Magnetic Reconnection

Magnetic reconnection is a fundamental dynamical process in highly conductive plasmas. It can be regarded as the process that removes the following difficulty. On typical dynamical time scales a sufficiently hot spatially extended PLASMA behaves approximately as an ideal fluid in the sense that resistive effects are ignorable. As a consequence, the magnetic field is 'frozen' to the plasma motion and magnetic topology is conserved. This sets strong limitations on the accessible dynamical states. Large-scale magnetic flux tubes, which are strongly stretched out by the plasma pressure, as for instance observed in PLANETARY MAGNETOSPHERES or in stellar CORONAS, would be unable to release large amounts of their energy and return to a correspondingly relaxed state, as long as the plasma is trapped in the flux tubes. In other words, efficient transformation of magnetic to kinetic energy would largely be ruled out in ideal plasmas. There would be no obvious process that could counteract the generation of magnetic flux by dynamo processes and the magnetic fields in many space and astrophysical situations would grow secularly. Also, plasmas with magnetic fields of different origin would not be able to mix. Beginning in the late 1950s, several authors, including P A Sweet, E N Parker, H E Petschek and J W Dungey, introduced magnetic reconnection as the central process allowing for efficient magnetic to kinetic energy conversion in SOLAR FLARES and for interaction between the magnetized interplanetary medium and the MAGNETOSPHERE OF EARTH.

How does reconnection circumvent the difficulty associated with frozen-in magnetic fields? Resistive dissipation is more effective the more the electric current is localized to regions with a small spatial scale length. Thus, in reconnection a small-scale structure is generated in some region, such that there the constraint of ideal dynamics is broken. The interesting aspect is that a *local* non-ideality can have a *global* effect. Under such circumstances highly conducting plasma structures are able to transform magnetic to kinetic energy in an efficient way and the magnetic topology can change. According to a major line of present thinking, this is what happens in solar flares or magnetospheric substorms, and possibly in many other plasma processes in the universe.

Basic model

The formal description of reconnection requires the choice of a dynamical model. Here we confine the discussion to magnetohydrodynamics, where we allow for a finite resistivity (resistive MHD or 'RMHD') as the only non-ideal transport process. The corresponding basic equations consist of a combination of fluid dynamics and electrodynamics:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \qquad (1)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} \qquad (2)$$

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = j/\sigma \qquad (3)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e\boldsymbol{v}) = -p\nabla \cdot \boldsymbol{v} + j^2/\sigma \qquad (4)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad (5)$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} \qquad (6)$$

$$\nabla \cdot \boldsymbol{B} = 0. \qquad (7)$$

Here, ρ, $\boldsymbol{v}$, p, $\boldsymbol{j}$, $\boldsymbol{B}$, $\boldsymbol{E}$, σ^{-1}, e and μ_0 denote respectively mass density, velocity, pressure, current density, magnetic field, electric field, resistivity, plasma energy density and vacuum permeability (see the article on MAGNETOHYDRODYNAMICS). Here we mention only the fact that the equations (1)–(7) imply the conservation of energy. The balance of mechanical and electromagnetic energy, respectively, take the form

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2} + u\right) + \nabla \cdot \left(\left(\frac{\rho v^2}{2} + u + p\right)\boldsymbol{v}\right) = \boldsymbol{j} \cdot \boldsymbol{E} \qquad (8)$$

$$\frac{\partial}{\partial t}\left(\frac{B^2}{2\mu_0}\right) + \nabla \cdot \left(\frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}\right) = -\boldsymbol{j} \cdot \boldsymbol{E}. \qquad (9)$$

Adding these two equations gives conservation of energy

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2} + u + \frac{B^2}{2\mu_0}\right) + \nabla \cdot \left(\frac{\rho v^2}{2}\boldsymbol{v} + (u+p)\boldsymbol{v} + \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}\right) = 0. \qquad (10)$$

For some purposes it has proved useful to impose the condition of incompressibility on the flow velocity

$$\nabla \cdot \boldsymbol{v} = 0 \qquad (11)$$

replacing (4). This simplifies the problem significantly. It should, however, be kept in mind that for an incompressible flow, an energy conservation law of the form of (10) is not available. However, mass conservation and momentum balance are still described appropriately.

In a resistive fluid the importance of resistivity is measured by the Lundquist number

$$S = \frac{v_A L}{\eta} \qquad (12)$$

where $v_A = B/\sqrt{\mu_0 \rho}$ is the Alfvén velocity, $\eta = (\mu_0 \sigma)^{-1}$ the magnetic diffusivity and L a typical (global) scale length. Alternatively, one uses the magnetic reynolds number $R_m = vL/\eta$, where the Alfvén velocity is replaced by a typical plasma velocity v. (In the literature the expression 'magnetic Reynolds number' frequently is also used for the quantity S.) Large values of S or R_m, which are typical for space and astrophysical plasmas, correspond to small resistive effects. In the limit of large S or R_m, the terms involving resistivity can be neglected (unless singularities form) and equations (1)–(7) reduce to the equations of ideal magnetohydrodynamics (IMHD).

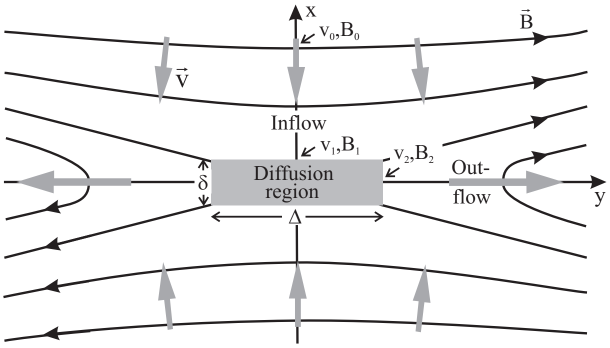

Figure 1. Qualitative pattern of two-dimensional reconnection.

Importantly, IMHD implies conservation of magnetic field line topology. In IMHD this fact can also be expressed by the property that two plasma elements that are connected by a magnetic field line at one time are connected by a magnetic field line at any later time (magnetic line conservation). Furthermore, the magnetic flux through an arbitrary contour transported by the plasma velocity field is also conserved. These properties provide the quantitative background for the dynamical constraints of IMHD mentioned above. In particular, they imply that large-scale topological reconfigurations of the magnetic field structure, as assumed to be associated with stellar and magnetospheric activity, are ruled out. In the following, we summarize in what sense magnetic reconnection resolves that dilemma and what is known at present about that process.

Two-dimensional reconnection

The simplest geometry in which reconnection may be described has two spatial dimensions, requiring the presence of an ignorable coordinate in three-dimensional physical space. In this section Cartesian coordinates x, y, z are used and it is assumed that the physical quantities are independent of z. We will first consider steady states and then introduce time dependence.

Steady-state reconnection

The basic configuration of two-dimensional steady-state reconnection is shown in figure 1. All field quantities are independent of time. Also, the magnetic field $\boldsymbol{B}$ and the plasma velocity $\boldsymbol{v}$ are assumed to lie in the x, y-plane, while for the electric field a non-vanishing z-component is admitted. The plasma is highly ideal such that the Lundquist number S (12) is much larger than 1.

To obtain an efficient conversion of magnetic to kinetic energy (along the trajectories of fluid elements) it is appropriate to assume a stagnation-type flow field $\boldsymbol{v}$ and oppositely directed magnetic fields in the upper and lower part of the inflow region (figure 1). The magnetic field vanishes at the origin (neutral point); viewed three-dimensionally a neutral line (line on which $B = 0$) extends along the z-axis.

Since S is large, for a smooth plasma flow with maximum gradients associated with the global length scale L the frozen-in condition would not allow annihilation of magnetic flux to any significant extent. This difficulty is avoided by the presence of a ‘diffusion region’ near the neutral line, where the resistive term j/σ in Ohm’s law is much larger than in the approximately ideal environment (‘external region’), typically by an enhancement of j_z. The diffusion region has length scales δ and Δ (figure 1) with $L \geq \Delta \geq \delta$. A locally defined Lundquist number, where L is replaced by δ in (12) can be considerably smaller than the global Lundquist number, indicating that in the diffusion region resistive diffusion can play an important role. There, the plasma and magnetic fields may decouple effectively, so that field annihilation along the fluid path becomes possible.

Under the present conditions (5) implies that E_z is a positive constant, say E_0. The presence of the diffusion region allows for a non-vanishing value of E_0, because otherwise (i.e. under ideal conditions with $\boldsymbol{j}/\sigma$ negligible) the z-component of equation (3) would require $E_z = 0$ at the neutral point, such that E_0 would have to vanish.

Another important property of the present geometry (shown in figure 1) is that $\partial B_y/\partial x > \partial B_x/\partial y$ or $j_z > 0$. Therefore, $\boldsymbol{E} \cdot \boldsymbol{j} = E_0 j_z > 0$ holds, which by (8) or (9) implies that magnetic energy is converted to kinetic energy. In fact, from (8) one finds

$$\frac{\partial}{\partial s}\left(\frac{v^2}{2} + \frac{u+p}{\rho}\right) > 0 \tag{13}$$

where (1) was used assuming that $\rho v \neq 0$, and s denotes the arc length of the trajectory of the plasma element (increasing in the direction of $\boldsymbol{v}$). Note that the thermal part on the left-hand side of (13) is enthalpy per unit mass rather than internal energy per unit mass, because the work done by the pressure force is included.

For a discussion of the consequences of mass and momentum conservation we specialize the resistive MHD equations (1)–(7) further, using the incompressibility condition (11) with constant density ρ_0 instead of (4). Then the resistive RMHD equations for a steady state assume the form

$$\rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} \tag{14}$$

$$E_0 + \boldsymbol{v} \times \boldsymbol{B} \cdot \boldsymbol{e}_z = j_z/\sigma \tag{15}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{16}$$

$$(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{e}_z = \mu_0 j_z \tag{17}$$

$$\nabla \cdot \boldsymbol{B} = 0. \tag{18}$$

Quantities in the outer inflow region will be characterized by their magnitudes at the point $(x_0, 0)$ where the positive x-axis crosses the boundary, and are labeled by the subscript zero, in particular (in addition to ρ_0, E_0)

$$p_0 = p(x_0, 0), \quad B_0 = B_y(x_0, 0), \quad v_0 = -v_x(x_0, 0)$$

(v_0 and B_0 are indicated in figure 1). Analogously, the subscripts '1' and '2' refer to the center inflow and outflow points on the boundary of the diffusion region (figure 1) and the subscript 'nl' is used for quantities on the neutral line.

Keeping the shape of the boundary fixed, except for the global scale length L, it can be expected that under the present conditions ρ_0, p_0, v_0, B_0, L and σ is a set of control parameters. Note, however, that in view of the nonlinearity of the problem a solution is not guaranteed for arbitrary parameter choices.

To disregard configurations that are merely the result of a similarity transformation, it is of interest to note that from these parameters three independent dimensionless quantities may be formed, which are conveniently chosen as

$$M_0 = \frac{v_0}{a_0}, \quad S_0 = \frac{a_0 L}{\eta}, \quad \beta_0 = \frac{2\mu_0 p_0}{{B_0}^2} \tag{19}$$

where a_0 is the (inflow) Alfvén velocity $B_0/\sqrt{\mu_0\rho_0}$. The earlier discussion of steady-state reconnection in the literature largely ignores the parameter β_0. This seems justified if β_0 is negligibly small or if pressure is constant in the external region. (It is only the gradient of the pressure that counts.) Then, reconnection is a two-parameter process, for instance described by M_0 and S_0. The parameter M_0 is regarded as of particular interest and is usually called reconnection rate. It measures the velocity with which the plasma enters the region of consideration (normalized by the local Alfvén velocity). The so-defined reconnection rate should not be confused with the rate of magnetic flux reconnection, which is defined by the rate at which flux conservation is violated in the reconnection process, which, in the present case, is given by the electric field component E_z along the neutral line, which equals $E_0 = v_0 B_0$.

There is no fully satisfactory analytical treatment of the system of equations (14)–(18). There are solutions for the external (ideal) region and solutions for the diffusion region, based on singular asymptotic expansions. However, a rigorous matching of such solutions has not yet been achieved. In this situation one introduces intuitive assumptions or simplifications. Much of the discussion in the literature is based on the following approximate picture.

Consistent with $j_z > 0$, let us assume that the aspect ratio $\kappa = \Delta/\delta$ is large compared to 1, that derivatives with respect to x are large compared with derivatives with respect to y and that $|B_x| \ll B_0$. Pressure is treated as constant in the external region. Then approximate relations are obtained in the following way:

Condition of incompressibility (11):

$$v_1 \Delta = v_2 \delta.$$

x-component of momentum balance (14) at $y = 0$:

$$p_1 + \frac{{B_1}^2}{2\mu_0} = p_{\rm nl}.$$

y-component of momentum balance at $x = 0$, ignoring B_x:

$$\frac{\rho_2 {v_2}^2}{2} + p_2 = p_{\rm nl}.$$

Ohm's law:

$$E_0 = v_1 B_1 = v_2 B_2 = j_{\rm nl}/\sigma.$$

Ampère's law (6) (replacing the derivative by a difference quotient)

$$j_{\rm nl} = \frac{B_1}{\mu_0 \delta}.$$

Combining these equations and using that, in view of the assumptions, $\rho_2 = \rho_1 = \rho_0$, $p_1 = p_2 = p_0$ one obtains

$$v_2 = a_1 \tag{20}$$

$$M_1 = \frac{1}{\kappa} = \frac{1}{\sqrt{S_1}} \tag{21}$$

$$\frac{M_1}{M_0} = \left(\frac{B_0}{B_1}\right)^2 \tag{22}$$

$$\frac{\Delta}{L} = \frac{S_1 B_0}{S_0 B_1}. \tag{23}$$

This system of equations has to be completed by an equation for the ratio B_0/B_1 which requires a more complete solution of equations (14)–(18). In the absence of such a solution one introduces an additional condition as an *ad hoc* assumption, or from the external solution alone, or on the basis of numerical computations. We give three examples.

(a) Sweet–Parker model. Here it is assumed that the diffusion region is a thin extended structure such that Δ becomes of the order of L. For simplicity, let us set $\Delta = L$. The external region is largely homogeneous such that approximately $B_1 = B_0$ and $S_1 = S_0$. Under these conditions, (21) gives the reconnection rate as

$$M_0 = \frac{1}{\sqrt{S_0}}.$$

This rate is generally regarded as too low to be relevant for typical conditions in stellar atmospheres and space plasmas because of their large Lundquist numbers.

(b) Petschek's model. In this model it is assumed that $\Delta \ll L$. In that case, it is necessary to consider the presence of slow-mode shock waves (here in the limit of incompressibility) which implies that B_1 may be considerably smaller than B_0. Approximately, one finds $B_1/B_0 = 1 - 4M_0/(\pi \ln(R_{m_0}))$. The maximum reconnection rate occurs near $B_1/B_0 = 1/2$, such that

$$M_0 < \frac{\pi}{8}\frac{1}{\ln R_{m_0}}.$$

Typically this reconnection rate is considerably larger than that of the Sweet–Parker process.

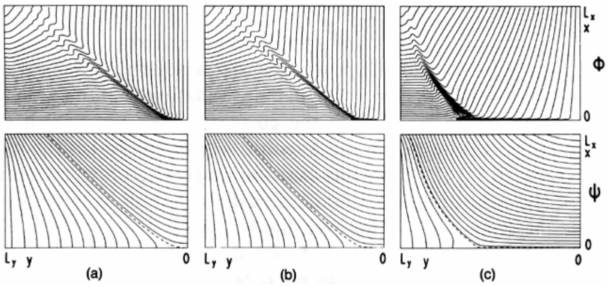

Figure 2. Numerical solutions of equations (25) and (26) with $\Psi = \hat{A}$ and $\Phi = \hat{D}$ by Biskamp. In (*b*) and (*c*) the Lundquist number of (*a*) is increased by factors of 2 and 4, respectively (from Biskamp 1986).

(c) Further reconnection models. Several authors (e.g. W I Axford, B U Ö Sonnerup, E R Priest, T G Forbes) have generalized the models by Sweet and Parker and by Petschek in various respects. The most general are the fast reconnection models of Priest and Forbes. They included electrical currents in the external region and obtained a description that contains the Sweet–Parker and Petschek models as particular cases.

For numerical studies (as for other purposes) it is convenient to represent $\boldsymbol{v}$ and $\boldsymbol{B}$ by single flux functions $D(x, y)$ and $A(x, y)$. This is possible because of the vanishing divergence of both fields and the absence of z-components,

$$\boldsymbol{B} = \nabla A \times \boldsymbol{e}_z, \quad \boldsymbol{v} = \nabla D \times \boldsymbol{e}_z. \tag{24}$$

Then, one eliminates the electric current density by using (17), and the pressure by taking the curl of the momentum equation. The remaining equations of the system (14)–(18) are usually written in non-dimensional form (here non-dimensional quantities carry the hat-label), such that A is normalized by $B_0 L$, the velocity potential D by $a_0 L$ and coordinates by L

$$[\Delta\hat{D}, \hat{D}] = [\Delta\hat{A}, \hat{A}] \tag{25}$$

$$M - [\hat{A}, \hat{D}] = -\frac{1}{S_0}\Delta\hat{A}. \tag{26}$$

For functions $f(x, y)$, $g(x, y)$ the symbol $[f, g]$ is defined by

$$[f, g] = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}.$$

Equations (25) and (26) have been solved numerically for a variety of boundary conditions by several groups. Figure 2 shows, for example, a result by Biskamp, demonstrating that a Sweet–Parker current sheet rapidly develops for increasing S_0.

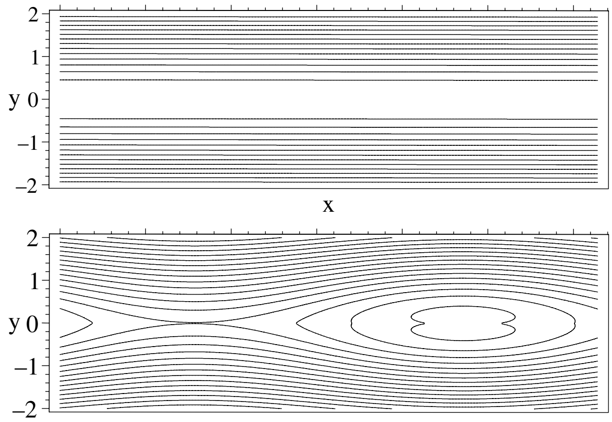

Figure 3. Magnetic field lines of the unperturbed Harris sheet (upper panel) and the linear tearing mode (lower panel).

Time-dependent reconnection

Although, historically, steady-state reconnection has been given great deal of attention, it seems that in many cases magnetic reconnection occurs as a time-dependent process. Several features of steady-state reconnection are also present in typical time-dependent (two-dimensional) cases, such as a neutral line and an associated stagnation-flow pattern. This analogy is particularly close for driven reconnection, where—as in steady states—the plasma inflow is determined by boundary conditions.

A qualitatively different case arises when reconnection occurs as an unstable process. The prototype of an instability involving reconnection is the tearing mode (suggested by H P Furth, J Killeen and M N Rosenbluth). A plane current sheet located in an infinite domain undergoes spontaneous formation of magnetic islands (figure 3). Resistivity plays a similar role as in steady states: it is important only in regions of strong current concentration. Assuming that the unperturbed configuration does not involve such concentrations, it can be described in the limit of $S \to \infty$. The classical example is the Harris sheet, where the unperturbed magnetic field $\boldsymbol{B}$, the flux function A and the plasma pressure p are given (in dimensionless form) by

$$\boldsymbol{B} = -\tanh(x)\boldsymbol{e}_y, \quad A = \ln(\cosh(x)), \quad p = \frac{1}{\cosh^2(x)}$$

which is a static solution of (25) and (26) for infinite S.

The instability generates the required current concentration spontaneously. The dynamical evolution is described by equations (25) and (26), if generalized to include time dependence. In view of the time dependence, it is appropriate to derive the electric field from the time dependence of the flux function A, rather than from an electric potential. In dimensionless form one obtains the following linearized equations for the perturbations ϕ and ψ of the velocity potential D and the flux function A

$$\Delta\frac{\partial\psi}{\partial t} = [\Delta a, A] + [\Delta A, a] \tag{27}$$

$$\frac{\partial a}{\partial t} = \frac{1}{S}\Delta a - [A, \psi]. \qquad (28)$$

Choosing modes of the form

$$\psi(x, y, t) = \hat{\psi}(y)\, \mathrm{e}^{\mathrm{i}\alpha y + qt}$$

with a corresponding expression for ϕ, (27) and (28) give two ordinary differential equations for $\hat{\psi}$ and $\hat{\phi}$. These equations are solved analytically by a singular perturbation method for the regime

$$\frac{1}{S} \ll |q| \ll 1, \quad |q^2| \ll \alpha^2 < 1.$$

The essential aspect is the occurrence of a thin region around $y = 0$ of width $\epsilon = (q/(\alpha^2 S))^{1/4}$, where the current density becomes large. Using appropriate scaling in this region and in the external region, one finds explicit solutions to lowest significant order in ϵ. The matching condition determines the dispersion relation, i.e. q as a function of α

$$\hat{q} = \frac{\sqrt{\lambda}}{\pi} \frac{\Gamma\left(\frac{\lambda+1}{4}\right)}{\Gamma\left(\frac{\lambda+3}{4}\right)} (1-\alpha^2)(1-\lambda^2)$$

where

$$\lambda = \frac{\hat{q}^{3/2}}{\hat{\alpha}}, \quad \hat{q} = qS^{1/2}, \quad \hat{\alpha} = \alpha S^{1/4}.$$

The tearing mode develops a series of magnetic islands with corresponding X-type and O-type neutral lines (figure 3). The local structure near the X-line resembles the steady-state reconnection pattern of figure 1.

For the reconnection processes associated with solar flares and magnetospheric substorms (see MAGNETOSPHERE OF EARTH: SUBSTORMS) more realistic two- and three-dimensional models have been developed (pioneered by J Birn, A Otto, T G Forbes, Z Mikic and others). Figure 3 gives a qualitative sketch of the magnetic field structure as it develops with time. The original equilibrium configuration becomes unstable by a process which is a generalization of the tearing mode shown in figure 3. During its nonlinear evolution a plasmoid forms, which grows, becomes accelerated and eventually leaves the system, carrying a substantial amount of energy that was stored in the original equilibrium. Processes of this kind have been suggested to be relevant for magnetospheric substorms, solar flares and SOLAR CORONAL MASS EJECTIONS. For the magnetosphere it is believed that the onset of the non-ideal (e.g. resistive) process is related to the formation of a thin current sheet late in phase (a) in figure 4.

In the case of three-dimensional modeling one encounters new aspects, as compared with reconnection in two dimensions, which are discussed in the following section.

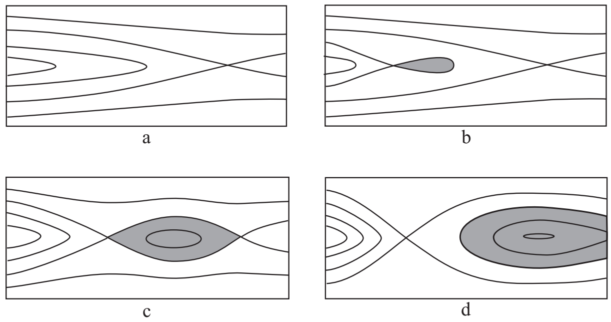

Figure 4. Plasmoid formation and ejection in a stretched magnetic field configuration.

Three-dimensional reconnection

The two-dimensional models discussed so far seem to be realistic for reconnection occurring in three-dimensional space only if the z-dependence is small and if the extent of the reconnection region along the perpendicular direction (z-direction) is large enough that effects of the edges can be neglected. Moreover, it requires that magnetic flux of exactly opposite direction is convected along the x-axis into the reconnection region. Each of these assumptions is doubtful, and so a generalization with a component of the magnetic field along the invariant direction is required which allows for magnetic flux to approach the reconnection region with a non-vanishing z-component. This is most simply realized by adding a constant B_z-component in the model given by equations (14)–(18). This requires an additional (E_x, E_y) component of the electric field, which has the form of a gradient ($\nabla(B_z D)$ for the representation of $\boldsymbol{v}$ given in equation (24)). It therefore does not destroy the stationarity of these models nor does it modify the momentum equation.

Although the additional B_z-component seems to be a minor modification, it gives rise to several fundamental questions about the notion of reconnection. In two dimensions ($B_z = 0$) reconnection is usually defined by the existence of an X-type neutral point and a flow of stagnation type which transports magnetic flux across the separatrices, i.e. the field lines which end at the neutral point and separate the magnetic flux of the inflow and outflow regions (see figure 1). With the additional B_z component, the former neutral line of the two-dimensional models now becomes an ordinary magnetic field line and the former separatrices, or separatrix surfaces, respectively, do not exist anymore or, if the notion of a separatrix is applied to the projection of the field onto the plane perpendicular to the field line, they are not unique. (The latter can be shown by the example $\boldsymbol{B} = (y, x, 1)$, where every field line possesses separatrices in this sense, i.e. has an X-type magnetic field in the plane perpendicular to the field line.) These difficulties become even more serious for fully three-dimensional magnetic fields without translational invariance. Several methods have been proposed to solve these difficulties of localizing and defining reconnection.

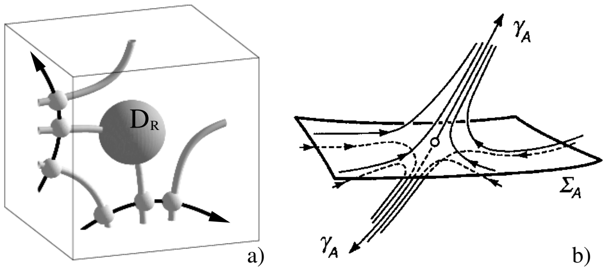

Figure 5. (*a*) Sketch of a breakdown of magnetic line conservation at a localized non-ideal region D_R. Two plasma elements (small spheres) which originally share a field line end up on different field lines. (*b*) Topology of field lines in the vicinity of an A-type generic null, showing the spine (γ_A) and the fan (Σ_A); B-type nulls have reversed field directions (from Lau and Finn 1990).

First, it is tempting to use the plasma flow in addition to the structure of the magnetic field to identify reconnection. However, this quantity is not independent of the frame of reference used, and for instance the location of the stagnation point depends on the observer. In another approach Hesse and Schindler therefore used the original meaning of reconnection, i.e. a breakdown of magnetic field line conservation (first suggested by Axford). They introduced the notion of general magnetic reconnection to occur if

$$\int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} \neq 0 \tag{29}$$

where the integral is evaluated for field lines passing through a localized non-ideal region D_R embedded in an otherwise ideal plasma (see figure 5). The criterion (29) is sufficient for a breakdown of magnetic line conservation, provided all magnetic field lines start and end in the ideal region outside D_R. This is a consequence of the general form of magnetic field line conservation

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{w} \times \boldsymbol{B}) = \lambda \boldsymbol{B} \tag{30}$$

where $\boldsymbol{w}$ is the transport velocity of the field lines, which can be identified with the plasma velocity $\boldsymbol{v}$ in the ideal region but may differ from it in non-ideal processes. Equation (30) implies $\boldsymbol{B} \cdot \nabla\lambda = 0$, and therefore λ is constant on magnetic field lines. Moreover, in the ideal region we have $\boldsymbol{w} = \boldsymbol{v}$ and $\lambda = 0$ and hence $\lambda = 0$ across D_R as well. In this case equation (30) together with the induction equation implies

$$\boldsymbol{E} + \boldsymbol{w} \times \boldsymbol{B} = \nabla\Psi \tag{31}$$

and therefore $\int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = 0$ along all magnetic field lines, because $\nabla\Psi$ vanishes in the ideal region so that Ψ is constant outside D_R. A non-vanishing integral (equation (29)) therefore requires a breakdown of magnetic line conservation. Vice versa, if $\int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = 0$ holds for all magnetic field lines crossing D_R, the potential Ψ can be integrated within D_R from

$$\boldsymbol{E} \cdot \boldsymbol{e}_B = \nabla\Psi \cdot \boldsymbol{e}_B \tag{32}$$

and the field line velocity, given by

$$\boldsymbol{w} = (\tilde{\boldsymbol{E}} \times \boldsymbol{B})/B^2 \tag{33}$$

with $\tilde{\boldsymbol{E}} = \boldsymbol{E} - \nabla\Psi$, exists provided there is no magnetic null within D_R. In this case (29) is also necessary for a breakdown of magnetic line conservation.

Magnetic null points

The existence of $\boldsymbol{w}$ given by (33) is critical if there are magnetic nulls within D_R. Using $\boldsymbol{E} = -\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t}$ for a given evolution of an electromagnetic field, (32) can be restated by the existence of a potential $\tilde{\varphi} = \varphi + \Psi$ with

$$\boldsymbol{B} \cdot \nabla\tilde{\varphi} = -\boldsymbol{B} \cdot \frac{\partial \boldsymbol{A}}{\partial t} \tag{34}$$

where $\boldsymbol{A}$ is a vector potential for $\boldsymbol{B}$. Given the potential $\tilde{\varphi}$ on a surface crossed only by non-recurring field lines this condition defines $\tilde{\varphi}$ along these field lines. This method, called potential mapping, does not necessarily lead to a smooth potential $\tilde{\varphi}$ if field lines from separated regions join at magnetic nulls. For instance, smooth boundary conditions on $\tilde{\varphi}$ given for all field lines entering a surface enclosing the null, may lead to discontinuities of $\tilde{\varphi}$, and if the boundary is part of the ideal region the condition on $\tilde{\varphi}$ corresponds to boundary conditions on the plasma velocity $\boldsymbol{v}$. Therefore, Greene, followed by Lau and Finn, argued that in an almost-ideal plasma magnetic nulls are the site where non-ideal terms, especially the resistive term in Ohm's law, become important, and hence a breakdown of magnetic field line conservation may take place.

Magnetic nulls can be classified in terms of the eigenvalues of the tensor $\nabla\boldsymbol{B}$. They have either one real and two complex conjugated eigenvalues or three real eigenvalues. For the latter case they are called type A for $(+ - -)$ signs of the eigenvalues and type B for $(- + +)$ (see figure 5). The eigenvectors of the complex conjugated eigenvalues, or of the real eigenvalues with the same sign, span a magnetic surface called the *fan* surface by Priest and Titov. The third eigenvector defines the spine as shown in figure 5. In the the case of more than one magnetic null the fan surfaces of an A-type and B-type null intersect at a structurally stable magnetic field line called separator. It can be shown that this field line is also a potential site of reconnection due to discontinuities in $\tilde{\varphi}$ or singularities of $\boldsymbol{w}$ for corresponding boundary conditions.

The topological structure of magnetic nulls led Priest and Titov to propose two additional mechanisms of reconnection called spine and fan reconnection. They showed that certain prescribed motions of the field lines on a surface enclosing the null produce singular field line velocities according to equation (33), and hence require a breakdown of field line conservation. In spine and fan reconnection the current tends to concentrate along the the spine and fan respectively.

Reconnection without nulls

Magnetic nulls are not the only places where magnetic reconnection may occur. For instance if equation (34) is integrated over a closed field line with a non-vanishing contribution of the right-hand side, one finds that the potential $\tilde{\varphi}$ may not exist and therefore processes breaking the magnetic line conservation have to be present. This is also reflected by the criterion (29) which does not require nulls. For a non-vanishing magnetic field the method of potential mapping always leads to a smooth potential $\tilde{\varphi}$ and transport velocity $\boldsymbol{w}$. However, the latter might be very large, much higher than the Alfvén velocity, which excludes under realistic conditions an ideal evolution. This may happen, as noted by Priest, Forbes and Demoulin, in layer-like regions where the potential mapping or mapping of foot points of field lines shows strong gradients and which are therefore called magnetic flipping layers or quasi-separatrix layers.

While the method of potential or field line mapping aims at finding potential sites of reconnection and thus adds to the general criterion (29) certain conditions on the structure of the magnetic field, Hornig gave a more restricted definition of reconnection by generalizing the observation that in two dimension the field line velocity $\boldsymbol{w}$ has a singularity at the X-point. A covariant description shows that this singularity is a special type of null of the corresponding four-vector field W^4. This property is structurally stable in the transition from two to three dimensions, where now the site of reconnection is determined by a line of finite length along which W^4 vanishes. Within this definition it is in particular possible to distinguish a simple local slippage of plasma relative to the field lines, which also may satisfy (29) but which is not usually called reconnection, from reconnection itself.

Another aspect of reconnection is the dynamics of MAGNETIC HELICITY. While in two dimensions the source of magnetic helicity ($-2\boldsymbol{E} \cdot \boldsymbol{B}$) vanishes, this is not necessarily the case in three dimensions. Hence magnetic reconnection in three dimensions does not necessarily conserve magnetic helicity.

Collisionless reconnection

Magnetic reconnection can also occur in the absence of a collisional resistivity. Collisionless reconnection processes are based on non-ideal terms that in a more refined macroscopic picture appear on the right-hand side of Ohm's law (3) in addition to the resistive term. For instance, a current-driven microinstability may lead to fluctuations that on the macroscopic level have an effect similar to resistivity based on particle collisions. Also, resonant wave–particle interaction, off-diagonal terms of the electron pressure tensor or electron inertia have been suggested for magnetic reconnection. The final assessment of the role that each of these processes plays in reconnection requires a full three-dimensional kinetic description. Although such a kinetic point of view is crucial for the understanding of the small-scale plasma physics of reconnection, it less crucial for the overall dynamics, which in many of its features seems to be largely independent of the type and details of the non-ideal process. Thus, even highly collisionless reconnection processes, as for instance occurring in the Earth's magnetosphere, have been successfully simulated by using a simple resistive model of the form (3). Often, the resistivity is empirically adapted, for instance by spatial localization or by introducing an *ad hoc* dependence of η on the electric current density. The formation of thin current sheets in the pre-reconnection dynamics seems to play an important role in the onset of collisionless reconnection processes.

Bibliography

Biskamp D 1993 *Nonlinear Magnetohydrodynamics* (Cambridge: Cambridge University Press)

Parker E N 1979 *Cosmical Magnetic Fields* (Oxford: Clarendon Press)

Priest E R and Forbes T G 1999 *Magnetic Reconnection* (Cambridge: Cambridge University Press)

Tsinganos K C (ed) 1996 *Solar and Astrophysical Magnetohydrodynamic Flows* (Dordrecht: Kluwer)

Vasyliunas V M 1975 Theoretical models of magnetic field line merging *Rev. Geophys. Space Phys.* **13** 303

Karl Schindler and Gunnar Hornig