DETERMINATION OF THE GEOCENTRIC GRAVITATIONAL CONSTANT FROM LASER RANGING ON NEAR-EARTH SATELLITES

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Abstract. Laser range observations taken on the near-earth satellites of Lageos (a = 1.92 e.r.), Starlette (a = 1.15 e.r.), BE-C (a = 1.18 e.r.) and Geos-3 (a = 1.13 e.r.), have been combined to determine an improved value of the geocentric gravitational constant (GM). The value of GM is 398600.61 km^3/sec^2, based upon a speed of light, c, of 299792.5 km/sec. Using the IAG adopted value of c equalling 299792.458 km/sec scales GM to 398600.44 km^3/sec^2. The uncertainty in this value is assessed to be ± .02 km^3/sec^2. Determinations of GM from the data taken on these four satellites individually show variations of only .04 km^3/sec^2 from the combined result.

The Lageos information dominated the combined solution, and gave the most consistent results in its data subset solutions. The value obtained for GM from near-earth laser ranging compares quite favorably with the most recent results of the lunar laser and interplanetary experiments.

Background. Most reductions of data from lunar and interplanetary probes have included reestimation of the gravitational constant times the mass of the earth, GM (earth plus atmosphere). These missions include the Ranger, Surveyor, Lunar Orbiter, Pioneer, Mariner, and Viking series of flights, as well as a number of Russian interplanetary probes. The results of these GM estimations have been summarized by Esposito, 1974; Esposito and Ng, 1976; and Esposito, 1978. The GM obtained from the best available interplanetary investigations has been adopted by the XVI General Assembly of the IUGG/IAG (Moritz, 1975). This value, based upon the IAG adopted value of the speed of light (299792.458 km/sec) is 398600.5 km^3/sec^2.

Laser ranging to the retroreflectors left on the moon during the Apollo Missions has been carried out at the McDonald Observatory in Texas since 1969. Williams (1974) has reported a value of GM obtained from these observations, which is in good general agreement with the interplanetary estimates. This value of GM and its uncertainty that he reported is 398600.65 ± .1 km^3/sec^2 for c equalling 299792.5 km/sec. This translates into a value of 398600.48 km^3/sec^2 using the latest IAG speed of light. King et al (1976) combined lunar laser and Alsep VLBI observations to obtain the ratio of the mass of the sun to that of the earth plus moon $M_s/(M_e + M_m)$ of 328900.50 ± .03. Assuming the AU and the IAG value of c, this yields a value of GM of 398600.51 ± .03 when using an earth to moon mass ratio of 81.3007 (Wong and Reinbold, 1973).

In this paper we use near-earth laser ranging in a new determination of GM. These results basically confirm those obtained from interplanetary and lunar laser experiments, but further reduces the uncertainty of GM.

Near Earth Laser Ranging Experiment. The experiment reported here was performed in the development of the recent Goddard Earth Models (GEMs) 9, 10, 10A and 10B (Lerch et al, 1977; Lerch et al, 1978). For the first time in the GEM series, these gravity models included an adjustment of GM.

At Goddard Space Flight Center (GSFC) the emphasis has been on using as much precise near-earth satellite data as possible for our geodetic investigations. The Goddard (GSFC) laser systems currently deployed have errors (bias and noise) of from 5 to 8 cm (Vonbun, 1977) while the SAO lasers have accuracies of 40 cm (Pearlman et al, 1977). Although the Goddard lasers are more accurate, the combination of the two systems provide global distribution. The precision data provided by these lasers on Geos-3, Starlette, BE-C and Lageos have shown that improved force modeling is necessary to achieve better orbit accuracies. The recovery of GM along with the geopotential became a major objective. The laser systems are the most successful instrument for providing accurate near-earth orbital positioning. The GEM models now contain nearly 213,000 laser observations taken on 9 satellites. However, only the four satellites used in this study have been extensively tracked by the newest laser systems.

Results. Table 1 presents combined as well as individual solutions of GM using observations taken on four satellites. In all cases, the laser ranges were reduced in arcs of 5 days length. The Geodyn program (Putney, 1977) was used for the orbital recovery and generation of epoch orbit, station coordinate and GM normal equations. The normal equations were then analyzed using the Solve Program (Putney et al., 1976) to perform the various solutions for GM and the tracking station coordinates. The force modeling and coordinate system displacement included:

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TABLE 1. ESTIMATION OF THE GEOCENTRIC GRAVITATIONAL CONSTANT,*[1] GM, FROM INDIVIDUAL SATELLITE LASER OBSERVATIONS

SATELLITE NAME	SEMI-MAJOR AXIS (Earth Radii)	ECCENTRICITY	INCLINATION (Deg.)	NO. OF 5^D ARCS	NO. OF OBS.	RECOVERED GM (Km^3/Sec^2)
LAGEOS	1.92	.004	109.85	32	88,800	398600.60
STARLETTE	1.15	.020	49.80	20	23,000	398600.61
BE-C	1.18	.026	41.19	20	18,500	398600.65
GEOS-3	1.13	.001	114.98	37	90,900	398600.60
		COMBINED RESULT		109	221,200	398600.61 ± .02

*[1] Based on c = 299792.5 Km/Sec

- the full GEM-9 geopotential
- luni-solar perturbations
- BIH polar motion and UT1 variations
- atmospheric drag and
- solar radiation pressure.

The value of c used was 299792.5 km/sec. Relativistic effects were modeled for light time correction (Moyer, 1971) and reduced GM by .024 km^3/sec^2.

The individual satellite estimates of GM varied by less than .05 km^3/sec^2 from the combined result. The Lageos data dominated the combination solution as it has an order of magnitude greater capability than the other satellite data to separate GM from the orbital semi-major axis and station coordinate adjustments. Subset solutions of GM for Lageos are presented in Table 2. In Table 2, the uncertainty shown is the formal least square error estimates based upon 1 meter range accuracies. Although the accuracy of the laser systems is far better than 1m, other error sources are present which cause these formal statistics to be optimistic. These errors include range biases, geopotential error, and errors in modeling drag and solar radiation pressure. We have tested the sensitivity of the solutions to different gravity fields and have concluded that this error and the other error sources result in total errors which are within our quoted uncertainty of GM. Lageos has significantly more consistent results for its subset solutions than did similar results obtained from the other satellites. This is because the high altitude of Lageos provides for better geometry and dynamics in estimating GM. Geos-3 had the greatest variability in subset solutions for GM, and also had the poorest statistical conditioning factors. Because of this problem, the solution of normal equations required refinement (Brownd, 1978) in the numerical procedures for computing GM correctly.

Adjustment of GM for Speed of Light. Our value of GM can be adjusted to correspond to the IAG value of the speed of light (c) by using the following relationship:

$$\delta GM = 3GM \frac{\delta c}{c} \qquad (1)$$

With δc = 42 m/s, δGM = 0.17 km^3/sec^2, GM = 398600.44 km^3/sec^2 for c = 299792.458 km/sec. This scale factor is also reported by Esposito and Ng, (1976).

TABLE 2. LAGEOS SUBSET SOLUTIONS FOR GM

SOLUTION	ARCS	NO. OF OBS.	ESTIMATED GM
L1	507,512,517,522,527	12,003	398600.657 ± .010
L2	531,606,611,616,621	11,812	.665 ± .007
L3	924,930,1005,1011,1016	8,102	.637 ± .008
L4	1109,1115,1120,1125,1130	30,538	.618 ± .005
L5	1205,1210,1214,1220,1225	9,094	.601 ± .007
L6	1016,1021,1026,1031,1105	15,576	.592 ± .005

TABLE 3. COMPARISON OF RECENT DETERMINATIONS OF THE GEOCENTRIC GRAVITATIONAL CONSTANT, GM

INVESTIGATORS	VALUE OF GM* (Km^3/Sec^2)	TYPE OF INVESTIGATION
(A) ESPOSITO AND WONG, 1972	398600.63 ± .4	INTERPLANETARY - MARINER 9
(B) WILLIAMS, 1974	.48 ± .1	LUNAR LASER RANGING
(C) MARTIN ET AL, 1975	.66 ± .06	INTERPLANETARY - MARINTER 9
(D) ESPOSITO AND Ng, 1976	.55 ± .2	INTERPLANETARY - MARINTER 9
	.45 ± .2	INTERPLANETARY - MARINER 10
(E) KING ET AL, 1976	.51 ± .03	LUNAR LASER RANGING AND ALSEP VLBI
(F) ESPOSITO, 1978	.40 ± .2	INTERPLANETARY - VIKING 1
	.60 ± .2	INTERPLANETARY - VIKING 2
(G) MARTIN AND OH, 1978	.35 ± .15	NEAR EARTH - SATELLITE TO SATELLITE TRACKING ON ATS-6/GEOS-3
(H) THIS PAPER	.44 ± .02	NEAR EARTH LASER RANGING ON FOUR SATELLITES

*ASSUMING c = 299792.458 Km/Sec

Summary. Table 3 intercompares the near-earth laser determination of GM with other recent experimental results. The results obtained from our analysis (GM = 398600.44 ± .02 km^3/sec^2) generally confirms the latest results obtained from radiometric data taken when numerous interplanetary missions departed earth and those obtained from lunar laser ranging. Moreover, the uncertainty assessed for GM determined from the near-earth laser observations reduces the known uncertainty on GM (Figure 1). These results are important for satellite missions requiring submeter radial orbital accuracies.

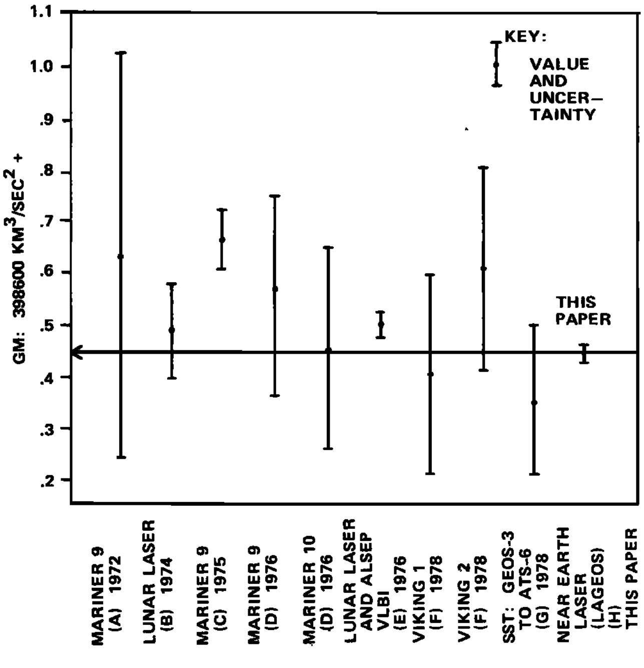

FIG. 1. COMPARISON OF RECENT DETERMINATIONS OF THE GEOCENTRIC GRAVITATIONAL CONSTANT GM (C = 299792.458 KM/SEC)

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