PROGRESS IN THE DETERMINATION OF THE GRAVITATIONAL COEFFICIENT OF THE EARTH

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Abstract. In most of the recent determinations of the geocentric gravitational coefficient (GM) of the Earth, the laser ranging data to the Lageos satellite have had the greatest influence on the solution. These data, however, have generally been processed with a small but significant error in one of the range corrections. In a new determination of GM using the corrected center-of-mass offset, a value of 398600.4415 km$^3$/sec$^2$ (including the mass of the atmosphere) has been obtained, with an estimated uncertainty (1 $\sigma$) of 0.0008 km$^3$/sec$^2$.

Introduction

The best values of the gravitational coefficient of the Earth (GM) are generally accepted as those determined by observing the influence of this parameter on the motion of near-Earth satellites. The satellite best suited for this purpose is Lageos [Cohen and Smith, 1985]. Lageos was designed to minimize the effects of nongravitational forces, and the gravitational effects of all but the longest wavelength components of the Earth's geopotential are greatly attenuated because of its high altitude (5900 km). Thus the modeling of the forces on the Lageos satellite is more accurate than for any other satellite. In addition, the laser ranging measurements to Lageos are of very high accuracy. For the higher quality stations, normal points with millimeter precision are routine, and biases for some stations have been shown through collocation experiments to be at the sub-centimeter level [Degnan, 1989]. Errors in the modeling of the tropospheric refraction are also estimated to be at or below the centimeter level. The result is that satellite solutions for GM that include laser ranging to Lageos tend to be dominated by these data because of the greater accuracy of the modeling and the observations. Any error in processing that data set will also strongly influence the estimate.

In 1978, a solution for GM using laser ranging to four near-Earth satellites produced a value of 398600.44 ± 0.02 km$^3$/sec$^2$ [Lerch et al., 1978]. In 1985, a value for GM of 398600.44 ± 0.002 km$^3$/sec$^2$ was determined from eight years of laser ranging to Lageos by the University of Texas Center for Space Research (UT/CSR) [Tapley et al., 1985; Chovitz, 1987]. Most recently, UT/CSR reported a solution for GM obtained from a 3-year fit to Lageos laser ranging and also from a multi-satellite solution to be 398600.4405 ± 0.001 km$^3$/sec$^2$ [Ries et al., 1989]. The error analysis for that solution indicated that the primary source of uncertainty in the estimate of GM was the possibility of biases in the laser range measurement.

Optical tests on the Lageos-II satellite, which was built under the direction of the Agenzia Spaziale Italiana (ASI) to be a replica of the Lageos satellite, prompted a second look at the tests conducted on Lageos. It was discovered that the
resonance with the orbital motion. The orbit and parameter adjustment resulted in range residuals with a weighted root mean square (rms) of approximately 5 cm. The value obtained for GM was 398600.4415 km$^3$/sec$^2$ (in TDT units).

Covariance analysis indicates that the dominant error sources which influence the estimate of GM are biases in the laser ranges and, to a lesser degree, biases in the modeled effect of tropospheric refraction. For this solution, a range bias of 2 cm and a troposphere bias of 0.2% was considered for each station, with the exception of 1 cm for two stations for which collocation tests results were available (Matera and the Moblas-7). These biases are considered to be reasonable, if not conservative, estimates of the 1-σ errors in the tracking station range calibrations and in the Marini and Murray [1973] model for the troposphere refraction correction. Assuming a 100% error in the Earth radiation pressure model resulted in a negligible effect on GM. The resulting uncertainty (1-σ) considering all of the indicated errors was 0.0008 km$^3$/sec$^2$. Various combinations of estimated and considered parameters never resulted in a change in the estimate greater than 0.0005 km$^3$/sec$^2$, and even a very pessimistic assumption about the range biases (4 cm for all stations) did not increase the uncertainty above 0.0013 km$^3$/sec$^2$.

An additional solution for GM was performed to investigate the sensitivity of the solution to the arc length and the parameters estimated. Instead of a single 5-year arc, the same data were fit with 3-day arcs in which GM, the geopotential, and station coordinates were estimated as parameters common to all of the arcs. Unlike the other solutions described here, polar motion and UT1 were adjusted, and a mean range bias for each station was estimated with a 2 cm a priori standard deviation. The exceptions were two stations with possible data problems for which the range bias was estimated every 15 days with 1 m a priori standard deviation. Troposphere errors identical to the long-arc were considered. The value of GM obtained was also 398600.4415 km$^3$/sec$^2$, with an estimated uncertainty of 0.0005 km$^3$/sec$^2$. The rms of the residuals from the short-arc fit was 2.8 cm.

Multi-Satellite Solution

An effort to develop an improved model for the Earth's gravity field has been undertaken jointly by the Space Geodesy Branch at NASA/GSFC and UT/CSR in support of the orbit determination requirements of the joint NASA/CNES TOPEX/POSEIDON altimetric satellite mission. In the latest UT/CSR geopotential solution (TEG-2B), data sets from 17 satellites including Lageos, spanning inclinations from 15 to 115 degrees, were used to obtain a solution of the gravity field complete to degree and order 50 [Tapley et al., 1991]. The measurements include laser range and doppler data, satellite altimeter data from Seasat and Geosat, and terrestrial surface gravity observations. In addition to the spherical harmonic coefficients of the geopotential and of the quasi-stationary sea surface topography, the set of estimated parameters included GM, selected ocean tide parameters, and various satellite-dependent parameters such as initial position and velocity, drag coefficients, and solar reflectivity coefficients. For the doppler data, pass-dependent frequency and tropospheric biases were estimated also. In the solution for this analysis, the coordinates for all laser tracking stations were also estimated so that the solution would not be constrained by the coordinates derived from the Lageos analyses. The Lageos data in this solution were processed using a series of 15-day arcs spanning 1979 through 1985. The estimated value for GM obtained in the multi-satellite solution was 398600.4415 km$^3$/sec$^2$.

The covariance matrix for the UT/CSR gravity field solution was calibrated in order that it would be representative of the true errors in the solution. The calibration procedure is accomplished by the comparison of the geopotential coefficient differences with their predicted uncertainty as various data sets are excluded from the solution. This method increases the formal covariance of the gravity model solution to account for unmodeled effects. If it is assumed that the same calibration applies to the estimate of GM, the uncertainty for GM was found to be 0.0005 km$^3$/sec$^2$.

**Discussion of Results**

The multi-satellite and Lageos solutions for GM are in remarkable agreement with each other. The improvement in the agreement over the previous UT/CSR solution, where the two solutions differed by 0.0004 km$^3$/sec$^2$, may be due to fully accounting for general relativity in the new multi-satellite solution. The error estimate from the previous UT/CSR solution may be conservative since a worst-case error (a bias common to the data from all stations) resulted in only a 1-σ change in the estimate. Based on the improved agreement of the various solutions with each other and using realistic assumptions in the error models, the uncertainty of the new solution is estimated to be no greater than 0.0008 km$^3$/sec$^2$.

While the direct effects of general relativity were taken into account in the data analyses described above, there is an indirect effect on the units being used that must be considered. The effect of general relativity on time is that coordinate time in different reference frames may run at different rates. Because the International Astronomical Union (IAU) definition of the time coordinate of the solar system barycentric reference system requires that only periodic differences exist between Barycentric Dynamical Time (TDB) and TDT, the spatial coordinates in the barycentric frame have effectively been rescaled to keep the speed of light unchanged between the two systems. Thus, when barycentric (or TDB) units of length are compared to geocentric (or TDT) units of length, a scale difference appears. Noting that the mass parameter GM/c$^2$ has units of length, the value for the mass parameter of a body in TDB units will be related to its value in TDT units by a scale factor. A similar scale change is introduced between the TDB and the International System (SI) units of length by the requirement that TDB run at the same rate as International Atomic Time (TAI), which is defined on the surface of the Earth. The scale difference of 1.4808 x 10^-8 between the TDB and TDT units of length results in a change in GM of 0.0059 km$^3$/sec$^2$; the scale difference of 6.969 x 10^-10 between the TDB and SI units results in a change in GM of 0.0003 km$^3$/sec$^2$ [Guinot and Seidelmann, 1988; Huang et al., 1990]. Thus the value of GM determined here would be 398600.4418 km$^3$/sec$^2$ in SI units. The value of GM in TDB units is 398600.4356 km$^3$/sec$^2$, which is in good agreement with the value of 398600.437+0.006 km$^3$/sec$^2$ determined from analysis of lunar laser ranging to the Moon [Newhall et al., 1987].

Recommendations to modify the IAU definition for coordinate times to eliminate the rescaling were adopted by
been determined from a 5-year fit to laser range data to the IAU in August of 1991 (D. McCarthy, personal communication, 1992). Under the new time definitions, described by Guinot [1991], the units of time and length will remain SI units in all reference frames. Consequently, the SI value of GM is appropriate when using the new time transformations.

Summary

The value of the gravitational coefficient of the Earth (GM), including the mass of the Earth's atmosphere, has been determined from a 5-year fit to laser range data to Lageos and from a combined solution with a variety of satellites using laser range, doppler, and altimeter data. The value of GM (398600.4415 km$^3$/sec$^2$ in TDT units and 398600.4418 km$^3$/sec$^2$ in SI units) obtained with these analyses is larger than the previous determination due to the correction of a small but significant error in the processing of the Lageos laser ranging data. The new estimate is also in good agreement with the LLR determination. The standard deviation of this solution for GM is estimated to be no greater than 0.0008 km$^3$/sec$^2$. The accuracy is sufficient to require careful consideration of all direct and indirect relativistic effects. These results are important aspects of the continuing improvement of the satellite reference frame modeling and its relationship to reference frames established by other techniques.

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References


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