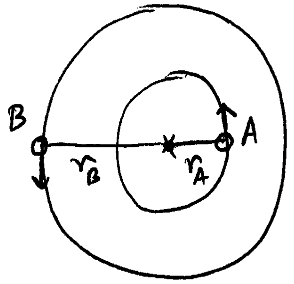
Binary star orbits – Kepler's Law



If two stars, of mass m_A and m_B, orbit their common center of mass, and they execute circular orbits of radii r_A and r_B, respectively, then the "Kepler's Law" that applies depends only on the total mass $m_A + m_B$ and the separation $r_A + r_B$.

<u>Proof</u>: The force on m_B is $F_B = \dfrac{G\, m_A m_B}{(r_A+r_B)^2} = m_B \dfrac{v_B^2}{r_B}$

where $v_B = \dfrac{2\pi r_B}{T}$

Similarly, the equation of motion for m_A is

$$F_A = \frac{G\, m_A m_B}{(r_A+r_B)^2} = m_A \frac{v_A^2}{r_A} = m_A \frac{4\pi^2 r_A}{T^2}$$

We therefore have [note T is the same for both masses]

$$(1)\quad \frac{G\, m_B}{(r_A+r_B)^2} = \frac{4\pi^2 r_A}{T^2}$$

$$(2)\quad \frac{G\, m_A}{(r_A+r_B)^2} = \frac{4\pi^2 r_B}{T^2}$$

dividing $\frac{(1)}{(2)}$ gives $\dfrac{m_B}{m_A} = \dfrac{r_A}{r_B}$ which is just the definition of the center of mass.

Adding (1) + (2) gives

$$\frac{G\,(m_A+m_B)}{(r_A+r_B)^2} = \frac{4\pi^2}{T^2}(r_A+r_B) \quad \text{or} \quad \boxed{T^2 = \left[\frac{4\pi^2}{G(m_A+m_B)}\right](r_A+r_B)^3}$$

generalized Kepler's 3rd law

if $m_A \gg m_B$ then Kepler's law is the approximation.