Astronomical meaning of a tropical year

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A tropical year is usually described incorrectly as the actual interval between two successive passages of the Sun through the vernal equinox. These intervals vary from year to year because of the differing periods of the perturbations of the angular velocity of the Earth. A tropical year is correctly defined as the mean interval between two successive passages of the Sun through the mean equinox of date. The time of beginning of spring depends on (i) the perturbations of the angular velocity of the Earth, (ii) the nutation of the Earth's axis, and (iii) the aberration of light.

I. BEGINNING OF SPRING

In the northern hemisphere, spring begins when the Sun passes the vernal equinox (to be defined subsequently). At this time, night and day are of equal length everywhere on Earth. Newspapers often state the exact time of this event since it is of interest to many readers. The tropical year is directly related to the vernal equinox. The usual statement is[1] that "the tropical year is the interval of time between two successive passages of the Sun through the vernal equinox."

Table I shows the dates and the universal times of the vernal equinox as given by the *American Ephemeris and Nautical Almanac*[2] for the years 1953–1978. The column headed ΔT gives the time interval in mean solar days between the vernal equinox for the stated year and the vernal equinox for the preceding year. The last column gives the difference between ΔT and the mean value of ΔT.

Clearly, the interval between passages through one vernal equinox and the preceding vernal equinox is variable. Furthermore, since the length of a tropical year[2] is $365^{\mathrm{d}}\ 05^{\mathrm{h}}\ 48^{\mathrm{m}}\ 46^{\mathrm{s}}$, we can see that very few of the yearly intervals equal a tropical year. However, the average yearly interval for the period 1953–1978 happens to be $365^{\mathrm{d}}\ 05^{\mathrm{h}}\ 49^{\mathrm{m}}$, which is very nearly one tropical year.

Purpose of this paper. If a tropical year is not the time interval between two successive passages of the Sun through the (apparent or observed) vernal equinox, then what is a tropical year? Do we mean average yearly interval, or is something else implied? The purpose of this paper is to attempt to answer this question.

II. PHYSICAL MEANING OF A TROPICAL YEAR

It is possible to define a tropical year as 31 556 925.9747 sec, where the second, according to the definition adopted at the October 1967 meeting of the 13th General Conference on Weights and Measures,[3] is "the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the fundamental state of the atom of cesium 133." In practice, the time kept by an atomic clock satisfies this definition. An atomic clock consists of a crystal-controlled clock monitored by a resonant cavity containing cesium 133 vapor.

The above definition of a tropical year in terms of the second is clear and precise. We do not gain, however, any insight into the question as to why a certain number of seconds has been selected to specify a tropical year. Furthermore, we would like to know whether an astronomical event such as the coming of spring can be predicted sufficiently precisely if we accept the above definition of a tropical year. We certainly need to understand the underlying astronomical basis.

III. ECLIPTIC COORDINATE SYSTEM

The geocentric view. To describe the motion of the Sun, we adopt a geocentric view; that is, we place an observer at the center of the Earth. Relative to this observer, the entire universe can be mapped onto the interior surface of a sphere of indefinitely large radius. This sphere is called the celestial sphere. The path of the Sun on the celestial sphere is essentially confined to a great circle. The Sun never departs from this path by more than 1″ of arc.

The ecliptic. The mean path of the Sun on the celestial sphere is called the ecliptic. Actually, the ecliptic is not fixed. It oscillates about a slowly rotating diameter of the instantaneous ecliptic, the present angular velocity of the plane of the ecliptic being 47″ per century. Since the ecliptic will be our prime reference circle, we shall speak of an ecliptic of date, that is, the ecliptic as of a certain date.

The vernal equinox. The rotational axis of the Earth precesses and nutates because of the gravitational action of the Sun and the Moon on the equatorial bulge of the Earth. The precession of the rotational axis may be described by saying that this axis in the absence of nutation traces a conical surface around the pole of the ecliptic. The semivertical angle of the cone is 23° 27′ 8″.26, and the period of the precession is 25 800 years.

If we disregard nutation for the moment, we may consider the precessing rotational axis to be the mean pole of the Earth (as of a certain date). The mean equator lies in a plane perpendicular to the mean pole.

The mean equator and the ecliptic intersect at two points on the celestial sphere. The point which the Sun crosses in going from south to north is called the mean vernal equinox. Since the position of the mean vernal equinox changes from day to day because of precession, we speak of a mean equinox of date.

The true rotational axis of the Earth, which both precesses and nutates, constitutes the true pole of the Earth. The true pole circumscribes the mean pole somewhat irregularly with an amplitude of 9″ and a main period of 18.6 years. The true equator of date lies in a plane normal to the

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Table I. Dates and times of the apparent vernal equinox for the years 1953–1978.

Year	Day	Universal time	ΔT	$\Delta T - (\Delta T)_{\rm av}$
1953	3/20	$22^{h}\,01^{m}$		
1954	3/21	$03^{h}\,54^{m}$	$365^{d}\,05^{h}\,53^{m}$	4^{m}
1955	3/21	$09^{h}\,36^{m}$	42^{m}	-7^{m}
1956	3/20	$15^{h}\,21^{m}$	45^{m}	-4^{m}
1957	3/20	$21^{h}\,17^{m}$	55^{m}	6^{m}
1958	3/21	$03^{h}\,06^{m}$	49^{m}	0^{m}
1959	3/21	$08^{h}\,55^{m}$	49^{m}	0^{m}
1960	3/20	$14^{h}\,41^{m}$	46^{m}	-3^{m}
1961	3/20	$20^{h}\,32^{m}$	51^{m}	2^{m}
1962	3/21	$02^{h}\,30^{m}$	58^{m}	9^{m}
1963	3/21	$08^{h}\,20^{m}$	50^{m}	1^{m}
1964	3/20	$14^{h}\,10^{m}$	50^{m}	1^{m}
1965	3/20	$20^{h}\,05^{m}$	55^{m}	6^{m}
1966	3/21	$01^{h}\,53^{m}$	48^{m}	-1^{m}
1967	3/21	$07^{h}\,37^{m}$	44^{m}	-5^{m}
1968	3/20	$13^{h}\,22^{m}$	45^{m}	-4^{m}
1969	3/20	$19^{h}\,08^{m}$	46^{m}	-3^{m}
1970	3/21	$00^{h}\,57^{m}$	49^{m}	0^{m}
1971	3/21	$06^{h}\,38^{m}$	41^{m}	-8^{m}
1972	3/20	$12^{h}\,22^{m}$	44^{m}	-5^{m}
1973	3/20	$18^{h}\,13^{m}$	51^{m}	2^{m}
1974	3/21	$00^{h}\,07^{m}$	54^{m}	5^{m}
1975	3/21	$05^{h}\,57^{m}$	50^{m}	1^{m}
1976	3/20	$11^{h}\,50^{m}$	53^{m}	4^{m}
1977	3/20	$17^{h}\,43^{m}$	53^{m}	4^{m}
1978	3/20	$23^{h}\,34^{m}$	51^{m}	2^{m}
		$(\Delta T)_{\rm av} = 365^{d}\,05^{h}\,49^{m}$		

true pole. The true equator intersects the ecliptic in two points. The point crossed by the Sun in going from south to north is called the true (vernal) equinox of date.

Latitude. We are now prepared to establish the *ecliptic coordinate* system. We may locate a celestial object by noting that it lies on a certain meridian (great circle passing through the pole of the ecliptic). The number of degrees north or south of the ecliptic is the positive or negative latitude.

Geometric and apparent longitude. The actual position of an object in space is the geometric position. The observed or apparent position differs from the actual position because of aberration and refraction of light. The angle measured in the direction of motion of the Sun along the *ecliptic* from the mean equinox of date to the actual meridian of the object is the geometric longitude. The angle measured in the direction of motion of the Sun along the ecliptic from the true equinox of date to the observed meridian of the object is the apparent longitude. The apparent longitude differs from the geometric longitude mainly because of the effects of aberration and nutation.

IV. PATH OF THE SUN

Motion in latitude. Insofar as motion in latitude is concerned, the path of the Sun resembles a roller coaster. The Earth-Moon plane of rotation is inclined 5° 9′ to the ecliptic. Figure 1 shows that, as the Earth-Moon system rotates about its center of mass, the center of the Earth rises a maximum of 419 km above the ecliptic. As a result, the latitude of the Sun appears to vary with an amplitude of $0''.6$ and a period of approximately 27 days.

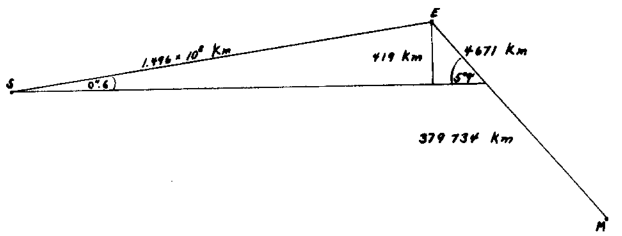

Fig. 1. As the Earth-Moon system rotates about its center of mass, the center of the Earth rises 419 km above the plane of the mean orbit. Because of this motion, the latitude of the Sun appears to vary with an amplitude of $0''.6$.

Of course, there are additional complications. The Earth-Moon plane of rotation both precesses and nutates, and within this plane, the perigee-apogee line rotates. Also, the other planets, particularly Jupiter, perturb the motion of the Earth. Consequently, there are various other periodic terms in the geocentric description of the motion in latitude of the Sun. For the purpose of understanding the meaning of a tropical year, however, the motion in latitude of the Sun is of relatively little importance.

Motion in longitude. The Sun is said to have executed one revolution with respect to Earth when the geometric longitude has increased by 360°. The question is: What kind of path does the Sun follow in longitude?

The evection of Earth. We now come to the central point of this paper. We know that the Earth travels approximately in an elliptic orbit relative to an observer at the center of the Sun. The orbit of the Earth is perturbed because of the gravitational forces exerted by the other planets in the solar system. In particular, the eccentricity of the orbit and the position of the perihelion vary with time. As a result, the angular velocity of the Earth differs at all times from what it would be in an undisturbed Keplerian orbit. The perturbation of the angular velocity may be approximated by a series of periodic terms. The different periods are incommensurable with each other and with the basic period of revolution of the Earth around the Sun. Thus, the interval between two successive passages of the Sun through the vernal equinox fluctuates because of the perturbations of the angular velocity of the Earth. This is the main reason for the variation of the period of revolution of the Sun relative to the Earth. The variation of the period of revolution is called the *evection*[4] of Earth.

V. TROPICAL YEAR

The tropical year is the *average* interval between two successive passages of the Sun through the mean equinox of date. The problem is how to determine the average. Since the variation in the period of revolution of the Sun relative to Earth contains some very long-period terms, the length of time used for averaging would have to be impractically long.

Actually no simple averaging is performed at all. A value for the tropical year is adopted such that "the observed motions of celestial bodies are in agreement with the rigorous dynamical theories of these motions."[5] In essence a tropical year represents a best fit to the observed astronomical data. The accepted value of the tropical year is[6]

$$365^{d}\,05^{h}\,48^{m}\,46^{s}.0 - 0^{s}.530T,$$

where T is the number of (Julian) centuries of 36 525, days that have elapsed since 12:00 noon December 31, 1899.

The small time-dependent term in the above expression represents a secular or extremely long-period change in the interval between two successive passages of the Sun through the mean equinox of date. The present trend of the tropical year is to decrease slowly. After many centuries, the year will lengthen. The same gravitational forces that give rise to the short-period changes cause the secular change in the length of the tropical year.

We can now understand the relation of a tropical year to the second as defined in Sec. II. The proper statement is that the tropical year *as of the beginning of 1900* contains 31 556 925.9747 sec. Clearly, we must attach a date to the length of a tropical year.

VI. ARRIVAL OF SPRING

By definition, spring begins when the *apparent* longitude of the Sun is 0°. Thus, the arrival of spring is the time when the Sun is observed to be at the true equinox of date. Since the true equinox is one of the intersections of the true equator with the ecliptic, the arrival of spring depends on the nutation of the rotational axis of the Earth. Moreover, since the beginning of spring is an observed event, the exact time of this event is affected by aberration of light. Thus, the time of arrival of spring depends on (i) the evection of Earth, (ii) the nutation of the Earth's axis, and (iii) the aberration of light. The variability of the intervals shown in Table I is therefore not surprising. On the other hand, since periodic variations tend to average to zero in the long run, the *average* interval between the arrival of spring one year and the arrival of spring the next year is one tropical year.

VII. SUMMARY

In this paper, we have considered the astronomical meaning of a tropical year. We have found that a tropical year is not simply the interval between two successive passages of the Sun through the vernal equinox. On the contrary, a tropical year is the *average* interval between two successive passages of the Sun through the mean equinox of date. The reason why we must consider an average interval is that the angular velocity of Earth is perturbed by the gravitational forces exerted by the other planets.

The time of arrival of spring in any one year depends on the evection of Earth, the nutation of the Earth's axis, and the aberration of light. However, the average interval between the arrival of spring one year and the arrival of spring the next year is a tropical year.

Perhaps the most surprising aspect of our consideration of the meaning of a tropical year is that it is not constant. When stating a time interval in years, we must recognize that the length of a tropical year decreases very slightly at present as time increases. In general, the fluctuation of the yearly period of revolution of the Sun with respect to Earth may be described by the sum of many terms, some with short periods and some with very long periods.

[1] John Robson, *Basic Tables in Physics* (McGraw-Hill, New York, 1967), p. 239.

[2] *The American Ephemeris and Nautical Almanac* (U.S. GPO Washington, D. C., 1953–1978).

[3] Robert Besancon, *The Encyclopedia of Physics,* 2nd ed. (Van Nostrand Reinhold, New York, 1974), p. 950.

[4] Harry Pollard, *Mathematical Introduction to Celestial Mechanics* (Prentice-Hall, Englewood Cliffs, 1966), p. 106.

[5] *Explanatory Supplement to the Astronomical Ephemeris and American Ephemeris and Nautical Almanac* (Her Majesty's Stationery Office, London, 1961), p. 68.

[6] See Ref. 5, p. 99.

LETTERS TO THE EDITOR

Letters express personal opinions and may critically examine any aspect of physics or physics instruction. They need not conform to our regular editorial policy and ordinarily are not reviewed. From the large number submitted, published letters are selected for their expected interest for our readers. They must be brief and are subject to editing, with the author's approval of significant changes. Running controversies among letter writers will not be published.

PHYSICS REQUIREMENTS IN UNDERGRADUATE ENGINEERING EDUCATION: REPORT OF A PRIVATE SURVEY

Referring to an earlier survey,[1] this article[2] began by noting that during the last decade many engineering schools have reduced the number of credit hours in physics required to obtain a B.S. in engineering. To determine if this trend continues, Kaup and Czanderna investigated the present situation among 77 schools which produced 81% of the engineering B.S.'s in the U.S. in 1975. The findings were: (i) the mean number of credit hours in physics (taught by physics department) required for a B.S. in engineering ranges from 9.3 to 11.2 h, including 1.6 to 1.9 h of laboratory, and (ii) for the last four years the number of schools increasing (9), decreasing (10), and not changing (58), the number of required credit hours in physics has reached a steady state.

Kaup and Czanderna reported that of the ten schools decreasing their physics requirements, eight either elimated or reduced the requirement for Modern Physics. They noted that where this had happened most engineering students still chose Modern Physics as an elective. These observations raise several questions.

For those students who elect Modern Physics even when it is not required, what reasons do they give for their actions? Surely they could have found an "easier" course to elect; what intrinsic worth did they perceive in Modern Physics? Are present day Engineering students better able to perceive the value of a Modern Physics course than their Engineering Professors? What percentage of students that took Modern Physics as an elective were sorry they did so? What percentage of students that decided NOT to elect Modern Physics were sorry they didn't elect it? After differences in IQ, math pretest scores, etc., are factored out, does the taking of Modern Physics in any way contribute to: upper division and/or graduate course success, laboratory/research performance, future employment success or flexibility, providing a hedge against obsolescence? Are certain engineering curricula so obsolete that Modern (post 1905) Physics is *really* not necessary?

[1] R. Alley, Jr., Am. J. Phys. **40**, 1063 (1972).

[2] D. J. Kaup and A. W. Czanderna, Am. J. Phys. **47**, 235 (1979).

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RESPONSE TO "PHYSICS REQUIREMENTS IN UNDERGRADUATE ENGINEERING EDUCATION: REPORT OF A PRIVATE SURVEY"

Yes, it is indeed true that several significant questions have been raised by this survey, not the least of which is Dr. Frink's last question. We think it would be extremely enlightening if these questions could be posed to recent engineering graduates. Such questions were not included in our survey, but we would certainly encourage an undertaking to obtain responses to them."

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RESPONSE TO "ASTRONOMICAL MEANING OF A TROPICAL YEAR"

Reuben Benumof's article on the "Astronomical meaning of a tropical year"[1] is an interesting discussion of how even technical definitions are often slightly inadequate. Another example happens to occur in his first paragraph, in which it states that when the Sun passes the vernal equinox, "night and day are of equal length everywhere on Earth." Actually, this would be the case only if the sun were a point in the sky and if the earth had no atmosphere to refract sunlight. Because the top of the real sun rises ahead of the bottom of the sun and sets later than the bottom of the sun does, and because of atmospheric refraction, the days on which daytime and nighttime are of equal length are a few days away from the equinoxes.

[1] Am. J. Phys. **47**, 685 (1979).

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REPLY TO PASACHOFF

Pasachoff's comment on the exact times when day and night are of equal length raises the question of precisely what is meant by the length of day or night. Pasachoff evidently implies that the duration of daylight is the interval between the instants when the actual angular distance of the center of the Sun from the zenith is ±90° 50′. Since the effect of refraction is 34′, and the angular radius of the Sun is 16′, the observed (but not the actual) position of the *upper limb* of the Sun at these instants is on the horizon for an observer at sea level.[1] Meteorologists consider the daylight interval to be bounded by the instants when the *observed center* of the Sun is on the horizon and hence use[2] a value of ±90° 34′ for the extremes of the distance of the Sun's center from the zenith. Climatologists, on the other hand, consider the proper bounds for the sunshine interval to be the instants

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when the Sun's center is *actually* on the horizon and hence use a value of ±90° for the extremes of the zenith distance. When the latter definition of the sunshine or daylight interval is employed, the beginning of spring does coincide with the time when the duration of sunshine is exactly 12 mean solar hours. We must bear in mind, however, that the system of definitions presented in my paper to define the tropical year does not in any way involve the duration of daylight, and consequently the above discussion is at best tangential to the astronomical meaning of a tropical year.

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[1] *Explanatory Supplement to the Astronomical Ephemeris and American Ephemeris and Nautical Almanac* (Her Majesty's Stationery Office, London, 1961), p. 401.

[2] Reference 1.

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IS ANYONE PAYING ATTENTION?

As teachers, we all must have asked ourselves that frequently regarding our students, but about our *colleagues*?

It is probably true as Current[1] claims that the formula for generators connected in parallel is "relatively unknown." Further, he claims his result is "certainly not new." One wonders where he might have seen it before—perhaps Am. J. Phys. **36**, 639 (1968)?

What makes the note by Current particularly noncredible is his comment that he had discovered in a book "a result only for $N = 2$ and (written) in such a way that the extension to arbitrary N is not obvious"; then he references Halliday and Resnik, *Physics*, p 172 (1978), problem 16, 24, Fig. 32-18 (b). If Current would have simply looked at the next page (713, problem 29) he would have found the result written in such a way that the extension to arbitrary N *is* obvious (and a proper credit[2] given). I find it astonishing that the note was accepted for publication. In addition to Current, one can only conclude that the referee was "not paying attention."

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[1] D. H. Current, Am. J. Phys. **47**, 463 (1979).

[2] J. S. Wallingford and H. W. Jones, Am. J. Phys. **36**, 639 (1968).

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RESPONSE TO WALLINGFORD'S "IS ANYONE PAYING ATTENTION?"

My apologies are due to Professor Wallingford for overlooking his Note[1] and the reference to it in Halliday and Resnick. I suspect that one time or another all of us have been guilty of not digging deeply enough in the literature. My purpose in publishing the piece was the same as Wallingford's was eleven years ago, namely to point out the pedagogical utility of this kind of calculation in elementary physics courses.

In addition, my derivation serves as a simple example of the power of the Norton representation in circuit problems and produces the equivalent parallel resistance expression directly. Perhaps this exchange will eliminate the possibility that someone else will feel compelled to work the problem over again in another eleven years.

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[1] Am. J. Phys. **36**, 639 (1968).

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A RESPONSE TO "A RESPONSE TO 'DILEMMA OF THE PRIMARY COLORS' "

Marvin Hoshino, in his letter,[1] missed our point. We did not wish to show, physically, why cyan, magenta, and yellow are the best subtractive primary colors—this is well known and mentioned in our article. We wished to demonstrate experimentally, for pedagogic reasons, why artists use yellow rather than green for their primary. If we also want to establish why artists do not use magenta and cyan, examine Fig. 6 of our article[2] and note the points labeled M and T have a low perceptual brightness, or relative luminance. As Hoshino says, this is in the nature of the pigment used. One point Hoshino makes is in error, however. He states that pigments "make" color by subtraction. This is, at best, a partial truth, the process being much more complex because scattering and surface reflection must be included. To see the effect of scattering, suppose we mix a uniformly grey pigment with a white pigment such as titanium dioxide but the particles of which scatter more effectively in the red than the blue. For the mixture, the blue component of incident white light will travel through a greater thickness of the grey pigment than will the red before being reemitted from the surface. Consequently, the blue component will be absorbed more by the pigment, and the paint will appear red. Correspondingly, if the white pigment scattered more effectively in the blue than the red, the mixed paint would appear blue. It appears paradoxical for a mixture of grey and white pigments to produce either a blue or red color. This problem has been dealt with recently using Kubelka–Munk analysis.[3] Surface texture affects color appreciably also. For example, specular Fresnel reflection from a surface will generally desaturate colors in the direction where this reflection is most intense. All in all, the color of paints is a fascinating subject of which, to make our point, we barely scratched the surface.

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[1] M. Hoshino, Am. J. Phys. **47**, 573 (1979).

[2] R. D. Edge and R. Howard, Am. J. Phys. **47**, 142 (1979).

[3] Ruth M. Johnston in *Pigment Handbook*, edited by T. C. Patton (Wiley, New York, 1973), p. 229.

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