reader to judge its adequacy.

Since the Gregorian calendar is based on the Julian calendar, which, in turn, depends on the Egyptian calendar adapted to the Roman system of time measurement, it is necessary first to develop the history of the calendar and some of the associated problems that finally led to the Gregorian revision.

2. The Calendar from Ancient Times to the Julian Era

From earliest times, people have adapted themselves to the daily alternation of light and dark following sunrise and sunset. Survival, whether as hunter or herder, fisher or farmer, also depended on the seasonal variations in the plant kingdom and the associated migration cycles of animal and marine life.

Primitive calendars appear to have arisen from the observations of the correlation between such survival activities and changes associated with the apparent motions of such celestial objects as the sun, moon, and stars. Indeed, on a small limestone tablet discovered in 1908, now known as the Gezer calendar, which is inscribed in ancient Hebrew and is dated approximately 950 B.C., there is a description of a sequence of agricultural activities associated with the successive months—lunar calendars are probably the most ancient. But the Greek writer Hesiod (app. eighth century B.C.) in his *Works and Days* associates harvesting and seeding with the rising and setting of constellations and stars. Similarly, sailors early learned to derive navigational information from observations of celestial bodies.

The need for some type of time reckoning, to determine the order of events or the intervals between them, likely led to the development of more formal types of calendars, the rules for which would generally be prescribed by religious authorities. The earliest celestial measures of time were associated with the daily motion of the sun, the cycle of waxing and waning of the moon, and the annual revolution of the sun relative to the fixed stars. A fundamental fact—that the durations of the day, the month, and the year are incommensurable—was likely recognized in prehistoric times.

The oldest Egyptian calendar was a lunar one of 12 months consisting alternately of 29 and 30 days. But by at least the fifth millenium B.C. it was replaced by a calendar of 12 months each consisting of 30 days. Each day was divided into 24 "hours," 12 for daylight and 12 for night, the duration of an hour thus varying daily. (Only later, in Hellenistic astronomical works, was the day divided into 24 uniform time periods. The 60-minute hour and 60-second minute are Babylonian in origin.)

To adjust the calendar, five days, the epagomenes, were added at the end of the 360-day year. The Egyptians apparently recognized that this adjustment was not exact—the solar year was about 365¼ days. In the remarkable trilingual Decree of Canopus by Ptolemy III Euergetes dated 238 B.C. (rediscovered in 1866), a sixth epagomenal day was introduced every fourth year. But this Alexandrian calendar was soon rejected as "Greek" by the conservative Egyptians who did not even accept its successor, the Julian calendar, until well into the Christian era. (In modern times the Alexandrian calendar survives in the calendars of the Coptic and Ethiopian Churches.)

Julius Caesar became the Pontifex Maximus in Rome in 63 B.C. Following his Egyptian military campaigns in 48–47 B.C., he returned to Rome accompanied by Sosigenes, an Alexandrian astronomer. To resolve the problems caused by the divergences between the calendrical months proclaimed by the pontifices and the months in their normal sequence in the year, Caesar, with the advice of Sosigenes, made drastic changes in the old 12 month Roman calendar which began in

The Gezer Calendar. Museum of the Ancient Orient, Istanbul.

March. The year 46 B.C. (the 708th after the founding of Rome according to the old calendar), was extended to 445 days—it was later termed "the year of confusion." The calends of 45 B.C. fell on January 1 of the new Julian calendar.

The second change was the adoption of the mean length of the year as 365¼ days, three successive common years of 365 days and the fourth year, a leap year, of 366 days. The preexisting calendar was thus replaced by a solar calendar. In 44 B.C. the fifth month, Quintilis, was renamed July. Caesar also redistributed the lengths of the months within the year in a more regular manner, except for February, which had 29 days. His successor, in 8 B.C., had Sextilis extended by one day at the expense of February, and renamed August. Other minor changes in the lengths of months from September to December were also made by Augustus.

The Julian calendar did not proceed according to the intended scheme. This was due to misinterpretations by the pontifices concerning the intercalation of one day every fourth year. The errors thus introduced were finally compensated for, and by A.D. 8, the originally intended Julian calendar was in normal operation—to remain unchanged in Europe until the Gregorian correction of 1582. The Julian calendar, with its subdivision according to the Roman month, was also used in the Near East and North Africa, indeed throughout the Roman empire.

The most detailed modern reference on the history of ancient calendars in the Mediterranean area is that of V. Grumel [1]. An older but valuable work is the three-volume treatise of F. K. Ginzel [2]. A good summary is given in the Explanatory Supplement to the Astronomical Ephemeris [3; Ch. 14], which is based on earlier work by the British scholar J. K. Fotheringham. Additional references on special topics will be found in these works. Finally, the monumental treatise of O. Neugebauer [4], the dean of historians of ancient science and mathematics, is replete with historical, mathematical, and astronomical detail not available in any other single source.

3. The Controversy over the Date of Easter

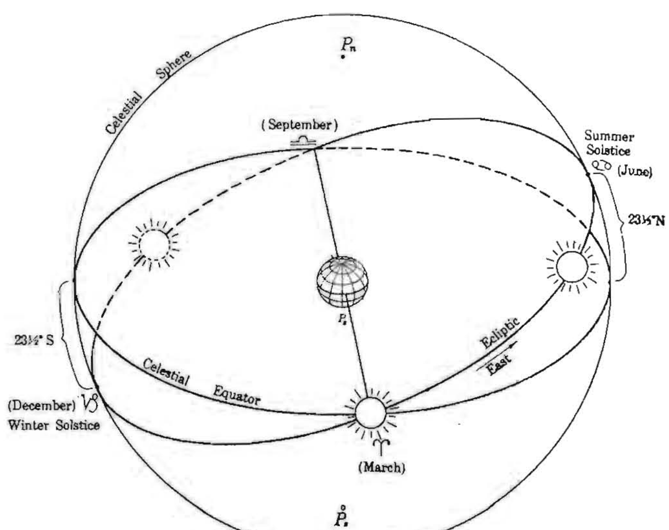
Easter is the most important holiday in the Christian ecclesiastical calendar, and the determination of the date of Easter is significant not only for this reason but because the dates of the movable holidays are dependent on it. The formulation of a better set of rules for its annual celebration was the principal motivation for the Gregorian revision of the Julian calendar.

The earliest Christians, who had been born Jews, continued to observe the Jewish Passover, but gave a transfigured symbolic meaning to the traditional sacrifice of the Paschal lamb. As Christianity gained Gentile adherents, there was increasing agitation within the Christian communities to distance their religious observance of Easter from the Jewish celebration of Passover. By about A.D. 150 bitter controversies increased between Western Christian communities based in Rome and Alexandria, who celebrated Easter on a Sunday, and those who followed the Jewish practice of observing Passover beginning with the evening of the 14th day of the lunar month of Nisan. (About the fourth century B.C., following their return from the Babylonian exile, the Jews adopted the later form of the Babylonian calendar with the same 19-year cycle as had been introduced by the astronomer Cidenas in 383 B.C. This cycle consists of 12 ordinary or regular years of 12 lunar months and 7 leap years of 13 lunar months in which a year begins with the autumnal month of Tishri. This calendar replaced the previous calendar, mentioned in Exodus, whose first month, Abib, corresponded to the spring month of Nisan.)

Numerous ecclesiastical assemblies failed to settle the Easter question until Constantine made Christianity the state religion and convened the Council of Nicaea (in Bythnia) in A.D. 325. The Council was charged with settling the Arian schism and the Easter question. In its epistle to the Church of Alexandria, the Council expressed its approval of the practice of the Romans who observed Easter on a Sunday and henceforth enjoined all Christians to do so on the same day. It also requested the Alexandrian church, whose mathematical and astronomical skills it praised, to compute yearly the date of Easter and to communicate this to other Christian churches. The Church of Alexandria complied, but its directions were not accepted by Eastern churches, which follow their own customs to this day. Many Western churches in the more remote parts of the Roman Empire clung tenaciously to local customs for determining Easter for centuries.

By about the sixth century the practice was established among those who followed the doctrine of the Roman church of a fixed rule: Easter is the first Sunday after the first full moon occurring on or next after the vernal equinox. This was based on a Julian (solar) calendar for the date of the vernal equinox and a (lunar) calendar with a 19-year cycle, the Metonic cycle, for the determination of the date of the full moon. (In 432 B.C., Meton of Athens introduced a cycle in which the dates of the new moons repeat every 19 years.)

At this point some astronomical definitions and constants will be useful. The tropical (mean solar) year is the interval between two successive passages of the mean sun through the mean vernal equinox and is about 365.2422 (mean solar) days. (It varies slowly over time.) The synodic month is the interval between two successive new moons (conjunctions), the latter occurring when the (geocentric) longitudes of the sun and moon on the celestial sphere are equal, and is about 29.53059 (mean solar) days.



Apparent motion of the sun on the celestial sphere.

Let a and b denote the number of 365- and 366-day years such that

$$1 \text{ tropical year} =$$
$$365.2422 \text{ days} = \frac{365a + 366b}{a + b} = 365 + \frac{b}{a + b}.$$

Thus, expressing $b/(a + b)$ as a simple continued fraction, one gets

$$.2422 = \frac{1}{4+}\,\frac{1}{7+}\,\frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{1\tfrac{2}{3}}$$

with the first six convergents

$$\frac{1}{4}, \frac{7}{29}, \frac{8}{33}, \frac{31}{28}, \frac{132}{545}, \frac{163}{673}$$

expressed in decimals and rounded off as .25, .24135, .24242, .24219, .24220+, .24220−, and alternately in excess and defect. The first of these is the Julian intercalation, and the third is that suggested by the Persian astronomer, mathematician, and poet, Omar Khayyam, in about A.D. 1079. It is a more accurate approximation of the tropical year than the Gregorian calendar with the approximation 97/400 = .2425. The problem facing the Calendrical Commission of Gregory XIII, however, included constraints imposed by lunar and solar calendars, as well as those imposed by religious traditions, custom, and usage. The compromise actually adopted, described in Section 1, has many conveniences.

The ratio of the tropical year to the synodic month, expressed as a continued fraction, is

$$\frac{365.2422}{29.53059} = 12 + \frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{17+}\cdots$$

with the convergents

$$\frac{12}{1}, \frac{25}{2}, \frac{37}{3}, \frac{99}{8}, \frac{136}{11}, \frac{235}{19}, \frac{4131}{334}, \ldots$$

Such approximations were extensively investigated by astronomers of antiquity. The fourth is associated with Cleostratus and the sixth with Meton and Cidenas in Greece and Babylon. The Metonic cycle is one of the outstanding achievements of ancient astronomy. It may be interpreted as equating 235 lunations (= 6939.69 days) to about 19 Julian years (= 19 · (365.25) = 6939.75 days), the error involved being about 1.48 hours.

We do not know how this remarkable approximation of 235/19 to the ratio of the tropical year to the synodic month was obtained. A. Pannekoek [5; 51ff.], an astronomer of the Netherlands, suggested that it arose from a long series of Babylonian observations. Others have suggested that ancient astronomers may have employed numerical algorithms now lost, like continued fractions, to obtain close approximations to ratios with large numerators and denominators by means of simpler fractions with much smaller numerators and denominators. Perhaps the most remarkable examples of this type in the West occur in a treatise of Aristarchus of Samos, who was active about 280–264 B.C. and maintained a heliocentric theory of the universe and the rotation of the earth on its axis. In this treatise, *On the Sizes and Distances of the Sun and the Moon*, he obtains grossly inaccurate results, but his methodology is mathematically valid. In his Proposition 13, Aristarchus states that 7921/4050 is greater than 88/45. The latter ratio is the fourth convergent in the continued fraction expansion of the former. In his Proposition 15, Aristarchus states that 71755875/61735500 is greater than 43/37. The latter ratio is the third convergent in the continued fraction expansion of the former. (On this question see I. Thomas [6; Vol. II, 14–15] and O. Neugebauer [4; Part 2, 641ff.], who present contrasting views.)

An enormous literature exists in virtually every modern European and many non-European languages on questions concerning Easter. In addition to the general references on chronology and the calendar mentioned at the end of Section 2, an excellent discussion in English of the controversies associated with Easter is in C. W. Jones [7], which is updated by R. R. Newton [8; Ch. II].

4. Prehistory of the Gregorian Revision

The two premises on which the ecclesiastical calendar of the Roman church had been based before the Gregorian revision, the Julian year length and the Metonic cycle of 235 lunations in 19 Julian years, were only approximations to the actual celestial phenomena. The Julian year errs in excess by .0078 per year; the synodic month obtained from the Metonic cycle is in excess by about .000255 per month. Within a few centuries after the Council of Nicaea, it was observed that the vernal equinox was occurring earlier each year than the nominal date of March 21 associated with the year A.D. 325. (According to Ginzel [2; Vol. 3, 257], the mean solar time of the vernal equinox at Rome in 325 was at 12h 44m on March 20; Newton [8; 24–25] gives a range of 3h to about 21h on March 20 for the years around 325.) By the thirteenth century, the error was seven or eight days. These divergences were recognized by the Roman church, but because of external and internal circumstances, no calendrical revisions were made despite numerous plans for reformation. The publication of the *Alfonsine Tables* under the sponsorship of Alfonso X of Castile (begun much earlier but finally completed c. 1272), incorporating Arabic and contemporary observations, gave a much better foundation for medieval astronomy in Europe than had heretofore existed, and it became the standard astronomical work for centuries.

(Alfonso himself, on being given an explanation of the Ptolemaic system as set forth in the *Almagest*, is said to have remarked that if God had asked his advice, he would have made the universe less complicated.)

The first edition of the *Tables* was printed at Venice in 1483, and subsequently it went through many editions. (An interesting description of the circumstances of composition, the contents, the various editions, and the men involved was given by the Danish astronomer J. L. E. Dreyer [9].) In particular, according to the *Alfonsine Tables*, the tropical year is 365d 5h 49m 16s, some 10m 44s shorter than the 365d 6h of the Julian year. The value for the tropical year of 365.2422d is equal to 365d 5h 48m 46s, so the Alfonsine year errs in excess by 30 seconds per year. The value finally adopted for the (present) Gregorian calendar is 365 97/400d = 365d 5h 49m 12s, which is 26 seconds in excess per year.

The French cardinal, Pierre d'Ailly, proposed at a council in Rome in 1412 that one leap year in 134 Julian years be replaced by an ordinary or common year of 365d (and every 304 years one day be omitted from the 19-year moon cycle.) This proposal regarding the yearly correction appears to have been later accepted by the German cardinal Niclaus von Cusa, who moreover suggested the one-time elimination of seven days from the calendar (for the end of May 1439), which would have brought the solar and lunar years into consonance.

Aloysius Lilius. (From an engraving printed in Naples.)

These proposals for calendrical changes did not prove fruitful until about the 1560s. Petrus Pitatus of Verona wrote a number of works on the calendar and made detailed suggestions for calendrical revision (see, e.g., Pitatus [10]). In these works, he accepted the Alfonsine year, but he approximated the proposal of d'Ailly by replacing it by a far simpler and more convenient suggestion for the omission of three leap years in every 400-year cycle

$$\frac{1}{134} \simeq \frac{1}{133\frac{1}{3}} = \frac{3}{400}$$

Moreover, he proposed that the omissions occur in centurial years. Since 1500 had been a leap year in the (Julian) calendar, he proposed that 1600, 1700, and 1800 be common years, but that 1900 be a leap year—this is, except for the starting point of the centurial leap-year omissions, identical to the plan of the Gregorian calendar that was later proposed by Aloysius Lilius.

The principal architect of the Gregorian calendar was a physician and lecturer at the University of Perugia, Aloysius Lilius (Lilio, Giglio) of Calabria (1510?–1576). Few details concerning his life are known, but G. Aromolo [11] wrote a biography that summarizes the history of calendrical reform efforts and of the work of Lilius, who migrated to northern Italy after becoming a physician.

After the death of Lilius, his brother and (partial) collaborator, Antonius, a personal physician of Gregory XIII, in 1577 presented Aloysius's manuscript to a commission appointed by Gregory XIII to consider the reformation of the calendar. (Gregory XIII, Ugo Buoncompagni of Bologna, had been elected Pope in 1572 at age 70 and died in 1585. A history of his papacy was written by L. von Pastor [12]. A nineteenth-century descendant of the family, Prince B. Boncompagni of Rome, was the publisher of the *Bulletino di Bibliografia di Storia delle Scienze Matematiche e Fisiche*, and a patron of continental writers on the history of mathematics.) After some delays, a summary of the manuscript was prepared by an astronomer, Pedronus Chaconus of Toledo, perhaps initially for the use of the Calendrical Commission. The summary, Compendium [13], was reprinted in the works of Christoph Clavius [14; 3–12] in 1603 and 1612. Copies of the summary were sent by the order of Gregory to Catholic princes and universities in early 1578 to obtain support for the projected reform and invite comments. (Copies of the original Compendium were found a few years ago by Gordon Moyer and Th. B. Settle in Italian libraries in Florence, Rome, and Siena.) A number of comments and suggestions, some contradictory, were received and evaluated by the Commission.

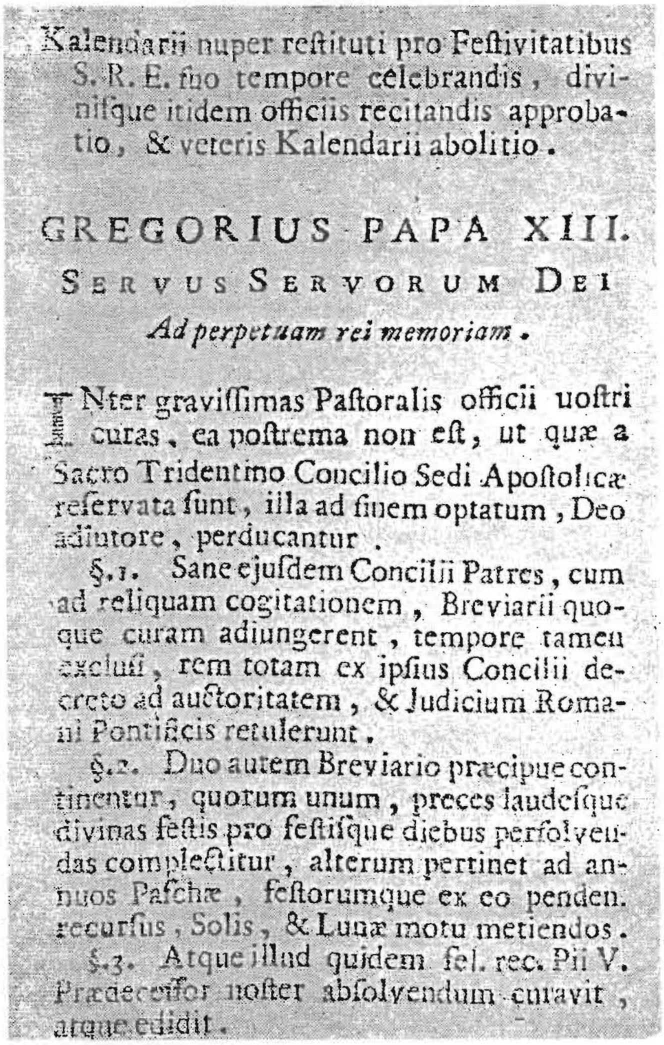
After much internal discussion of many different views, the Commission settled on the plan originally suggested by Lilius, with some modifications. The Commission had been charged with two basic tasks: to determine a method for computing the date of Easter, and to stabilize the vernal equinoctial drift in the calendar. An implicit assumption was that such changes as would be made would be in accord with the religious traditions and customs of the Roman church. Moreover, as the Jesuit Christoph Clavius of Bamberg was later to emphasize in his reply to the objections to the Gregorian calendar by Michael Maestlin of Tübingen, a teacher of Johann Kepler, the rules for the calendar finally adopted had to be convenient and reasonably simple—accordance to (existing) astronomical information was secondary.

For the calendar, Lilius suggested a plan equivalent to that previously put forward by Pitatus, also based on the Alfonsine year. While Pitatus had left open the question of the dates of the vernal and autumnal equinoxes, Lilius suggested March 21 for the date of the vernal equinox—(nominally) that of the Council of Nicaea in 325—and offered two alternatives to compensate for the equinoctial drift. One of these—the omission of 10 days in a single year—was accepted. This, and the change in the Julian four-year cycle of leap years to a modification of the Pitatus's scheme so that the 400-year cycle would begin with 1600 as a leap year, constitute the (present) Gregorian calendar.

Lilius's innovation was in the revision of the method of computing new moons so that the lunar calendar agreed more closely with the proposed solar calendar. This was the use of the epact, the age of the moon (in days) on January 1, reckoned from the last new moon of the preceding year; see Kaltenbrunner [15; (1), 490ff]. This linked the two calendars and enabled the date of the last full moon preceding Easter of the current year to be computed. Tables of epacts to cover the 30 possible ages of the moon on January 1 were included in the Compendium [13], as was a concordance, in tabular form, between the epact and the "Golden Numbers" previously used in the 19-year cycle (equal to [year/19] + 1). (One explanation of this name is that in illuminated medieval manuscripts these numbers were frequently written with gold ink.) An extended table of epacts in the form finally adopted by the Commission was given by Clavius [14; 110–111].

The Commission finally issued a report to the Pope (Cod. Vatic. 3685) (which was later printed in Kaltenbrunner [15; (3), 48–54]), about the beginning of 1581. The plan that Lilius had proposed was approved in a somewhat modified form. In regard to the Julian calendar, it was decided to omit 10 days in October and thenceforth three leap years in 400, as had been proposed. After further delay, the Papal Bull, *Inter Gravissimas,* was issued dated February 24, 1581/2 (reproduced by Clavius [14]) and was accompanied by a set of rules, the *Canones in Kalendarium Gregorianum perpetuum,* which is the present ecclesiastical calendar of the Roman church. No officially sanctioned explanation of the new calendar was made until the appearance of the massive tome of Clavius in 1603 (reproduced in Clavius [14]). It was based on an earlier work of Clavius [16] of 1588, which was a reply to the attack of Michael Maestlin on the new calendar.

In the literature, the influence of Nicolaus Copernicus on the final form of the Gregorian revision is frequently discussed. Pitatus [10] does mention Copernicus, but only in the sense that some observations of Copernicus and others confirm his own choice of the *Alfonsine Tables* for his basic astronomical data. In the Compendium [15], there is no mention of Copernicus—only the Alfonsine year length is mentioned as a



Kalendarii nuper restituti pro Festivitatibus S. R. E. suo tempore celebrandis, divinisque itidem officiis recitandis approbatio, & veteris Kalendarii abolitio.

GREGORIUS PAPA XIII.

SERVUS SERVORUM DEI

Ad perpetuam rei memoriam.

INter gravissimas Pastoralis officii nostri curas, ea postrema non est, ut quæ a Sacro Tridentino Concilio Sedi Apostolicæ reservata sunt, illa ad finem optatum, Deo adiutore, perducantur.

§.1. Sane ejusdem Concilii Patres, cum ad reliquam cogitationem, Breviarii quoque curam adiungerent, tempore tamen exclusi, rem totam ex ipsius Concilii decreto ad auctoritatem, & Judicium Romani Pontificis retulerunt.

§.2. Duo autem Breviario præcipue continentur, quorum unum, preces laudesque divinas festis pro festisque diebus persolvendas complectitur, alterum pertinet ad annuos Paschæ, festorumque ex eo penden. recursus, Solis, & Lunæ motu metiendos.

§.3. Atque illud quidem fel. rec. Pii V. Prædecessor noster absolvendum curavit, atque edidit.

The Papal Bull of 1582 proclaiming the new calendar.

Medallion of Gregory XIII commemorating the restored calendar. Obverse: A likeness of Gregory XIII. Reverse: A ram's head—the constellation Aries, adorned with a garland, denoting spring, encircled by a serpent biting its tail, denoting a cycle, with a motto.

(obverse) *(reverse)*

mean among several, as having lesser error. Finally, and most important of all the evidence *before* 1582, is the official report of the Calendrical Commission (of which Clavius was the most active member); see Kaltenbrunner [15; 52–54]. The report states that as of September 8, 1580, the Commission had reached a consensus to the effect that the Alfonsine year value, taken as 365d 5h 49m, etc., was assumed to be between the maximum and minimum, so that the vernal equinox is anticipated (in the calendar) by one day in about 134 years, which becomes three days in 400 years. In regard to the lunar-calendar aspect of Lilius's original plan, Clavius [14] [16] states that the *Prudentine Tables* (of Erasmus Reinhold, a disciple of Copernicus) were used in preference to the *Alfonsine Tables*.

As mentioned in Section 1, after the introduction of the new calendar, considerable partisan controversy concerning it arose. It lasted in some countries for centuries, but is now principally only of historical interest. People gradually came to accept that the Gregorian calendar was much more accurate, and thus ultimately more useful, than the Julian calendar it had replaced. The error in the Gregorian calender is less than one day in 3000 years, and Vatican astronomers, such as J. de Fort [17], occasionally reexamine the question in the light of more recent astronomical information, such as the slowing down of the earth's rotational rate by about ½ second per century.

After the elapse of more than a century, the studies of F. Kaltenbrunner [15] on the history of the Gregorian calendar remain the most extensive. They were supplemented and corrected in some detail by J. Schmid [18], J. G. Hagen [19], and others. Especially to be noticed is a commemorative volume recently published (G. V. Coyne, et al., eds. [20]) after the 400th anniversary of the Gregorian calendar.

Many articles and books on the Gregorian calendar are to be found in the literature, old and new. The writer has found the technical monograph (in Dutch) of W. E. van Wijk [21], with its illustrations, tables, and bibliography, particularly interesting.

5. The Misapplications of Continued Fractions

Raffaele Bombelli of Bologna is generally credited with having been the first European in modern times to introduce, in the first edition of his book on algebra of 1572, a particular type of continued fraction that was applied to the extraction of square roots. His work was improved upon in the seventeenth century, and simple continued fractions were introduced independently by various writers. But the practical development of this algorithm, in relatively modern form, occurred in the eighteenth century with the posthumous publication of a work of C. Huygens, and particularly with the far-reaching developments of L. Euler in many books and memoirs. For a brief summary of the early history, see D. J. Struik (ed.) [22; 111].

Euler was the most prolific and surely one of the great mathematicians of all time. Yet, as others have pointed out, in matters relating to the history of mathematics, he is often unreliable.

Euler [23; 390] takes as the length of the tropical year 365d 5h 48m 55s and proceeds to convert the fractional part of the day into seconds. Then he obtains a continued fraction expansion of 20935/86400 with the convergents:

$$\frac{0}{1}, \frac{1}{4}, \frac{7}{29}, \frac{8}{33}, \frac{55}{227}, \frac{63}{260}, \frac{181}{747}, \ldots$$

He says that from the first and second, the Julian calendar arises. More exact is 8 days in 33 years or 181 days in 747 years; "whence there follows in four hundred years 97 days. Whereby in this interval the Julian calendar inserts 100 days, the Gregorian converts three leap years into common [years] in four centuries." Euler lacks his customary clarity in the conclusion!

J. L. L. Lagrange, a younger contemporary and friend of Euler, was among the greatest mathematicians and astronomers of the eighteenth century and justly esteemed for the quality of his writing. He gave an exemplary presentation, from a mathematical standpoint, of the application of continued fractions to problems of diophantine analysis that is still worth reading. He is less careful, however, when it comes to historical detail. Lagrange [24; reprint, 524–533] says that in the Gregorian reformation, use was made of the determination of the year given by Copernicus, stated as 365d 5h 49m 20s. He converts the reciprocal of the fraction of the day into 86400/20960, but after obtaining approximations is still unable to get the corresponding fraction 86400/20952 = 400/97 that arises in the Gregorian year of 365d 5h 49m 12s.

George Peacock (1791–1858), Charles Babbage (1791–1871), and John Herschel (1792–1871) were members of a group at Cambridge University in the first quarter of the nineteenth century that was influential in bringing continental mathematics to Britain. The mathematical work of Peacock and his textbooks were highly regarded in Britain in his time. Peacock [25; Vol. I, 119] takes the length of the tropical year as $365.242263\dot{8}$d and converts it to a continued fraction:

$$.242263\dot{8} = \frac{1}{4+}\,\frac{1}{7+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{7+}\ \ldots$$

Illustration from book printed in 1586. Staats Museum, München.

whose first six convergents are

$$\frac{1}{4},\ \frac{7}{29},\ \frac{8}{33},\ \frac{39}{161},\ \frac{47}{194},\ \frac{418}{1519},\ \ldots$$

47/194 yields 47 days in 194 years, or 94 days in 388 years, or very nearly 94 + 3 or 97 days in 388 + 12 in 400 years: "this is the Gregorian correction of the Julian calendar." No comment is required.

Noel Swerdlow is a contemporary historian of astronomy who with O. Neugebauer recently published an important two-volume work on the mathematical astronomy of Copernicus. About a dozen years ago, Swerdlow [26] published a note on the Gregorian calendar, frequently referred to since, in which he introduces Lilius, gives the rules for intercalation in the calendar, its mean year length of 365 97/400d, and says: "Curiously this value for the length of the year has never been explained, and so the exact origin of our calendar is unknown." As references to this he lists Clavius [14], Kaltenbrunner [15], and Ginzel [2] (among others). He also lists a work of Pitatus (which the writer has not seen but that appears to be similar to Pitatus [10]) in his bibliography. Despite his references, Swerdlow appears to have missed the import of the prehistory at least from Pierre d'Ailly on, and of the unique role of the Alfonsine year as the historical basis for the mean year length adopted in "the Gregorian civil calendar."

The Compendium [13], based on the work of Lilius, was not lost—it has been reprinted in Clavius [14; 3–12], who confirms [14; 74] the role of the Alfonsine year as mentioned in the report of the Calendrical Commission, etc. Swerdlow sets up some plausible but historically unjustified hypotheses, computes with numbers and fractions in sexagesimal notation, and truncates the results to arrive at 97/400. This is at variance with historical facts. He also says, without evidence, ". . .(the mathematician of the sixteenth century could handle simple continued fractions)," etc.

6. Formulas and Tables for the Date of Easter

As pointed out previously, the Gregorian revision of the calendar was instituted primarily to deal with the problem of Easter. After some previous work by others, C. F. Gauss [27] gave a formula for the determination of the date of Easter that is the mathematical realization of the tables of Lilius. (See the article of J. Mayr [28], which shows the relation of Lilius's tables and Gauss's formula. A comprehensive and straightforward proof of the formula of Gauss was given by H. Kinkelin [29].)

A set of ten instructions for finding the date of Easter Sunday was given in the form of a table in *Nature*, April 20, 1876, p. 487, by an anonymous "New

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TO FIND EASTER

A NEW York correspondent sends us the following rule, which he states to be devised by himself, to find the date of Easter Sunday, perpetually :—

To find Easter "for ever."

Divide	By	And call the	
		Quotient	Remainder
The year of our Lord	19	—	a
" "	100	b	c
b...	4	d	e
$b + 8$	25	f	—
$b - f + 1$...	3	g	—
$19a + b - d - g + 15$	30	—	h
c...	4	i	k
$32 + 2e + 2i - h - k$	7	—	l
$a + 11h + 22l$	451	m	—
$h + l - 7m + 114$	31	n	o

n is the number of the month of the year and $o + 1$ is the number of the day of the month on which Easter falls.

A table for finding Easter Sunday.

York correspondent." (The writer owes this reference to the interesting book of T. H. O'Beirne [30; Ch. 10].)

The subject of Easter formulas and tables continues to arouse interest in modern times, particularly in popular astronomical literature. A useful survey was given by H. J. Felber [31] in 1977.

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