Two-body problem

$$\vec{r} = \vec{r}_O + \vec{r}_{\text{rel}} \qquad \vec{v} = \vec{v}_O + \vec{\Omega} \times \vec{r}_{\text{rel}} + \vec{v}_{\text{rel}} \qquad \vec{a} = \vec{a}_O + \dot{\vec{\Omega}} \times \vec{r}_{\text{rel}} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}_{\text{rel}}\right) + 2\vec{\Omega} \times \vec{v}_{\text{rel}} + \vec{a}_{\text{rel}}$$

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\,\vec{r} \qquad \vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{h} = \vec{r} \times \dot{\vec{r}} \qquad h = v_\perp r = r^2\dot{\theta}$$

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta} \qquad \frac{dA}{dt} = \frac{h}{2} \qquad \frac{v^2}{2} - \frac{\mu}{r} = \varepsilon = -\frac{\mu^2}{2h^2}(1 - e^2) \qquad \mu = G(m_1 + m_2)$$

$$v_r = \frac{\mu}{h} e\sin\theta \qquad v_\perp = \frac{h}{r} = \frac{\mu}{h}(1 + e\cos\theta) \qquad \tan\gamma = \frac{v_r}{v_\perp} \qquad r = R + z$$

Elliptical orbits $(0 \le e < 1)$

$$2a = r_a + r_p \qquad 2ae = r_a - r_p \qquad h^2 = \mu a\left(1 - e^2\right) \qquad \varepsilon = -\frac{\mu}{2a} \qquad T^2 = \frac{4\pi^2}{\mu}a^3$$

$$b = a\sqrt{1 - e^2} = \sqrt{r_p r_a} \qquad \bar{r}_\theta = a\sqrt{1 - e^2} = b \qquad \bar{r}_t = a\left(1 + \frac{e^2}{2}\right)$$

$$\tan\frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}}\tan\frac{\theta}{2} \qquad M_e = E - e\sin E \qquad M_e = \frac{\mu^2}{h^3}\left(1 - e^2\right)^{3/2} t = \frac{2\pi}{T}\,t$$

Parabolic orbits $(e = 1)$

$$v = \sqrt{\frac{2\mu}{r}} \qquad \gamma = \frac{\theta}{2} \qquad h^2 = 2\mu\, r_p$$

$$M_p = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2} \qquad M_e = \frac{\mu^2}{h^3}t \qquad \tan\frac{\theta}{2} = \left[3M_p + \sqrt{(3M_p)^2 + 1}\right]^{1/3} - \left[3M_p + \sqrt{(3M_p)^2 + 1}\right]^{-1/3}$$

Hyperbolic orbits $(e > 1)$

$$\varepsilon = \frac{\mu}{2a} \qquad v^2 - v_{esc}^2 = v_\infty^2 \qquad v_\infty = \frac{\mu}{h}\sqrt{e^2 - 1} \qquad 1 + e\cos\theta_\infty = 0 \qquad h^2 = \mu a\left(e^2 - 1\right)$$

$$\tanh\frac{F}{2} = \sqrt{\frac{e - 1}{e + 1}}\tan\frac{\theta}{2} \qquad M_h = e\sinh F - F \qquad M_h = \frac{\mu^2}{h^3}\left(e^2 - 1\right)^{3/2} t$$

Newton-Raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Classical orbital elements

$$r = \sqrt{\vec{r}\cdot\vec{r}} \qquad v = \sqrt{\vec{v}\cdot\vec{v}} \qquad v_r = \vec{v}\cdot\hat{r} \qquad \vec{h} = \vec{r}\times\vec{v} \qquad \boxed{h = \sqrt{\vec{h}\cdot\vec{h}}} \qquad \boxed{\cos i = h_z/h}$$

$$\vec{N} = \hat{K}\times\vec{h} \;\; (N_x = -h_y, \;\; N_y = h_x) \qquad \boxed{\cos\Omega = N_x/N} \;\; (\text{if } N_y < 0 \text{ then } \Omega = 2\pi - \cos^{-1})$$

$$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - r v_r \vec{v}\right] \qquad \boxed{e = \sqrt{\vec{e}\cdot\vec{e}}} \qquad \boxed{\cos\omega = \vec{N}\cdot\vec{e}/Ne} \;\; (\text{if } e_z < 0 \text{ then } \omega = 2\pi - \cos^{-1})$$

$$\boxed{\cos\theta = \vec{e}\cdot\vec{r}/er} \;\; (\text{if } v_r < 0 \text{ then } \theta = 2\pi - \cos^{-1})$$

Oblateness of the Earth

$$a = 6,378,137.0 \text{ m} \qquad f^{-1} = 298.257 \qquad J_2 = 1.0826\times 10^{-3}$$

$$\dot{\Omega} = -\Upsilon\cos i \qquad \dot{\omega} = -\Upsilon\left(\frac{5}{2}\sin^2 i - 2\right) \qquad \Upsilon = \frac{3}{2}\,\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2a^{7/2}}$$

Geocentric equatorial frame: $\vec{r} = r\cos\delta\left(\cos\alpha\hat{I} + \sin\alpha\hat{J}\right) + r\sin\delta\hat{K}$, where α and δ are the right ascension and declination.

Topocentric horizon frame: $\vec{\rho} = \rho\cos a\,(\sin A\hat{\imath} + \cos A\hat{\jmath}) + \rho\sin a\hat{k}$, where a and A are the altitude and azimuth.

Topocentric equatorial frame: θ and ϕ are the sidereal time ($\theta = \theta_G + \Lambda$) and *geodetic* latitude, and α and δ are the *topocentric* right ascension and declination.

$$\hat{\imath} = -\sin\theta\hat{I} + \cos\theta\hat{J} \qquad \hat{\jmath} = -\sin\phi\left(\cos\theta\hat{I} + \sin\theta\hat{J}\right) + \cos\phi\hat{K} \qquad \hat{k} = \cos\phi\left(\cos\theta\hat{I} + \sin\theta\hat{J}\right) + \sin\phi\hat{K}$$

$$\cos(\theta - \alpha) = \left(\frac{\cos\phi\sin a - \sin\phi\cos A\cos a}{\cos\delta}\right) \;\; (\text{if } \sin A > 0 \text{ then } (\theta - \alpha) = 2\pi - \cos^{-1})$$

$$\sin\delta = \cos\phi\cos A\cos a + \sin\phi\sin a \qquad \vec{\rho} = \rho\cos\delta\left(\cos\alpha\hat{I} + \sin\alpha\hat{J}\right) + \rho\sin\delta\hat{K}$$

$$\vec{r} = \vec{R} + \vec{\rho} \qquad \vec{R} = R_C\cos\phi\left(\cos\theta\hat{I} + \sin\theta\hat{J}\right) + R_S\sin\phi\hat{K}$$

$$R_S = (1-f)^2R_\phi + H \qquad R_C = R_\phi + H \qquad R_\phi = \frac{R_e}{\sqrt{1 - (2f - f^2)\sin^2\phi}}$$

Orbital maneuvers: Hohmann transfer, bi-elliptic is more efficient if $r_C > 16r_A$.

Phasing maneuver (satellite paradox).

Plane change: $\Delta v = \sqrt{(v_{r_1} - v_{r_2})^2 + v_{\perp_1}^2 + v_{\perp_2}^2 - 2v_{\perp_1}v_{\perp_2}\cos\delta}$, most efficient if Δv_r is small and $v_{\perp_1}$ is small .

At periapse/apoapse: $\Delta v = \sqrt{(v_2 - v_1)^2 + 4v_1v_2\sin^2(\delta/2)} \rightarrow 2v\sin(\delta/2)$ for a pure rotation).

Launch azimuth: $\cos i = \cos\phi\sin A$.

Orbit determination from the topocentric horizon frame

$$\vec{\Omega} = \omega_E \hat{K} \qquad \dot{\vec{R}} = \vec{\Omega} \times \vec{R}$$

$$\dot{\delta} = \frac{-\dot{A}\cos\phi\sin A\cos a + \dot{a}(\sin\phi\cos a - \cos\phi\cos A\sin a)}{\cos\delta}$$

$$\dot{\alpha} = \omega_E + \frac{\dot{A}\cos A\cos a - \dot{a}\sin A\sin a + \dot{\delta}\sin A\cos a\tan\delta}{\cos\phi\sin a - \sin\phi\cos A\cos a}$$

$$\dot{\hat{\rho}} = (-\dot{\alpha}\sin\alpha\cos\delta - \dot{\delta}\cos\alpha\sin\delta)\hat{I} + (\dot{\alpha}\cos\alpha\cos\delta - \dot{\delta}\sin\alpha\sin\delta)\hat{J} + \dot{\delta}\cos\delta\hat{K} \qquad \vec{v} = \dot{\vec{R}} + \dot{\rho}\hat{\rho} + \rho\dot{\hat{\rho}}$$

Plane change maneuvers

$$\cos i_2 = \cos\phi\sin A \qquad i_2 \geq \phi \qquad \Delta v = \sqrt{(v_{r2} - v_{r1})^2 + v_{\perp 1}^2 + v_{\perp 2}^2 - 2v_{\perp 1}v_{\perp 2}\cos\delta}$$

At apoapse or periapse, when $v_{r1} = v_{r2} = 0$, then $\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\delta}$
For a pure rotation, when $v_2 = v_1 = v$, then $\Delta v = 2v\sin\left(\frac{\delta}{2}\right)$

Relative motion/Tidal acceleration

$$\delta\ddot{\vec{r}} = -\frac{\mu}{R^3}\left[\delta\vec{r} - \frac{3\vec{R}}{R^2}\vec{R}\cdot\delta\vec{r}\right] \qquad \ddot{\vec{R}} = -\mu\frac{\vec{R}}{R^3}$$

Clohessy-Wiltshire

$$\begin{aligned} \delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} &= 0 \\ \delta\ddot{y} + 2n\delta\dot{x} &= 0 \\ \delta\ddot{z} + n^2\delta z &= 0 \end{aligned}$$