4. The Galilean transformation is needed here, which states that

$$\vec{v}_{SG} = \vec{v}_{SW} + \vec{v}_{WG},$$

or in words, "the velocity of the Swimmer relative to the Ground is equal to the velocity of the Swimmer relative to the Water plus the velocity of the Water relative to the Ground." Note that this is a vector equation, so the magnitudes don't necessarily add. In this problem, let's let $\vec{v}_{WG} = -v\hat{y}$ and $|\vec{v}_{SW}| = c$.

(a) While swimming upstream, the swimmer's speed relative to the ground is reduced $\vec{v}_{SG} = (+c - v)\hat{y}$ so that

$$\Delta t_u = \frac{d}{c - v}.$$

Similarly, swimming downstream the swimmer goes faster $\vec{v}_{SG} = (-c - v)\hat{y}$ and $\Delta t_d = d/(c + v)$. The total time taken is

$$\Delta t = \Delta t_u + \Delta t_d = \frac{d}{c - v} + \frac{d}{c + v} = \frac{2cd}{c^2 - v^2}.$$

(b) If the swimmer wants to move *directly* across the river, they must angle slightly upstream so they don't drift downstream. In this case, c is the hypotenuse of the right triangle, v is one side, and therefore $|\vec{v}_{SG}| = \sqrt{c^2 - v^2}$ is the speed of the swimmer relative to the ground. The time taken to swim across the river is the same as that to swim back (same speed), so that the total time taken is

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}}.$$

(c) The ratio shows that it is quicker to swim across and back

$$\frac{\Delta t_{up}}{\Delta t_{across}} = \frac{c}{c^2 - v^2}\sqrt{c^2 - v^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \geq 1.$$

This, in fact, is exactly the analysis needed to interpret the Michelson-Morley experiment. The "swimmer" in that case is light, and the "river" is the ether. Michelson and Morley measured the two travel times and tried to detect a difference, which would have allowed them to determine the speed of the Earth relative to the ether. However, since their result was that the two times were identical, Lorentz proposed that objects (i.e., their measuring apparatus) *contracted* in length by a factor γ in the direction of motion. This *ad hoc* proposal would result in the two travel times being identical. Of course, there is a "Lorentz contraction," but for reasons having to do with observers in different reference frames (i.e., special relativity), rather than an actual contraction of objects.

5. From (1.8) we have that

$$E_3 - E_2 = E_1\left(\frac{1}{9} - \frac{1}{4}\right) = \frac{5}{36}\left(-E_1\right) \approx 1.89 \text{ eV}.$$

From (1.13), the frequency of the photon is

$$\nu = \frac{E_3 - E_2}{h} \approx 4.57 \times 10^{14} \text{ Hz}.$$

which results, after some algebraic rearrangement

$$\beta_y = \frac{1}{\gamma_r}\,\frac{\beta'_y}{1+\beta_r\beta'_x}.$$

98. The time dilation formula gives $\Delta t' = \Delta t/\gamma \approx \Delta t\left(1-\frac{1}{2}\beta^2\right)$, and the time difference is

$$\Delta t' - \Delta t = -\Delta t\left(\frac{1}{2}\beta^2\right),$$

where for LEO at $h = 300$ km, $v = \sqrt{GM_\oplus/(R_\oplus + h)} = 7.732$ km/s. Since $\Delta t = 3.156 \times 10^7$ s, the difference is $\boxed{-10.5\ \text{ms}}$.

However, as Hafele and Keating state in their 1971 report on a clock comparison between commercial airliners and the ground

> Special relativity predicts that a moving standard clock will record less time compared with (real or hypothetical) coordinate clocks distributed at rest in an inertial reference space.... Because the earth rotates, standard clocks distributed at rest on the surface are not suitable in this case as candidates for coordinate clocks of an inertial space. Nevertheless, the *relative* [my emphasis] timekeeping behavior of terrestrial clocks can be evaluated by reference to hypothetical coordinate clocks of an underlying nonrotating (inertial) space.... General relativity predicts another effect that (for weak gravitational fields) is proportional to the difference in the gravitational potential for the flying and ground reference clocks.[3]

We therefore need to compare two different "proper" time intervals

$$\Delta t_{ISS} - \Delta t_{Eq} = \Delta t\left(-\frac{1}{2}\beta_{ISS}^2 + \frac{1}{2}\beta_{Eq}^2\right),$$

where $\Delta t\left(\frac{1}{2}\beta_{Eq}^2\right)$ is a correction term. Given that the equatorial speed of the Earth's rotation is $v = 463.3$ m/s, this correction is only +0.038 ms. In addition, the general relativistic correction is $\Delta t(gh/c^2)$, where $g = 9.81$ m/s^2 and $h = 300$ km, which gives +1.03 ms. Including general relativity, the astronaut's clock will have lost $\boxed{9.5\ \text{ms}}$. It is probably impossible to tell (without a clock) that a person has aged not one year, but one year minus 9.5 ms.

What about the Earth's orbit? It is true that the (nonrotating) Earth does not qualify as an "inertial reference space" because of its orbit around the Sun. However, this effect will be much smaller than the two minor effects we have already included, and will not affect our final answer.

99. Pole-and-barn.

[3] Hafele and Keating, "Around-the-world atomic clocks: Predicted relativistic time gains," *Science* **177** (4044), 166-168, (1971).