Numerical Project

1. Read handout "The restricted 3-body problem (2D)".

2. Run the Matlab code "restricted2d.m" with the subroutine "fnewton.m". First, set the input parameters to the following values (these are the defaults):

   ```
   massratio = 0;

   x20   = 1;
   v2x0  = 0;
   y20   = 0;
   v2y0  = sqrt(1/x20);

   x30   = 0;
   y30   = 2;
   v3x0  = -sqrt(1/y30)*0.5;
   v3y0  = 0;

   tmax  = 1*period2;
   ```

   Answer the following questions:

   (a) What kind of orbit is m_2 in?

   (b) What is its orbital period (in terms of T, the natural time scale)?

   (c) If you were to change the value of `x20` would m_2 still be in a circular orbit? Why or why not? (Try it! You might want to adjust `tmax` so that you follow the orbits longer.)

   (d) What does `massratio = 0` mean physically?

   (e) What kind of orbit is m_3 in?

   (f) Calculate its period (in terms of the period of m_2).

3. Increase the time span to `tmax = 10*period2` and increase the effect of m_2 to `massratio = 0.01`.

   Answer the following questions:

   (a) What kind of orbit is m_3 in now?

   (b) Take the final (vector) value of `x`, which I have saved in the variable `xnew`, run the simulation again with this as the initial value `x0` and change `massratio` back to zero. Determine the period and semimajor axis of m_3 in this final state. Do they satisfy Kepler's third law?

4. Increase the perturbing effect of m_2 to `massratio = 0.1`. Notice the effect of the close approaches of the two "planets". (NOTE: Don't forget to comment out `x0 = xnew;`)

   Answer the following questions:

(a) What happens to m_3?

(b) Change the initial conditions very slightly, let `y30 = 2.01`. Now what happens to m_3? Explain the results.

(c) Take the final (vector) value of `x`, run the simulation again with this as the initial value `x0` and change `massratio` back to zero. Determine the period and semimajor axis of m_3 in this final state. Do they satisfy Kepler's third law?

5. Simulate the Earth and the Moon. (NOTE: Don't forget to comment out `x0 = xnew;`) Let m_2 be the Earth and m_3 be the moon.

   (i) Calculate the necessary value for `massratio`.

   (ii) Determine the initial conditions for m_3. One method is to use the defaults for m_2, but the following for m_3

```
x30   = x20 + emdistance;
y30   = 0;
v3x0  = 0;
v3y0  = v2y0 + deltav;
```

   where `emdistance` is the Earth-Moon distance (in the proper scaled units) and `deltav` is the speed of the Moon relative to the Earth (again, in proper dimensionless units).

   Answer the following questions:

   (a) What is the proper value for `massratio`?

   (b) What are the proper values of `emdistance` and `deltav`?

   (c) Prove that the Moon has a stable orbit around the Earth by extending the simulation to `tmax = 50*period2`, i.e., 50 "years." (You may find it useful to use `figure(3)` since it plots the moon centered on the Earth.)

   (d) Reduce `deltav` by 50% so that the satellite (no longer the Moon) is in an elliptical orbit. Does the line of apsides retain a constant orientation? Explain why or why not.

6. Investigate resonance between m_2 and m_3. Set `deltav` to zero, and try several different small values for `emdistance`. Describe what you find.

   This last part of the project is open-ended. You are to play around with the initial conditions of m_3 and see what happens. Try to obtain some interesting effects, such as a gravity assist that ejects m_3 from the solar system, or a gravity assist that sends m_3 onto an elliptical orbit.

   Type up your answers to all of the above questions, along with commentary explaining the results. Be sure to include orbital plots, since "a picture is worth a thousand words," but your plots *must* have properly labeled axes and enough notation for the reader (me) to understand.