Numerical Project

1. Read handout "The restricted 3-body problem (2D)".

2. Run the Matlab code "restricted2d.m" with the subroutine "fnewton.m". First, set the input parameters to the following values (these are the defaults):

   ```
   massratio = 0;

   x20   = 1;
   v2x0  = 0;
   y20   = 0;
   v2y0  = sqrt(1/x20);

   x30   = 0;
   y30   = 2;
   v3x0  = -sqrt(1/y30)*0.5;
   v3y0  = 0;

   tmax  = 1*period2;
   ```

   Answer the following questions:

   (a) What kind of orbit is m_2 in?

   (b) What is its orbital period (in terms of T, the natural time scale)?

   (c) If you were to change the value of `x20` would m_2 still be in a circular orbit? Why or why not? (Try it! You might want to adjust `tmax` so that you follow the orbits longer.)

   (d) What does `massratio = 0` mean physically?

   (e) What kind of orbit is m_3 in?

   (f) Calculate its period (in terms of the period of m_2).

3. Increase the time span to `tmax = 10*period2` and increase the effect of m_2 to `massratio = 0.01`.

   Answer the following questions:

   (a) What kind of orbit is m_3 in now?

   (b) Take the final (vector) value of `x`, run the simulation again with this as the initial value `x0` and change `massratio` back to zero. Determine the period and semimajor axis of m_3 in this final state. Do they satisfy Kepler's third law?

4. Increase the perturbing effect of m_2 to `massratio = 0.1`. Notice the effect of the close approaches of the two "planets".

   Answer the following questions:

   (a) What happens to m_3?

   (b) Change the initial conditions very slightly, let `y30 = 2.01`. Now what happens to m_3? Explain the results using concepts that we have discussed in class.

   (c) Take the final (vector) value of `x`, run the simulation again with this as the initial value `x0` and change `massratio` back to zero. Determine the period and semimajor axis of m_3 in this final state. Do they satisfy Kepler's third law?

5. Simulate the Earth and the Moon. Let m_2 be the Earth and m_3 be the moon.

   (i) Calculate the necessary value for `massratio`.

   (ii) Determine the initial conditions for m_3. One method is to use the defaults for m_2, but the following for m_3

```
x30   = x20 + emdistance;
y30   = 0;
v3x0  = 0;
v3y0  = v2y0 + deltav;
```

   where `emdistance` is the Earth-Moon distance (in the proper scaled units) and `deltav` is the speed of the Moon relative to the Earth (again, in proper dimensionless units).

   Answer the following questions:

   (a) What is the proper value for `massratio`?

   (b) What are the proper values of `emdistance` and `deltav`?

   (c) Prove that the Moon has a stable orbit around the Earth by extending the simulation to `tmax = 50*period2`, i.e., 50 "years."

      i. You may find it useful to use `figure(3)` since it plots the moon centered on the Earth.

      ii. By "stable" I mean that the moon remains in orbit and does not escape. You may notice, however, that it appears to lose energy. The reason is that energy is not conserved in the restricted three-body problem, but another quantity is. See § 2.12.2 on the Jacobi constant in Curtis's text for an explanation.

      iii. Calculate and plot the Jacobi constant C for m_3. The expression for this constant can be found in Eq. (2.212) on page 141 of Curtis. You will have to make it dimensionless as all the other quantities are.

   (d) Reduce `deltav` by 50% so that the satellite (no longer the Moon) is in an elliptical orbit. Does the line of apsides retain a constant orientation? Explain why or why not.

6. Investigate one of the scenarios listed below.

   (a) Simulate the Pluto system — a "restricted 4-body problem." Pluto has 4 moons, but I want you to simulate 3 of them: Charon, Nix, and Hydra. This article (http://arxiv.org/abs/astro-ph/0512491) lists the orbital elements of the three moons. Use the following masses:

Object	Mass
Pluto	1.3×10^{22} kg
Charon	1.5×10^{21} kg
Nix	1×10^{18} kg
Hydra	5×10^{18} kg

   You can let Pluto and Charon be m_1 and m_2 respectively, and Nix and Hydra will be the small masses that do not exert forces on Pluto or Charon, but do feel forces due to them.

   (b) Investigate the stability of asteroids in a Kirkwood gap. That is, m_2 is Jupiter, and m_3 is an asteroid with a period that is commensurate with Jupiter. You'll have to run the simulation for a long time to determine how long the asteroid is able to remain in that orbit. You can assume that both orbits are initially circular. Also, vary the initial period of the asteroid to see if you can get it to be stable. See `http://en.wikipedia.org/wiki/Kirkwood_gap` for more information.

   (c) Perform a satellite drag calculations. Start out m_3 in a circular orbit, but modify `fnewton.m` to include atmospheric drag.

   (d) Investigate the sensitivity to initial conditions for asteroid 2008EA9. See the article and the paper at `http://www.technologyreview.com/blog/arxiv/27112/`

   (e) Investigate the ejection from Earth of particles that might escape the solar system, or reach Jupiter. See the article
   and the paper at `http://www.technologyreview.com/blog/arxiv/27092/`

   (f) Investigate the sensitivity of the asteroid Apophis hitting the Earth.
   See `http://en.wikipedia.org/wiki/99942_Apophis` for more information.

   (g) Investigate "horseshoe" or "tadpole" orbits for near-Earth asteroids. See the article and the paper at `http://www.technologyreview.com/blog/arxiv/26608/`

   (h) Investigate the stability of any possible "trojan asteroids" of the Earth. How close to the L4 or L5 points must they be in order to remain there for a given length of time.

   (i) Choose your own problem. But you must clear it with me first.

   NOTE: When I say "investigate," I want you to try a variety of initial conditions, for example, and see which ones give you the resulting trajectory that you're looking for, and which do not. Also, are there any interesting trajectories that you weren't expecting to find?

7. Type up your answers to all of the above questions, along with commentary explaining the results. Use a research journal format, with an abstract and a two-column text.

I've posted some examples on the course web page, as well as a LaTeX template if you wish to use it (you don't have to, but equations and formatting are simpler than with Microsoft Word). Be sure to include orbital plots, since "a picture is worth a thousand words," but your plots *must* have properly labeled axes and enough notation for the reader (me) to understand. Also, even though I am the reader, you should pitch the level of your paper to your audience, which in this case is your fellow EP 393 students. Assume that they know everything that you do from class, but don't know about this specific project. So you'll have to explain this project in detail, using equations and concepts that you've learned in class.