Chapter 1

Introduction

A. Definition of a Plasma: The Scope of Modern Plasma Physics

To the classic three states of matter—solid, liquid, and gas—there was added, in the 20th Century, a new state, called "plasma," a term coined by the distinguished and highly inventive American experimental physicist, Irving Langmuir. It is a matter of common experience that heating a material which is in the solid state, in general, transforms it into a liquid and that heating the liquid will transform it into a gas, the phenomenological properties of these states of matter being significantly different from one another. Heating (i.e., supplying energy to) the gas will dissociate it, if its molecules are polyatomic, but this does not result in a qualitative change in behavior. With further heating, the molecules or atoms become ionized, and we have a gas composed of electrons, positive ions, and neutral particles. The forces between the neutral particles and the charged particles, or among the neutral particles, have a short range (of order of atomic dimensions), just as in a neutral gas, so they do not result in qualitatively different behavior of the system, although there will be ionization and recombination. However, when the long range Coulomb forces between the electrons and the ions dominate over the short range forces, as is the case, for instance, in a fully ionized gas at high temperature and low density, the behavior of the system becomes so different that it may properly be considered as a new state of matter.

Langmuir's research on gaseous discharges was carried out in the 1930's, and there had been, even earlier, experimental and theoretical investigations of plasma in connection with radio wave propagation in the ionosphere and with astrophysical problems. However, plasma physics as a well-defined discipline began only in the 1950's, spurred initially by attempts to create controlled thermonuclear reactions in the laboratory and, subsequently, by the burgeoning field of space physics and by modern developments in astrophysics (pulsars, neutron stars, etc.). The relatively late blooming of this field is the more surprising when we consider that the vast majority of the universe (99%, according to popular accounts) is in the plasma state, the occurrence of the three other states of matter, as here on earth, being a rarity, from a cosmic point of view.

In this book, we shall use the term "plasma" to mean a many body system whose dynamical behavior, both equilibrium and non-equilibrium, is dominated by electromagnetic forces (i.e., Coulomb and, to a lesser extent, Lorentz forces) between charged particles rather than nuclear, atomic, or molecular forces. In addition, collective or cooperative effects generally dominate binary collisional phenomena. Most of the work in this field has been concerned with gaseous plasmas, with densities ranging up to 10^{17} cm^{-3} and thermal energies of 0.1 to 10^5 eV; however some attention has been given to solid state plasmas, both classical and quantum-mechanical. The plasma is usually assumed to be neutral (equal electron and ion charge densities) but there is a considerable body of both theory and experiment dealing with non-neutral or even single component plasmas. The majority of effort has been devoted to non-relativistic plasmas; however, relativistic plasmas (electron kinetic energy comparable to or even larger than the rest energy, mc^2) are also of interest in view of applications to pulsars, quasars, relativistic electron beam machines (with energies of order 1 to 10 MeV and currents of order 10 to 1000 kA), and devices which use collective plasma effects to accelerate baryons to relativistic energies.

The study of plasmas forms an exiting and challenging branch of classical physics, characterized by an unusual richness of phenomena in which both non-linear and collective effects play a major role. Important areas of application include controlled fusion, ionospheric phenomena, space physics (the magnetosphere, solar wind, etc.), astrophysics, elementary particle acceleration, and plasma processing of materials. In addition, it provides an excellent vehicle for training students in modern classical physics, thanks to its combination of basic and applied work; the close relation between theory and experiment; and the "relevance" of some of its applications.

As a distinct field of physics, plasma physics is still relatively young, and many approaches to an exposition of the theory have been used. Ours differs from most previous ones in two respects:

(1) We emphasize first the collective effects, which are most easily explained for a plasma without an external magnetic field (an "unmagnetized" plasma), rather than the phenomena associated with single particle motion in a magnetic field (guiding center approximation, drifts, etc.). The latter are, of course, essential, but we feel that it is the cooperative phenomena which constitute the most important distinguishing characteristic of the physics of plasmas.

(2) We follow a deductive approach, starting with the most fundamental, microscopic formulation and deriving all others from it. While the inductive method, starting with the simplest, single fluid formulation and going on to successively more sophisticated representations, culminating in kinetic theory and a microscopic formalism, has undeniable pedagogic virtues, we believe this field has reached a state of maturity which justifies an exposition which puts in clear view the logical basis of the theory, illustrated, but not obscured, by specific applications.

Although no specific familiarity with plasma physics is assumed, many texts at the introductory level now exist, and the reader should consult these, as needed. However, two concepts in plasma physics are so fundamental that we shall briefly discuss the elementary theory of these before proceeding to the deductive exposition.

B. The Debye Length

The most important phenomenon in a plasma is Debye shielding; the most important parameter is the Debye length associated with this. We therefore begin with the simplest possible description of Debye shielding.

If a point "test charge," Q, is placed in a homogeneous plasma with density n and temperature T, it follows from elementary statistical mechanics that the equilibrium density will change from n to

$$\tilde{n} = ne^{-q\phi/T}, \tag{1.1}$$

where ϕ is the self-consistent potential due to the charges (both test and plasma). (We shall always measure temperature in energy units so the Boltzmann constant k will never appear.) Poisson's equation gives (assuming Q is at the origin)

$$\nabla^2\phi = -\left(4\pi Q\delta(\mathbf{r}) + 4\pi \sum \tilde{n}q\right) \tag{1.2}$$

where the summation is over the charge species of the plasma. If we can make the assumption, to be justified *a posteriori*, that

$$|q\phi/T| \ll 1 \tag{1.3}$$

then we have

$$\nabla^2\phi - 4\pi \sum \left(\frac{nq^2}{T}\right)\phi = -4\pi\left(Q\delta(\mathbf{r}) + \sum nq\right). \tag{1.4}$$

For a neutral plasma, the last term on the right side vanishes and we have

$$\nabla^2\phi - K_D^2\phi = -4\pi Q\delta(\mathbf{r}) \tag{1.5}$$

whose solution is

$$\phi = (Q/r)e^{-K_D r} \tag{1.6}$$

with $K_D^2 = \sum(4\pi nq^2/T)$.

In place of the Coulomb potential Q/r which the test charge Q would produce in vacuum, we have the Debye potential which drops off exponentially in a distance K_D^{-1}. The physics involved is clear. The test charge repels (attracts) plasma particles of like (unlike) sign, resulting in a neutralizing charge "cloud," called the **Debye cloud**,whose dimension increases with T since thermal effects tend to keep the density uniform. Both ions and electrons contribute equally to the effect if $T_e = T_i$. In general, it is convenient to define the **Debye wavenumber** on a single species basis:

$$k_D \equiv \sqrt{4\pi nq^2/T}. \tag{1.7}$$

Its inverse

$$L_D = k_D^{-1} \tag{1.8}$$

is the **Debye length** for that species. (Both r_D and λ_D are used by some authors in place of L_D, but the second of these is ambiguous, since λ connotes wavelength and so λ_D might reasonably be assumed to denote $2\pi/k_D$ rather than k_D^{-1}). When $T_e = T_i$, we have, of course, $K_D = \sqrt{2}k_D$.

We must now consider the validity of the approximation (1.3). We see from (1.6) that in a literal sense (1.3) cannot hold, since $\phi \to \infty$ as $r \to 0$. However, the region in which (1.3) fails can be expected to make a small contribution to the charge density provided that region is small compared to the Debye cloud, i.e., provided $|q\phi/T| \ll 1$ for $r \sim L_D$. This will be true provided

$$q^2 k_D/T = \left(k_D^3/4\pi n\right) = \left(4\pi n L_D^3\right)^{-1} \ll 1. \tag{1.9}$$

This condition has a simple and very reasonable physical significance: the whole Debye shielding picture makes sense only if there are many particles in the Debye cloud, i.e., if $nL_D^3 \gg 1$. In fact, the dimensionless quantity nL_D^3 is of transcendental importance in plasma physics. As in any many-body problem, development of a coherent theory is possible only if there is some small parameter in terms of which a perturbation expansion can be made. The plasma parameter

$$\varepsilon_p \equiv \left(nL_D^3\right)^{-1} \tag{1.10}$$

plays this role, and modern plasma theory is based on the assumption $\varepsilon_p \ll 1$. We note that ε_p orders the three basic length scales in an unmagnetized plasma, namely L_D; $L_T = e^2/T$, the "distance of closest approach"; and $L_n = n^{-1/3}$, the mean interparticle spacing

$$L_T : L_n : L_D = \varepsilon_p/4\pi : \varepsilon_p^{1/3} : 1. \tag{1.11}$$

Before leaving the subject of Debye shielding, we note that it is, of course, not restricted to the simplest case—point test charge, neutral plasma—discussed here. In fact, so long as the basic condition, $\varepsilon_p \ll 1$, which justifies (1.3), is satisfied, we can generalize (1.4) to

$$\nabla^2\phi - K_D^2\phi = -S(\mathbf{r}) \tag{1.12}$$

where S is a general source term, i.e., a superposition of external test charges plus terms due to possible lack of plasma neutrality. The particular solution of (1.12),

$$\phi(\mathbf{r}) = \int dr' \frac{e^{-K_D R}}{4\pi R} S(\mathbf{r}'), \tag{1.13}$$

$$R = \left|\mathbf{r} - \mathbf{r}'\right|, \tag{1.14}$$

shows how Debye shielding manifests itself for an arbitrary source term.

C. Plasma Oscillations

Debye shielding nicely illustrates the collective character of plasma phenomena, i.e., the simultaneous interaction of many particles, but it involves thermal effects in an essential way. **Plasma oscillations** are the simplest example of a collective phenomenon which can occur even when thermal effects are neglected, although, as we shall see later, their effect can be very important here also. We treat electrons as a simple fluid characterized by density $n(\mathbf{x}, t)$ and velocity $\mathbf{v}(\mathbf{x}, t)$, and linearize the equations in $\mathbf{v}$ and $n_1 = n - n_0$, where n_0

is the uniform density of background ions, whose motion we shall neglect. Then the usual continuity equation,

$$\partial n/\partial t + \nabla \cdot (n\mathbf{v}) = 0, \tag{1.15}$$

becomes

$$\partial n_1/\partial t + n_0 \nabla \cdot \mathbf{v} = 0, \tag{1.16}$$

while the momentum equation is just

$$\partial \mathbf{v}/\partial t = -(e/m)\mathbf{E} = (e/m)\nabla \phi. \tag{1.17}$$

Finally, Poisson's equation gives

$$\nabla^2 \phi = 4\pi e n_1. \tag{1.18}$$

By taking the divergence of (1.17) and the time derivative of (1.16), we can eliminate $\mathbf{v}$, leaving

$$\partial^2 n_1/\partial t^2 = -\left(n_0 e/m\right) \nabla^2 \phi = -\left(4\pi n_0 e^2/m\right) n_1 \tag{1.19}$$

so that n_1, $\mathbf{v}$ and ϕ all oscillate at the **plasma frequency**,

$$\omega_p = \sqrt{4\pi n_0 e^2/m} \approx 6 \times 10^4 \sqrt{n_0}\ \ \mathrm{s}^{-1}, \tag{1.20}$$

where n_0 is measured in cm^{-3}. Again, this is a collective effect, no hint of which can be gleaned from a single particle description. If we define the thermal velocity by a:

$$ma^2/2 = T \tag{1.21}$$

then we see that ω_p, k_D and a are related by

$$\omega_p = k_D a/\sqrt{2}. \tag{1.22}$$

A simple physical description of plasma oscillations, also called **Langmuir oscillations**, can be given. If charge neutrality is violated in some portion of the plasma, owing, say, to a deficiency of electrons, then the resulting electric field accelerates electrons from adjacent regions to fill the deficit. Once charge neutrality is achieved, the field and consequent acceleration disappear, but inertia keeps the previously accelerated electrons in motion and soon there is an excess of electrons where formerly there was a deficiency. Again, fields arise, this time accelerating the electrons out of the region; there is once more an overshoot; and so the oscillations continue, the frequency being just ω_p.

D. The Saha Equation

We shall concentrate on the physics of either fully ionized plasmas or those for which the processes of interest occur on a time scale short compared to ionization and recombination times, but it is useful to know what combination of density and temperature give a substantial degree of ionization. Conventional equilibrium statistical mechanics leads to the Saha equation, which we quote without proof:[1] The densities of electrons, ions, and neutral atoms for a monatomic system in equilibrium are related by:

$$\frac{n_e n_i}{n_0} = g e^{-I/T} \lambda_e^{-3} \tag{1.23}$$

where I is the first ionization potential and λ_e is the electron de Broglie wavelength

$$\lambda_e = \sqrt{h^2/2\pi m T}. \tag{1.24}$$

(We have assumed T large enough so that the molecule is dissociated if it is diatomic, like H_2, and small enough so that multiple ionization can be neglected for heavier atoms, like the rare gases.) The statistical factor g, is defined by

$$g = 2Z_I/Z_0 \tag{1.25}$$

where Z_I is the partition function for the ion,

$$Z_I = \sum_j e^{-E_j/T}, \tag{1.26}$$

the E_j being the energy levels of the ions, and Z_0 is the atomic partition function,

$$Z_0 = \sum_j e^{-W_j/T}, \tag{1.27}$$

the W_j being the excitation energy levels of the atom measured from its ground state. Since g is typically of order 1, the principal dependence on temperature comes through the factors λ_e^{-3} and $\exp(-I/T)$ in (1.23). While (1.23) can be written in many forms, one of the most useful is in terms of the degree of ionization

$$\alpha \equiv \frac{n_e}{n_e + n_0} = \frac{n_e}{n}. \tag{1.28}$$

for a neutral plasma we have

$$\alpha = \frac{\sqrt{\eta^2 + 4\eta} - \eta}{2} = \begin{cases} \sqrt{\eta} & \eta \ll 1 \\ 1 - \eta^{-1} & \eta \gg 1 \end{cases} \tag{1.29}$$

where

$$\eta = g \frac{e^{-I/T}}{n \lambda_e^3} \tag{1.30}$$

depends only on density and temperature. In particular, we note that the ionization can be substantial at low densities ($n\lambda_e^3 \ll 1$) even if $T < I$.

[1] A simple but rigorous derivation is given in Chapter 3 of "Plasma Physics in Theory and Application," by Burton D. Fried, ed. by Wulf B. Kunkel, 1966.

E. Plasma Beta

This section is taken from Chapter 3 of "Plasma Physics in Theory and Application," by B. D. Fried, ed. by Wulf B. Kunkel, 1966.

In the case of a plasma confined by a magnetic field, we should clearly abandon the single-particle picture when the magnetic field B_i due to current flow within the plasma becomes comparable with the external field B_0. If v is a typical particle velocity and L a characteristic distance for changes in B, then

$$B_i \sim \frac{4\pi nevL}{c} \geq 4\pi \frac{nev}{c}\frac{cvm}{eB_0}, \tag{1.31}$$

if we assume that L cannot be much smaller than the cyclotron radius, mvc/eB. (Here, e is the electron charge, m the electron mass, c the speed of light, and n the electron density.) Thus, replacing $mv^2/2$ by kT, we have

$$\frac{B_i}{B_0} \geq \frac{nmv^2/2}{B_0^2/8\pi} = \frac{nkT}{B_0^2/8\pi} \equiv \beta. \tag{1.32}$$

The ratio β of thermal-energy density to magnetic-energy density (or of kinetic pressure to magnetic pressure) is among the most important of the several dimensionless parameters used to characterize a plasma. As we see here, the single-particle description may be appropriate for the *low-*β plasmas found in such laboratory devices as the mirror machine or stellarator but is not adequate to deal with the *high-*β plasmas observed in a pinch or in an electromagnetic shock tube.

In general, we may expect the single-particle picture to fail whenever the internal fields, magnetic or electric, become comparable with those imposed externally. Of course, the dynamics of the many-body problem can always, in a sense, be reduced to that of a (representative) single particle moving under the influence of fields (which must eventually by determined in a self-consistent fashion) so that single-particle concepts such as guiding center, $\mathbf{E}\times\mathbf{B}$ drift, mirror action, and the like, continue to have heuristic[2] value even for high-β plasmas.

The simplest, most thoroughly studied, and best-understood aspects of the plasma are, naturally, those associated with the equilibrium state, which we shall consider first. We shall then study small deviations from equilibrium, i.e., the kinetic theory of the plasma. Finally, we shall discuss some features of the lowest approximation to the kinetic equations for a plasma, that which neglects correlations between particles. Our aim throughout will be to develop the basic equations, from which the various properties of a plasma needed for applications can (and in subsequent chapters will) be derived.

[2]heuristic — adj., encouraging the student to discover for himself or herself