Solitons

They are waves that do not disperse or dissipate but instead maintain their size and shape indefinitely. A recent finding is that solitons may appear as massive elementary particles

by Claudio Rebbi

Waves and particles have been intimately related in physical theory since the formulation of quantum mechanics in the 1920's. In the past few years another connection between them has emerged. It comes from a surprising source: the analysis of certain wave equations that are not part of quantum mechanics but instead derive from classical physics. Solutions to these equations describe waves that do not spread or disperse like all familiar waves but retain their size and shape indefinitely. The wave can be regarded as a quantity of energy that is permanently confined to a definite region of space. It can be set in motion but it cannot dissipate by spreading out. When two such waves collide, each comes away from the encounter with its identity intact. If a wave meets an "antiwave," both can be annihilated. Behavior of this kind is extraordinary in waves, but it is familiar in another context. Given a description of an object with these properties, a physicist would call it a particle.

Waves that propagate without dispersing have been known for some time in hydrodynamics, where they are called solitary waves or solitons. What has been discovered recently is that nondissipative waves also arise from some of the equations formulated to describe elementary particles. The name soliton has been borrowed for these objects. The solitons I shall discuss here are those that share a particular mechanism of confinement: they are prevented from dispersing by a topological constraint. They cannot decay by spreading out for the same reason a knot tied in an endless rope cannot be removed without cutting the rope.

For now the solitons of particle physics are entirely the creation of theorists, and it may yet turn out they do not exist in nature. On the other hand, if the equations that describe elementary particles are found to be among those that admit soliton solutions, then the solitons should appear as new particles. They would be very massive, perhaps thousands of times heavier than the proton. A soliton particle would also have certain distinctive properties; for example, one theory predicts that each soliton would be a magnetic monopole, an isolated north or south magnetic pole.

Even if such particles do not exist, solitons may enter the realm of elementary-particle physics in another role, as objects confined not only to a definite region of space but also to a moment in time. Such evanescent solitons have been named instantons. Like the soliton, the instanton is a classical object with a quantum-mechanical interpretation. It is viewed not as a particle but as a transition between two states of a system, a manifestation of the phenomenon called tunneling. Transitions facilitated by instantons have already been invoked to explain a pattern of particle masses that had been a long-standing puzzle to theorists.

In order to understand what is remarkable about a wave that does not disperse, one need only consider an ordinary, dissipative wave such as the one generated when a pebble is dropped into a still pond. Such a wave travels over the water surface as an expanding ring. Careful observation reveals that the disturbance becomes less pronounced as it moves away from the source, and eventually it dies out completely.

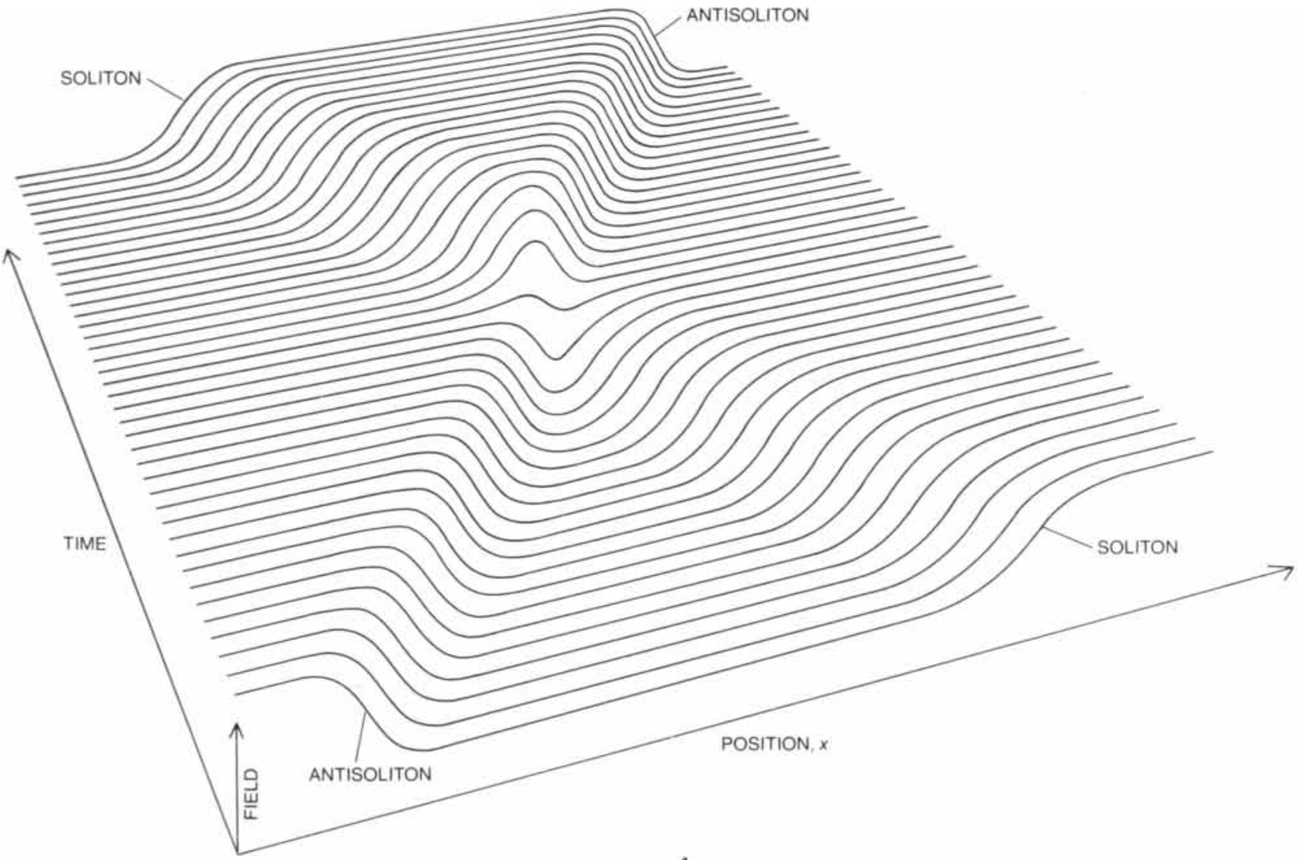
An important factor in the decay of water waves is the viscosity of the medium, a manifestation of friction. In some mediums, however, waves propagate without friction but nonetheless decay, and even waves in a pond filled with a frictionless fluid would die out. The reason is that components of the wave that have different wavelengths propagate with different speeds, spreading the energy of the wave over a larger area. When the wave is viewed in cross section, it becomes continually broader but of smaller amplitude. If it is allowed to continue to infinity, it disperses over an infinite area and thereby disappears.

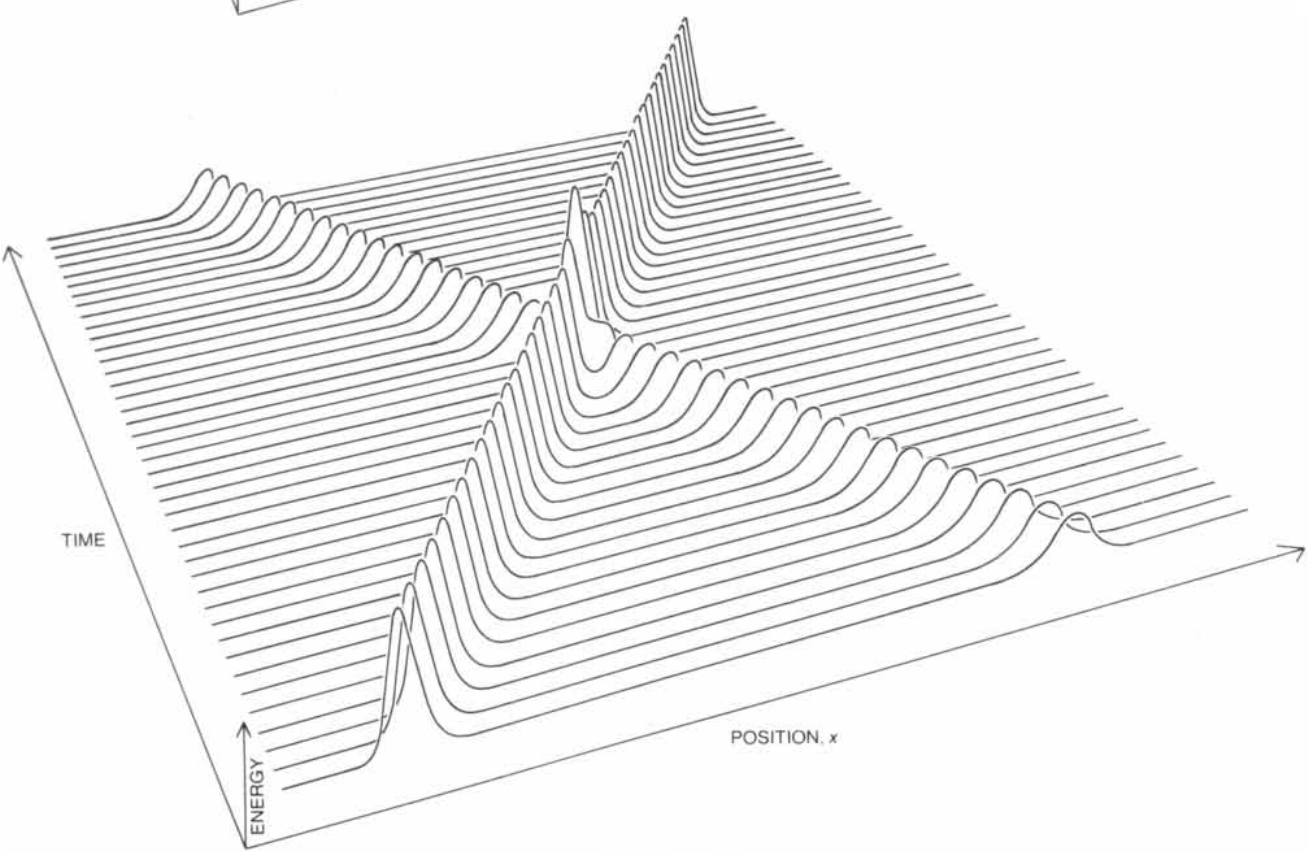
Solitons are not, strictly speaking, immune from dispersion; rather, they are waves in which the effects of dispersion are exactly canceled by some compensating phenomenon. The compensation is possible only in a certain class of waves, those whose equation of motion is of the kind said to be nonlinear. The propagation of such a wave is influenced not only by the shape of the disturbance but also by its magnitude.

The first recorded observation of a soliton was made almost 150 years ago by John Scott Russell, an engineer and naval architect. He reported to the British Association for the Advancement of Science: "I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large, solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles per hour, preserving its original figure some 30 feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel."

Scott Russell proposed that the stability of the wave he had observed resulted from intrinsic properties of the wave's motion rather than from the circumstances of its generation. This view was not immediately accepted. In 1895, however, D. J. Korteweg and Hendrik de Vries gave a complete analytic treatment of a nonlinear equation in hydrodynamics and showed that localized, nondissipative waves could exist. Solitons have since become a well-recognized phenomenon in various fields of engineering and applied mathematics.

The solitons of interest in particle physics arise from equations that describe fields, or physical systems that are extended in space. A field assigns to every point in space a value of some speci-

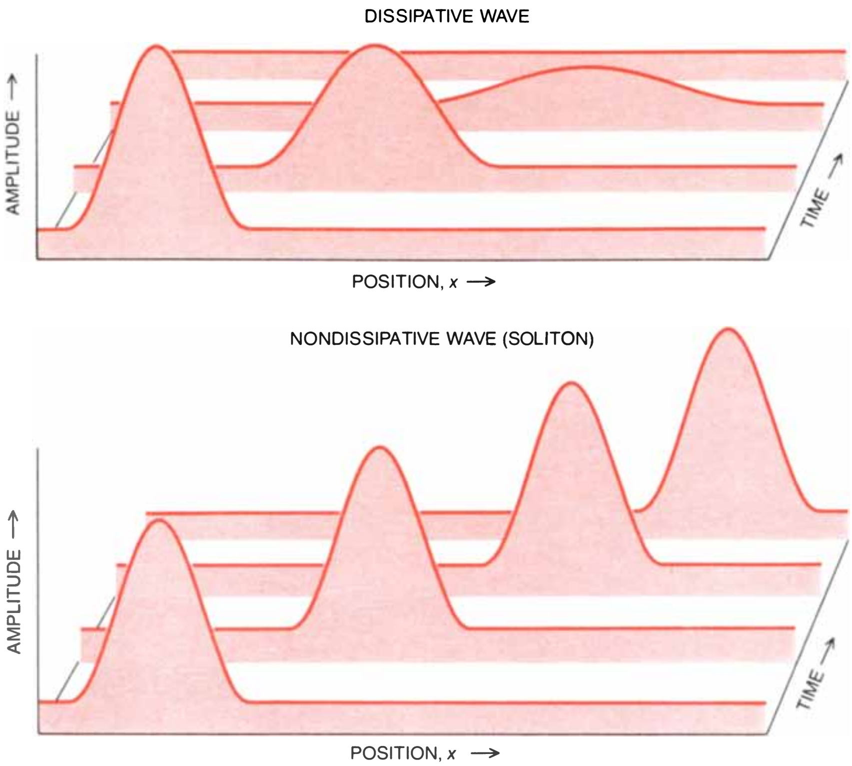




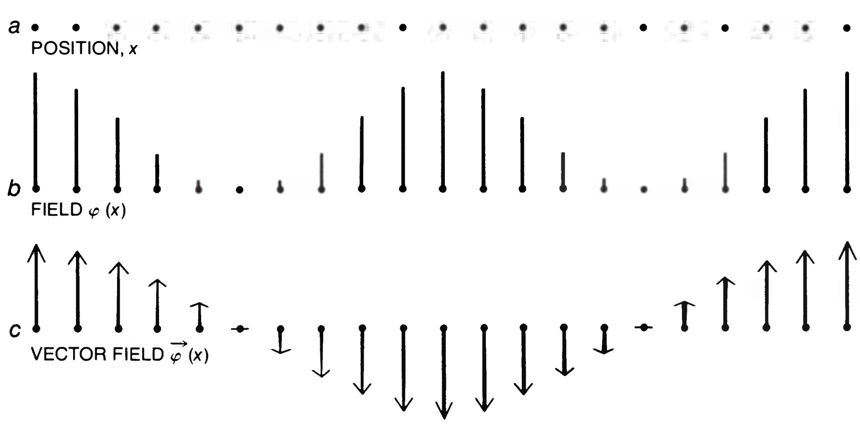
SOLITON AND ANTISOLITON appear as kinks of opposite direction in the structure of a field. In the upper illustration the evolution of the field itself is depicted; proceeding from front to back, each line represents a successive configuration of the field. The lower illustration shows the energy distribution corresponding to each configuration. At the beginning of the sequence the soliton and the antisoliton are approaching each other, the antisoliton moving faster and therefore having a higher energy. When the two waves meet, they are both annihilated and from their energy another soliton-antisoliton pair is immediately created. The one-dimensional field in which the waves propagate is called the sine-Gordon field. Everywhere outside the soliton and the antisoliton the field takes on values with zero energy.

fied quantity, such as electric potential. Often more than one quantity is defined at each point. The values can change from point to point and from moment to moment, but they must do so smoothly and continuously.

The most familiar field is the electromagnetic one described by the field equations of James Clerk Maxwell. In Maxwell's theory six values are assigned at each point in space; they represent the components of the electric and the magnetic fields along any three orthogonal axes. Gravitation is also described by field equations, those of Einstein's general theory of relativity. The surface of a pond can be regarded as a model of a two-dimensional field. The quantity to be specified at each point is vertical position, or height above some reference level. A wave passing through the water then becomes a perturbation of this field.



PERSISTENCE OF A WAVE is limited mainly by dispersion, the process whereby components of a wave that have different wavelengths propagate at different speeds. Because of dispersion, an ordinary wave (*upper graphs*) tends to flatten and spread as it moves and must eventually die out entirely. A soliton (*lower graphs*) is a wave that does not dissipate because the effects of dispersion are canceled by other features of the wave's motion. The soliton represents a quantity of energy that can move from point to point but cannot spread out in space.



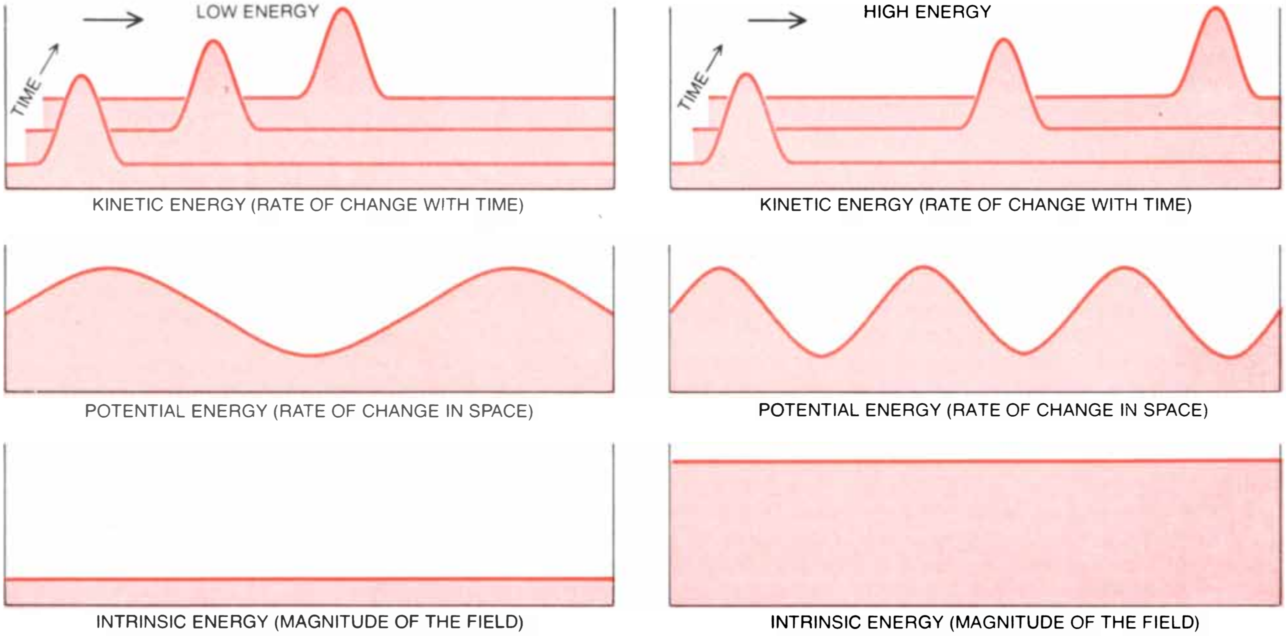
FIELD IN SPACE is the medium in which solitons propagate. A field assigns to each point in space some definite value of a specified quantity. Here the space (*a*) is one-dimensional, and the points shown are merely a few selected from an infinite continuum. In the simplest field (*b*) only a magnitude is measured at each point, represented here by the length of a line segment. A more complicated field, a vector field (*c*), has both a magnitude and a direction, shown here by arrows. For a field to be physically possible it must be defined everywhere (although its value can be zero), and if it changes from point to point, variation must be smooth and continuous.

An essential property of a field is that it can carry energy, just as a particle can. The energy of the field per unit volume is expressed mathematically as the sum of three quantities. One of these quantities is proportional to the square of the rate at which the field varies in time. The second term has a similar form but is proportional to the square of the rate at which the field varies in space. The third quantity is determined not by a rate of variation but by the actual magnitude of the field at each point. It is customary to call the first term kinetic energy and the sum of the other two terms potential energy. In this discussion, however, it will be convenient to have a name for each of the three quantities, and so I shall give the name intrinsic energy to the component that depends on the actual magnitude of the field, leaving the name potential energy for the component that is proportional to the square of the spatial rate of variation.

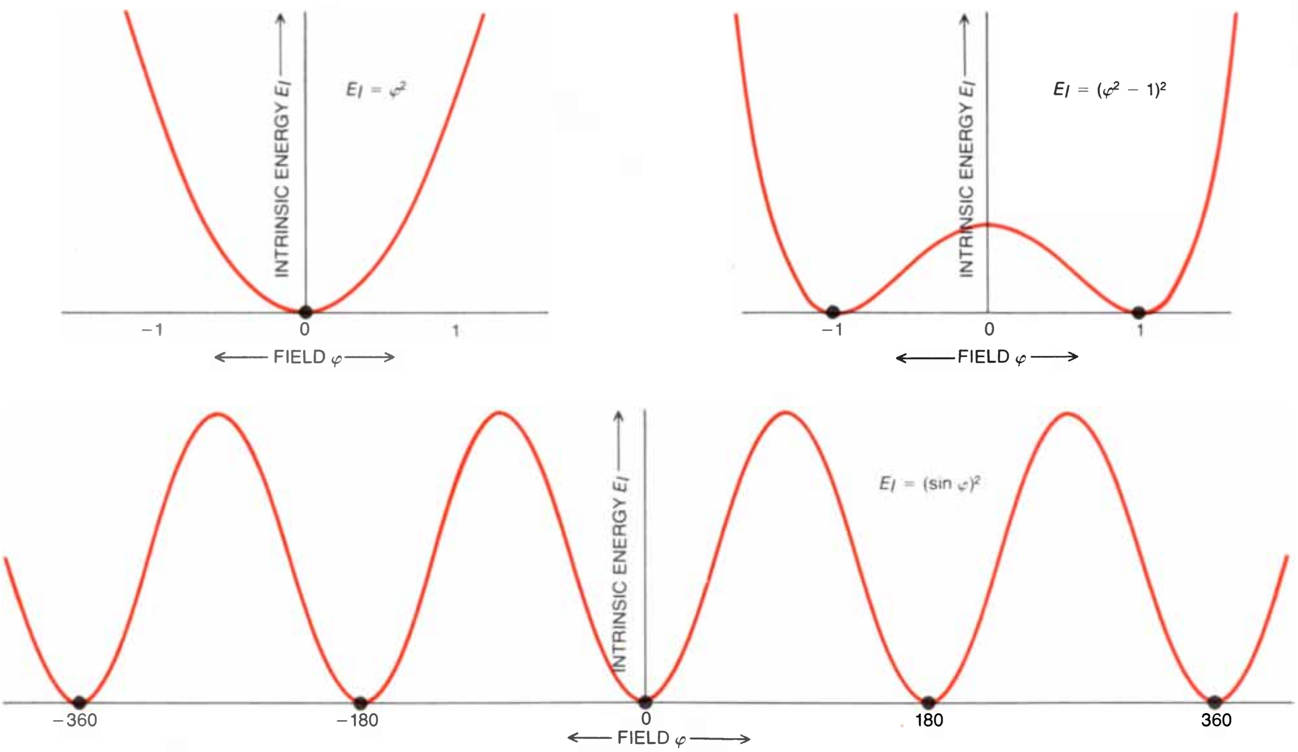
An intuitive rationale can be provided for this analysis of the energy of a field. The kinetic-energy term states that the total energy of the field rises when the value of the field (at each point) changes more quickly. In the water surface considered as a model of a field this relation implies that the kinetic energy is proportional to the square of a rate of change in position, or in other words to the square of a velocity. It is a matter of everyday experience that when something moves faster, it carries more energy.

The potential energy, as I have defined it, rises when a change in the state of the field is compressed into a smaller space. Intuition again confirms that the energy needed to bend or deform an object rises when the bend is made sharper. The interpretation of the intrinsic component of the energy is even more straightforward: it should hardly seem surprising that the energy of a field depends on the magnitude of the field. In the pond model the intrinsic energy is related to the overall surface level of the water. A higher waterline corresponds to a higher energy.

Because the kinetic energy and the potential energy of a field are both squared quantities they can never take a negative value. In evaluating the intrinsic energy of a field only energy differences need be considered, such as the difference between two water levels in a pond. The smallest observed intrinsic energy can therefore be set equal to zero, and so the intrinsic energy can also be defined in such a way that negative



ENERGY OF A FIELD is the sum of three components, all of them illustrated here on a water surface, which can be regarded as a model of a field. The kinetic energy is proportional to the square of the rate at which the field varies in time; a field with faster-moving waves embodies more energy. The potential energy is proportional to the square of the spatial rate of variation; a field whose fluctuations have a shorter wavelength has a higher energy. The intrinsic energy is determined by the magnitude of the field, although the rule relating these two quantities can take many forms. In the example of the water surface a higher water level corresponds to higher intrinsic energy. The energy of the field is zero when the field is static and uniform and in a configuration that yields the minimum intrinsic energy.



VACUUM STATES of a field are the zero-energy configurations. Because the kinetic and the potential energy can always be reduced to zero by making the field static and uniform, only the intrinsic energy need be considered in determining the vacuum states. Here the intrinsic energy is plotted as a function of the magnitude of the field for fields described by three simple equations. If the energy increases as the square of the magnitude of the field (*a*), there is only one vacuum state: the energy is zero when the field itself is zero. A variant of this equation (*b*) yields a field with two vacuum states. The energy is zero when the field has values of +1 or −1 but, significantly, the energy is greater than zero when the field itself vanishes. The energy of the sine-Gordon field (*c*) is a periodic function and has an infinite number of vacuum states. If the magnitude of the sine-Gordon field is interpreted as an angle in degrees, then the energy is zero whenever the magnitude is equal to zero, 180, 360 and so on. Topological solitons can exist only in fields that have multiple vacuum states.

values never appear. Thus the minimum total energy of a field is zero.

The state of a field that has minimum or zero energy is called the vacuum state. It is clear that in the vacuum state the field must be constant throughout both space and time, since any variation would give the kinetic or the potential energy a value greater than zero. To achieve the vacuum state it is thus necessary only to minimize the intrinsic energy. The vacuum is the state of a uniform field that has zero intrinsic energy.

Since the intrinsic energy is determined by the magnitude of the field, the most obvious configuration is one where the intrinsic energy is at its minimum when the field itself is zero everywhere. The vacuum is then the state with no field at all, which corresponds to the intuitive notion of a vacuum as being empty space. Surprisingly, however, that is not the only possibility. There are a number of field equations for which the intrinsic energy vanishes at some nonzero value of the field. Hence the state of the system with no field, which might at first seem to represent the vacuum, can actually have a higher energy than some alternative state with a nonzero field. That alternative state is the true vacuum, the point of zero energy, even though it describes a space permeated by a uniform field. An example of a field of this kind is the magnetic field of a ferromagnet. The state of minimum energy for a ferromagnet is not the demagnetized condition; on the contrary, a ferromagnet spontaneously generates and maintains a magnetic field because it can reduce its energy by doing so.

If the intrinsic energy can fall to zero for some value of the field other than zero, then it might be zero at more than one value. Indeed, certain field equations give rise to a multiplicity of vacuum states, each state associated with a different value of the field. All the vacuum states are equivalent (they have the same zero energy), but they are distinct. The existence of at least two vacuum states is a necessary condition for the creation of the solitons I shall describe here.

Suppose a wave, an excitation of some field, is observed at a given moment to occupy a finite volume of space. Outside this region there are no other excitations and the value of the field is such that the energy is zero everywhere. The overall state of the system cannot be the vacuum because there is energy in the wave. As the wave disperses, however, its energy will be distributed through ever larger volumes, and when it spreads to infinity, the energy per unit volume will have fallen to zero. The wave has then disappeared and the field has a constant vacuum value throughout space.

If the field is one that can have multiple vacuum states, there is another possible configuration of the system. In the regions surrounding an isolated wave there may be different values of the field, all of them corresponding to the vacuum but nonetheless distinct. If the topological arrangement of the vacuum states is such that the field cannot be extended to a consistent vacuum value everywhere in space, then the wave will be unable to expand and disperse. The result is a stable perturbation of the field: a soliton.

The topology of a soliton can be made clearer by considering a few examples of particular field equations. The simplest of these equations describes a field that exists in a one-dimensional space, a line of infinite length where each point can be specified by a single coordinate, x. A field in this space assigns to each point on the line some value, which I shall designate by the Greek letter φ (phi). A more formal description of the field is given by the mathematical expression $\varphi(x)$, which is read "φ is a function of x," or more briefly "φ of x," and which means that for every point on the line there is a unique value of the field φ.

In the same way that φ depends on x the intrinsic energy depends on φ. This dependence is recorded explicitly by the notation $E_I(\varphi)$: the intrinsic energy is a function of φ, and thus each possible value of the field has some definite intrinsic energy. The physical meaning of these mathematical formulas is easily deciphered. Given any point, x, in the one-dimensional space, the value of the field is specified at that point without ambiguity. Given the value of the field at each point, one can then calculate its intrinsic energy. Solitons appear when several values of the field have the same minimum intrinsic energy.

One expression for the intrinsic energy that can give rise to solitons is the equation $E_I = (\sin\ \varphi)^2$. The equation states that for any value, φ, of the field, the intrinsic energy can be found by taking the sine of φ and then squaring the result. The equation of motion for waves propagating in a field that has this property is called the sine-Gordon equation. It is a modification of another equation first discussed in 1926 by Oskar Klein and Walter Gordon.

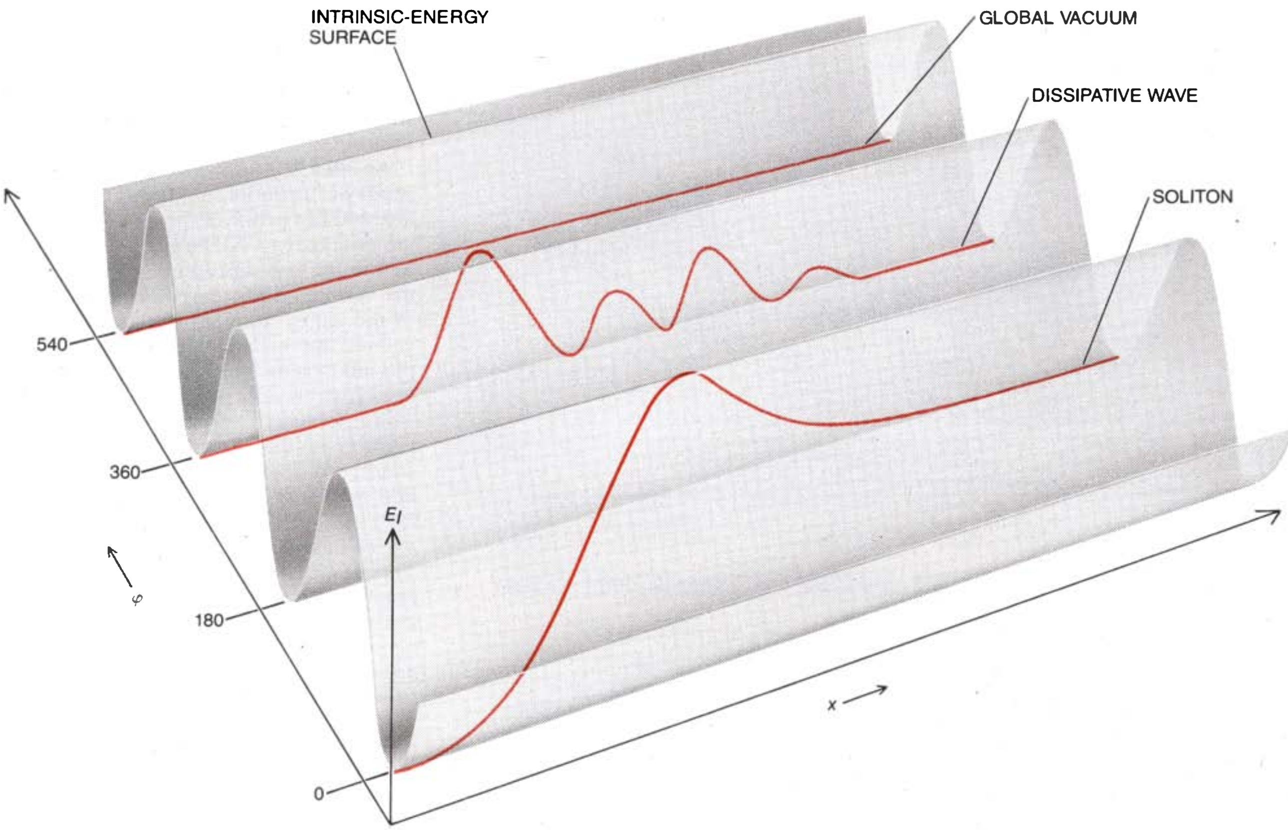
The properties of the sine-Gordon field can be explored by substituting numerical values for φ in the equation that defines the intrinsic energy. The values need not be assigned any dimensions. For the sake of simplicity I shall assume here that the units are chosen so that the intrinsic energy varies between zero and 1 and that φ is measured in degrees.

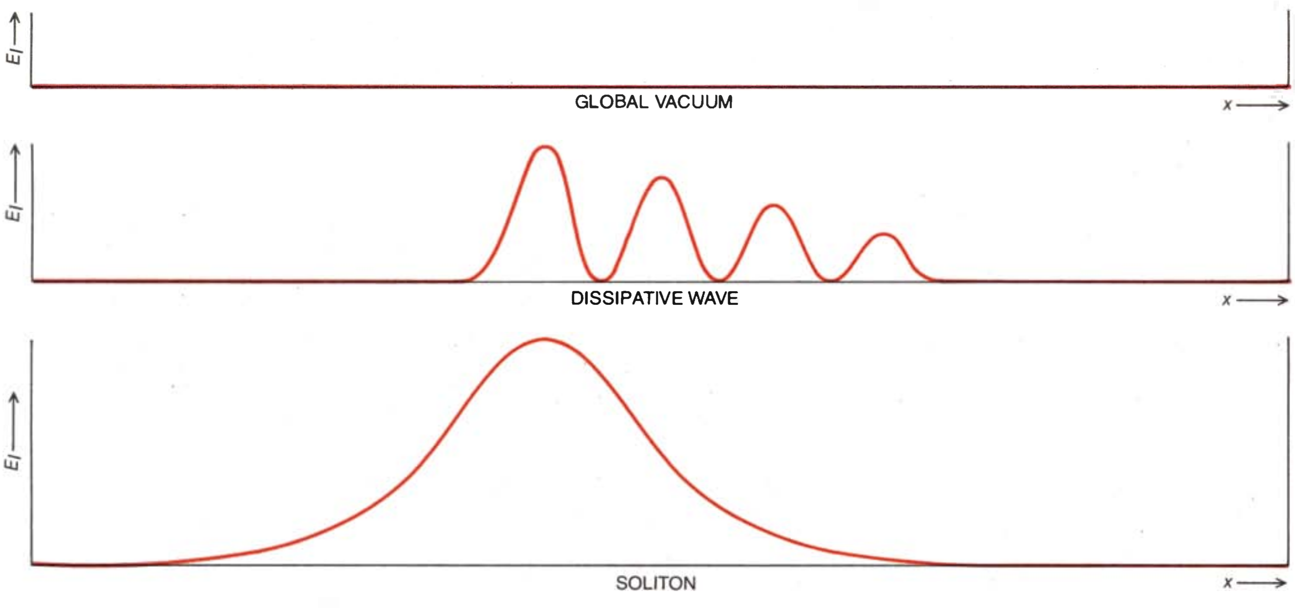
The sine is a trigonometric function whose value ranges from zero to $+1$ and from zero to -1, and so its square must be confined to the range of values between zero and $+1$. If φ is equal to zero, then the sine of φ and the square of the sine are also zero; hence the sine-Gordon equation is one in which the state with no field is a vacuum state. As φ increases from zero to 90 the sine and the square of the sine rise smoothly, reaching a value of 1 when φ is equal to 90. As φ continues to increase, however, the square of the sine decreases and returns to zero when φ is equal to 180. That value of φ therefore specifies another vacuum state of the system. With a continuing increase in φ the sine becomes negative, but the square of the sine, of course, remains positive and again has a value of 1 when φ is equal to 270. When φ is equal to 360, the square of the sine is zero again, representing another vacuum state. Additional states of zero intrinsic energy are found when φ is set equal to 540, 720, 900, 1,080 and so on. As φ increases there are infinitely many possible vacuum states, each separated from the neighboring vacuum by a hump where the intrinsic energy rises smoothly to a value of 1.

The configuration of a one-dimensional field can be represented by a two-dimensional graph, that is, by a graph on a plane. One axis is labeled with all the points, x, in the one-dimensional space; the other axis gives all possible values of the field, φ. For each point x a mark is made on the plane at a position corresponding to the value of the field at that point. Because the field must be defined at every point and because the values must form a continuum, the marks can always be connected to make a line or curve. If the line is straight and parallel to the x axis, it represents a state in which the field is constant at all points.

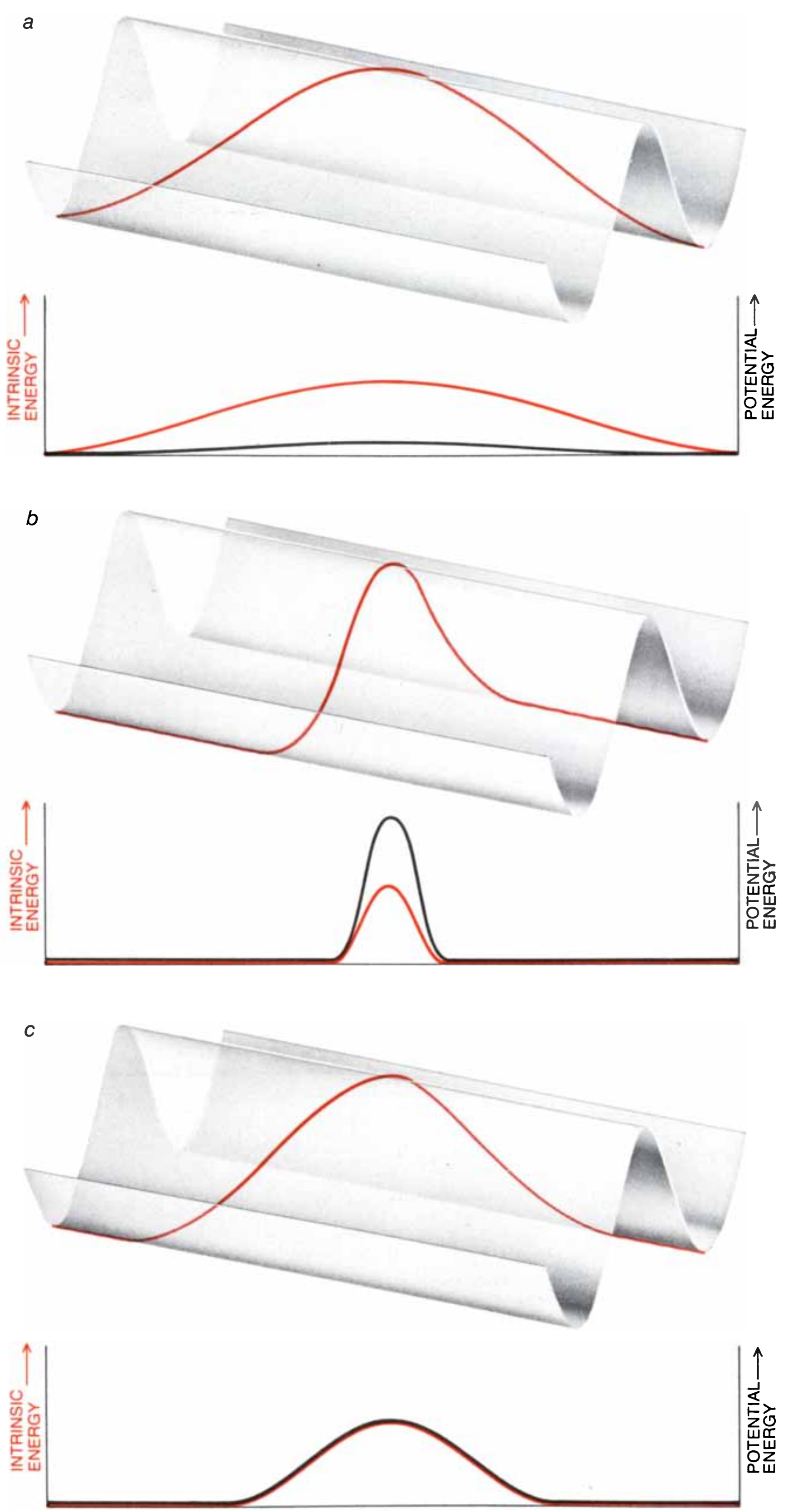
In order to display the intrinsic energy of the field a third dimension is added to the graph: intrinsic energy is made proportional to height above the plane. For the sine-Gordon field the points representing the intrinsic energy of every possible value of the field form a sinusoidally undulating surface. In this energy surface the undulations are parallel to the x axis and have troughs of zero energy where φ is equal to zero, 180, 360 and so on [*see illustration on opposite page*].

In this three-dimensional graph the configuration of the field is represented by a line laid on the undulating energy surface. One possible configuration is represented by a straight line that remains at the bottom of one of the troughs, say the trough at $\varphi = 180$. The meaning of the graph is that the field has a magnitude of 180 at every point on the line x. It does not vary in space and unless it is perturbed by some outside influence it cannot vary in time. Moreover, because $\varphi = 180$ is one of the values of





ENERGY SURFACE of the sine-Gordon field illustrates the topological constraint that confines a soliton. Distance along the x axis, which extends to infinity in both directions, gives the position of a point in a one-dimensional space. Distance along the axis labeled φ gives the magnitude of the field at the point x. The height of the energy surface above the plane (along the axis labeled E_I) gives the intrinsic energy of the field at this point. The sine-Gordon field has multiple vacuum states, namely the troughs in the energy surface. A line that lies in one of these troughs describes a global vacuum state: the field has the same magnitude everywhere, and it is a magnitude that corresponds to zero intrinsic energy. A line that oscillates within a valley represents a local excitation of the field, or an ordinary wave. Because the line wanders above the floor of the valley this field carries energy, but the wave will eventually disperse. A soliton forms when the field assumes different vacuum values along the two directions that lead to infinity. For the soliton shown here the field to the left is in the vacuum state at $\varphi = 0$, but to the right it has the alternative vacuum value of $\varphi = 180$. The field is required to be continuous, and so at some point along the x axis it must cross the hump in the energy surface that separates the two vacuum states. The kink at the transition zone is the soliton. It can be pushed from side to side, but if the surface is infinite, the kink can never be removed and the soliton can never disperse. At the bottom the intrinsic energy of the three field configurations is projected onto a two-dimensional graph. The dissipative wave and soliton also have potential energy, and if they are moving, they have kinetic energy, but these quantities are not shown.



OPTIMUM SHAPE for a soliton is the shape that yields the smallest sum of potential energy and intrinsic energy. The potential energy, being determined by the spatial rate of variation, is smallest when the transition is gradual and the line crosses the ridge at a shallow angle (*a*). A long segment of the line, however, is then near the peak in intrinsic energy. The intrinsic energy is minimized when the line crosses the ridge almost perpendicularly (*b*), but such an abrupt transition makes the potential energy large. The best compromise is achieved when the line crosses the ridge at an intermediate angle (*c*) and the potential and intrinsic energy are equal.

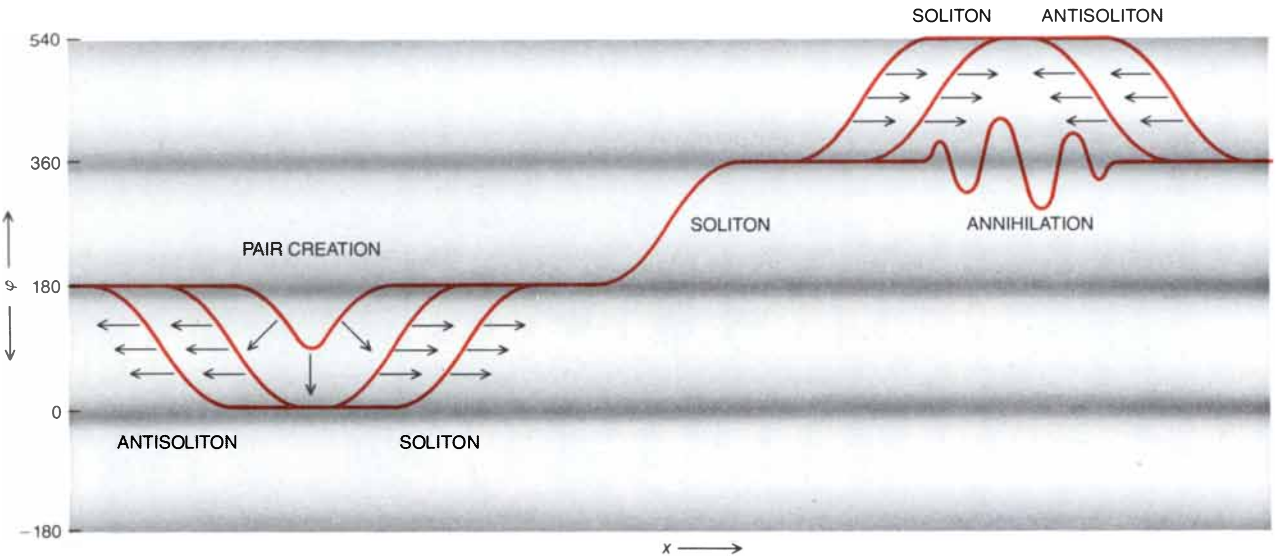
the field for which the intrinsic energy is zero, the total energy of the field is zero. The field is in one of its many equivalent vacuum states; because it is the same state everywhere it is called a global vacuum.

Another configuration results when a small perturbation is introduced into a line that lies in one of the troughs of zero intrinsic energy. At the site of the perturbation the line representing the state of the system climbs partway up one wall of the valley, plunges back to the floor and up the other wall, then continues this sinuous pattern in a series of oscillations that become progressively smaller. The graph corresponds to a wavelike excitation of the field, but one that will eventually disperse. Where the line wanders above the floor of the valley the field has intrinsic energy, and because it varies in space it has potential energy. The process of dispersion itself adds kinetic energy. As the wave spreads out over the infinite length of the line, however, the energy per unit length approaches zero and the field returns to a stable state of global vacuum.

A soliton of the sine-Gordon field appears when the line representing the state of the system climbs from the bottom of one valley in the energy surface, crosses a ridge and descends to the bottom of an adjacent valley. Extending in either direction from this point of transition the field is in a stable vacuum state but not the same vacuum state. For example, the magnitude of the field at all points to the left of the transition might be 180 and at all points to the right 360. In the transition region the field does not have a vacuum configuration. It has intrinsic energy because the line must surmount the hill in the energy surface, and it has potential energy because the value of the field changes from one point to the next.

The soliton represents a kink in the configuration of the field, and if the line x extends to infinity in both directions, the kink can never be removed. One end of the line is anchored in the vacuum state at $\varphi = 180$ and the other end at $\varphi = 360$. Since the line must be continuous, it must cross the energy maximum between these two vacuum states at some point along its length. The soliton can never disperse and its energy can never be dissipated.

The energy of a soliton in the sine-Gordon field depends on the geometry of the transition between vacuum states. The potential energy is minimized when the field varies as gradually as possible, and therefore when the line crosses the ridge at a shallow angle, nearly parallel to the x axis. In that configuration, however, the line is elevated above the bottom of the valley for a substantial distance and the intrinsic energy is high. The intrinsic energy assumes its minimum value when the line crosses



CREATION AND ANNIHILATION of solitons resemble the corresponding operations among quantum-mechanical particles. In the sine-Gordon field a section of the line that describes the state of the system can be lifted from the bottom of one valley and draped over a ridge into a neighboring valley. The result is the simultaneous creation of a soliton and an antisoliton, which can move away from the site. In the reverse process a soliton and an antisoliton meet and annihilate each other. Equations that govern the sine-Gordon field always create another pair immediately after the annihilation, but in other soliton systems energy of the waves can be dissipated in other ways.

the hill perpendicular to the x axis, making a very abrupt transition between vacuum states, but in that case the rate of spatial variation is high and the potential energy reaches an extreme value. The minimum total energy of the soliton is achieved when the crossing is smooth and at an intermediate angle, so that the intrinsic energy and the potential energy are equal.

Although the sine-Gordon field has a remarkably simple structure, the solitons that appear in it have several important properties in common with material particles. The stability of the soliton at rest has already been discussed. The soliton can also be set in motion without altering its form: the transition region between the two vacuum states rolls along the hill at a constant velocity. Even when the soliton is in motion, the width of the transition region remains constant, and the only change in the energy is the addition of a term for the kinetic energy.

There is nothing in the sine-Gordon equation that restricts the field to configurations with only one soliton, and in principle there could be an unlimited number of both solitons and antisolitons. By convention a soliton is a kink where the field increases (from one vacuum value to the next) as x increases and an antisoliton is a kink where the field decreases with increasing x. It is a simple matter to create a soliton-antisoliton pair from a state of global vacuum: simply pick the line up from the bottom of one valley and drape a loop over the ridge into an adjacent valley. The process corresponds to the creation of a particle and its antiparticle in a quantum field theory.

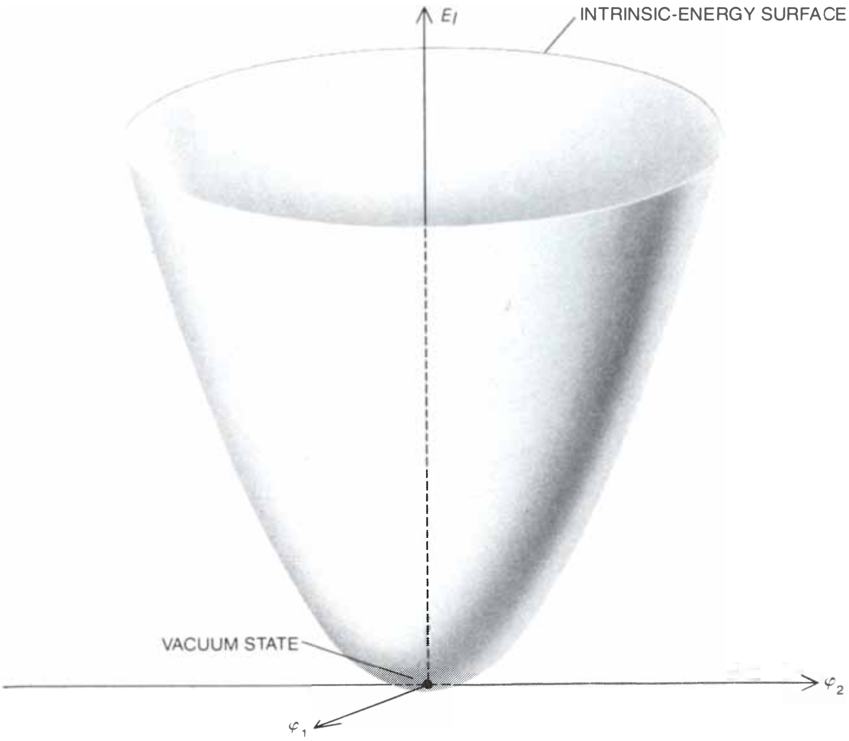
In the reverse process a soliton and an antisoliton collide. The sine-Gordon equation has the special property that both the soliton and the antisoliton emerge from the collision unchanged, but it is easy to modify the equation so that the waves are annihilated. Both kinks then disappear and their energy is dissipated in a wave that disperses into the global vacuum. Mutual annihilation is also observed among particles and antiparticles. Because any soliton-antisoliton pair can readily be created or annihilated, it is only the difference between the total numbers of these objects that is conserved. For example, a given field might go off to infinity in one direction at a value of $\varphi = 180$ and in the other direction at $\varphi = 360$, but there could be seven solitons and six antisolitons in the transition region between these two values. Six solitons and six antisolitons could be made to annihilate one another, but there is no way for the last soliton to be eliminated.

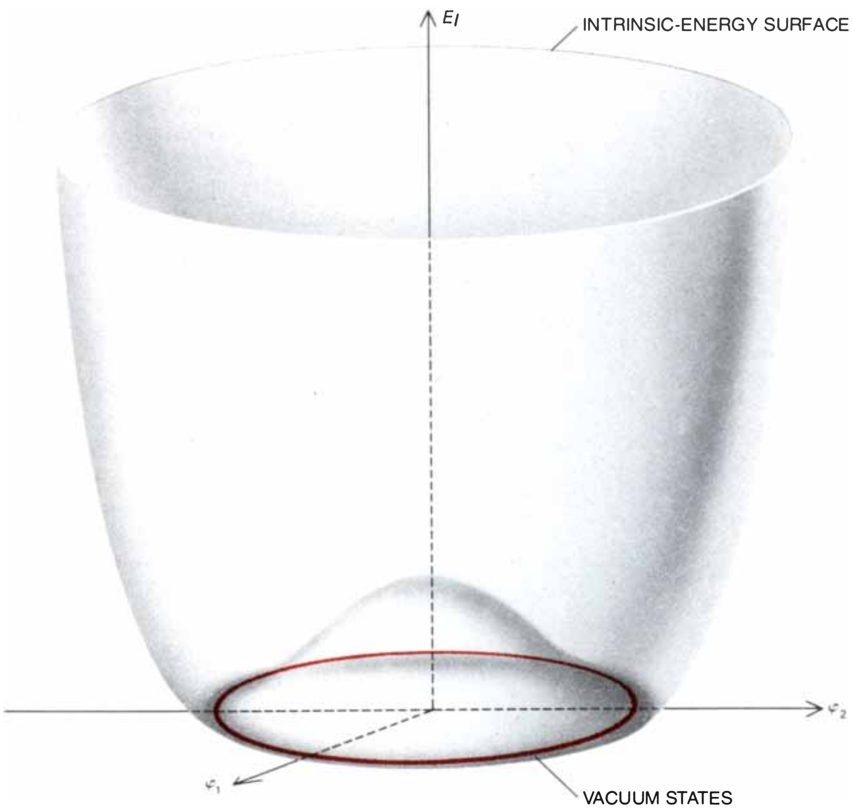
It is easy to make a working model of the energy surface of the sine-Gordon field. Fold a piece of heavy paper into several accordion pleats, or if possible into smooth undulations, then place a length of flexible key chain across the surface. If the chain begins in the bottom of one pleat and ends in another, a soliton must exist where the chain crosses from one vacuum state to the other. The absolute conservation of the soliton cannot be proved unless the surface and the chain are made infinitely long, but certain other properties can be investigated. By tilting the surface, for example, one can make the soliton move, and with careful manipulation it may even be possible to observe the creation and annihilation of soliton-antisoliton pairs. I first saw such a model demonstrated by Holger Nielsen, a Danish physicist who has done pioneering work in the study of solitons.

It may appear that the sine-Gordon field is an arbitrary construction that has no point of contact with the real world. As a one-dimensional field its descriptive power is necessarily limited. What is more, the multiple vacuum states that are essential for the existence of solitons are introduced by what may seem to be a rather artificial assumption, namely the assumption that the intrinsic energy is a periodic function of the field. There are physical phenomena in the three-dimensional world, however, that are effectively confined to one dimension. An example is the motion of electrons along a stack of molecules, and the sine-Gordon equation has been applied to the analysis of this system and similar ones. The periodic variation of the intrinsic energy is also not too remote from experience. Consider the energy of a pendulum expressed as a function of the angle that measures deviation from the vertical. As the angle increases from zero degrees the energy rises to a maximum when the angle measures half a turn, then returns to zero after a whole turn, and so on, passing through many equivalent maximums and minimums as the angle continues to increase.

The qualitative similarity between interacting solitons of the sine-Gordon field and certain interacting particles has been extended to a formal equivalence. In 1958 Walter E. Thirring of the University of Vienna formulated a quantum-mechanical model of particles and antiparticles moving in a one-dimensional space. Sidney R. Coleman of Harvard University has recently shown that the Thirring model and the solitons

America's health may be endangered by an unexpected threat.





TWO-DIMENSIONAL FIELDS can incorporate solitons if there are multiple vacuum states with an appropriate topology. The fields are defined by two quantities at each point in space, designated φ_1 and φ_2. Each combination of φ_1 and φ_2 implies a definite intrinsic energy, which is given by the energy surface for the field. In one possible field theory (*upper diagram*) the energy is zero only when both components of the field are zero; no solitons are possible in this field. Solitons can appear when the energy surface has a somewhat more complicated structure (*lower diagram*). Here a field whose components are both zero has a definite, nonzero energy. The vacuum states of the field are the combinations of φ_1 and φ_2 that describe the circumference of a circle with unit radius. If φ_1 and φ_2 are the components of a vector, then the field has zero energy when the magnitude of the vector is 1, regardless of its direction.

of the sine-Gordon equation describe the same phenomena. In the Thirring model the particles are assumed to exist and are then made to obey a postulated scheme of interactions; the solitons, on the other hand, are not introduced a priori but arise naturally from the equations of motion. Nevertheless, the two kinds of objects propagate and interact in the same way.

The notable successes of the sine-Gordon theory aside, there is no question that the real world has three spatial dimensions rather than one dimension. If solitons exist as real particles, they must be found in a three-dimensional theory. Two-dimensional waves with the properties of solitons have been known for a few years; the discovery of three-dimensional solitons is more recent. The latter may exist in nature as particles; the two-dimensional solitons are of interest in other branches of physics and are useful models for illustrating the properties of the three-dimensional ones. Indeed, once the step from the line to the plane is made, no further conceptual barriers are encountered in the passage to spaces of higher dimensionality.

The stability of many solitons in higher-dimensional spaces can be proved by topological arguments similar to those employed in the sine-Gordon field, although the geometric configuration of the fields becomes somewhat more difficult to visualize. In two dimensions the value of a field is a continuous function of position on a plane. A graph of the field can be constructed by drawing the plane and letting height above each point represent the value of the field at that point. The set of all values forms a surface erected over the plane; it is a flat surface parallel to the plane if the field is constant or a more complicated surface for a field that changes from point to point. In three dimensions the field must be defined at every point in ordinary space; it is not possible, however, to draw a graph of a three-dimensional field except in a hypothetical four-dimensional space. On the other hand, the properties of the two- and three-dimensional fields are very similar, so that for most purposes the simpler field can serve to illustrate both.

In one dimension the property that was found to be essential for the existence of solitons was the presence of multiple vacuum states, where the intrinsic energy vanishes at various values of the field. A soliton appears when the field approaches different vacuum values as one proceeds toward infinity in opposite directions on the line. On a plane, infinity can be reached by proceeding in any direction from a chosen origin; the number of directions that lead to infinity is itself infinite. The directions can be put into one-to-one correspondence with the infinity of points

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Viewfinder Eye-level pentaprism finder with instant-return mirror; green or yellow LEDs indicates correct handheld exposure, or need for flash/tripod.
Exposure Metering Center-weighted, TTL metering at full aperture via SPD cell; EV Range: 3-17 (24mm F 2.8 lens; ASA 100).
Shutter/Automatic Diaphragm Programmed electronic shutter with programmed body diaphragm; Range: F 2.8 to f/13.5.
Other Features Two-stroke film advance lever, synch terminal for automatic flash synch with AF 130P auto flash; tripod socket, lens bayonet release button, battery holder tray; exposure count via back cover window, winder capability.
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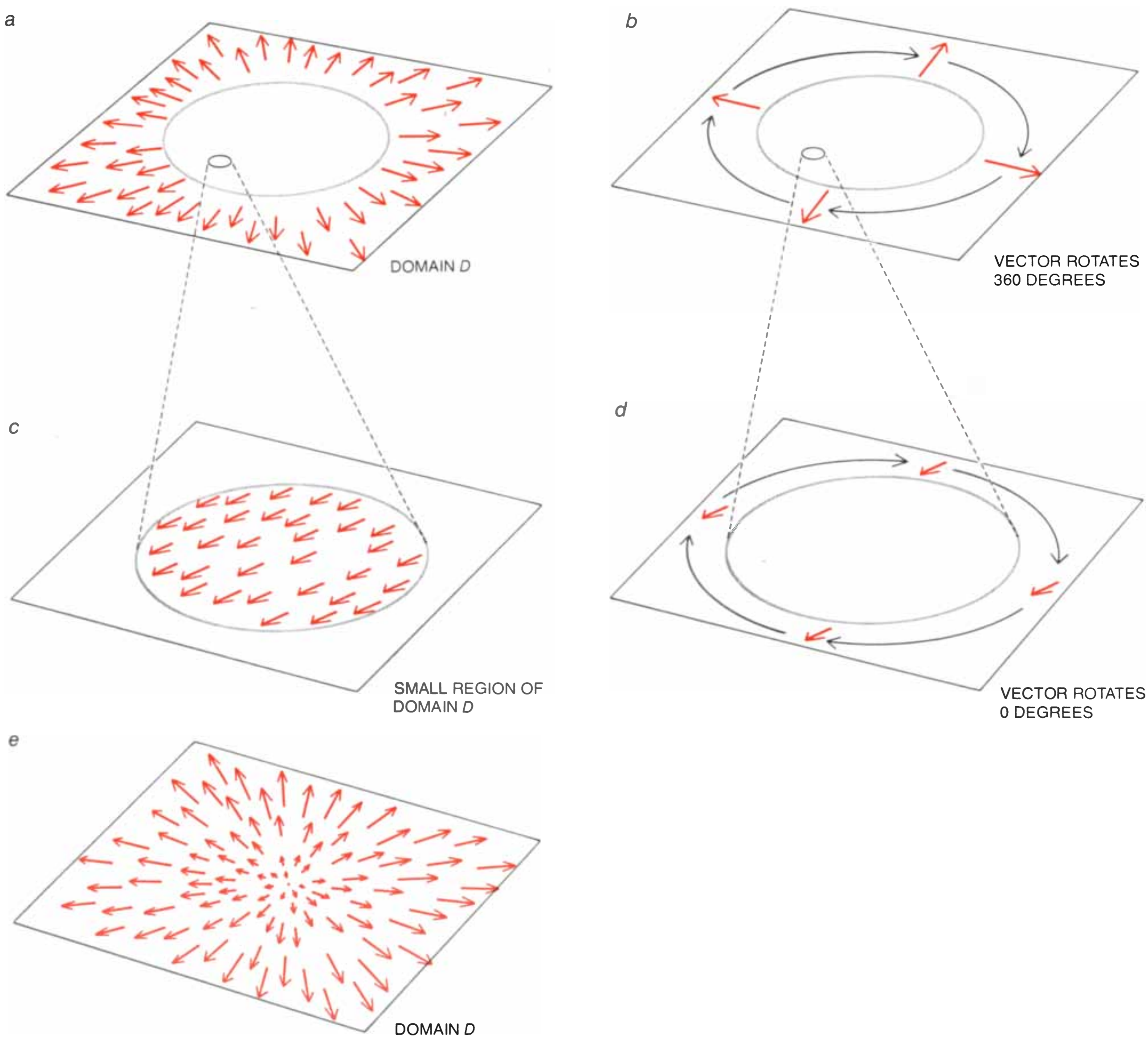
on the circumference of a circle. The situation is the same in three-dimensional space, where the directions toward infinity correspond to the points on the surface of a sphere. For a soliton to exist in either of these spaces the field must approach distinct vacuum states along each of the possible directions.

The topological peculiarity that gives rise to solitons is not possible for all two- and three-dimensional fields, or even for all those fields that have multiple vacuum states. One requirement is that the field itself be defined by more than one quantity at each point. The fields I shall discuss are specified by two values at each point in two-dimensional space and by three values in three-dimensional space. They are called vector fields because the two or three quantities can be regarded as giving the components of a vector along two or three orthogonal axes. The vector field is designated by writing the symbol for the field with an arrow over it: $\vec{\varphi}$. The intrinsic energy of a two-component vector field can be represented as a surface drawn over a plane. The plane is not that of the two-dimensional space itself but a plane giving all possible values of the vector field, or all possible combinations of magnitude and direction for the vector $\vec{\varphi}$.

One plausible equation for calculat-



CONFINEMENT OF A SOLITON in a two-dimensional space is demonstrated by a topological argument. The space (a plane) is filled with a vector field, a field defined at every point by a magnitude and a direction. Outside some arbitrary domain, *D*, the field is observed to radiate from *D* and its magnitude is everywhere equal to 1 (*a*). A field of that magnitude (but only one of that magnitude) is assumed to have zero intrinsic energy, and so everywhere outside *D* the field is in a vacuum state. At issue is whether the vacuum can continue inside *D*. That it cannot is proved by the following procedure. Imagine touring the perimeter of *D* while carrying an arrow that always remains parallel to the field (*b*). After a complete circuit the arrow resumes its original orientation, but it has rotated one turn. Now examine some microscopic region inside *D*; if the region selected is small enough, the field inside it must be uniform, with all the vectors parallel (*c*), since large variations in the field cannot be compressed into an arbitrarily small space. During a tour of this region (*d*) the arrow does not rotate at all. Many other circular tours can be imagined at scales intermediate between these two; for the larger tours the arrow would still rotate by a full turn and for the smaller ones it would not rotate. At some scale, however, there must be a transition, where the rotation of the vector field changes from 360 to zero degrees. A theorem in topology states that such a transition is possible only if at some point inside *D* the field itself vanishes, so that the direction of the arrow is undefined. It follows that somewhere inside *D* the field must be zero, and a smooth decline to zero at the center is the most plausible configuration (*e*). A field with a magnitude of zero, however, is not a vacuum state; it has energy greater than zero. The energy is that of a soliton, confined by a "twist" in the structure of the field.

ing the intrinsic energy as a function of the vector field is $E_I = (\vec{\varphi})^2$. The intrinsic-energy surface described by this equation is a paraboloid, the surface generated when a parabola is rotated around its axis. The intrinsic energy has just one minimum, at the apex of the paraboloid, which corresponds to the origin of the intrinsic-space plane. Therefore the intrinsic energy is zero only when both the magnitude and the direction of the field are zero. Because the field has only this one vacuum state, corresponding to no field at all, topological solitons are not possible.

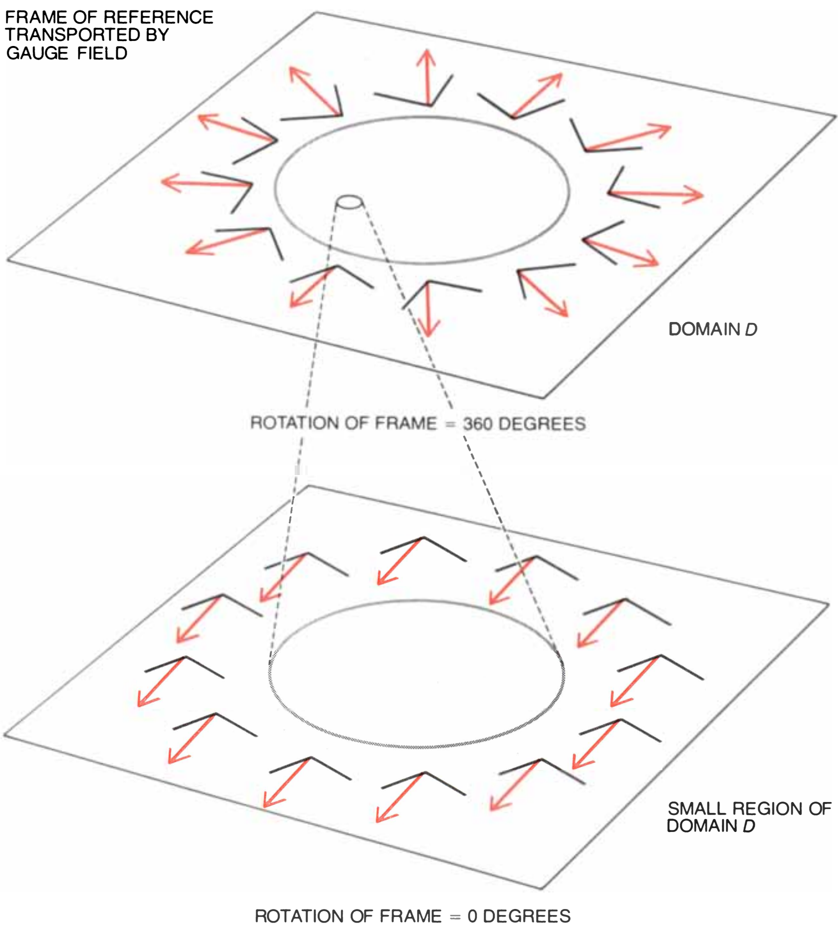
Suppose the intrinsic energy is given by a somewhat different equation, $E_I = (\vec{\varphi}^2 - 1)^2$. In this case the intrinsic energy is not zero at the origin of the plane; when $\vec{\varphi}$ is equal to zero, the intrinsic energy is equal to $(-1)^2$, or simply 1. Hence the state with no field is not a vacuum state. The true vacuum state is found when the magnitude of $\vec{\varphi}$ is equal to 1, since $(\vec{\varphi}^2 - 1)^2$ is then equal to zero. The equation $(\vec{\varphi}^2 - 1)^2 = 0$ is the equation of a circle; it is satisfied by all the points on the circumference of a circle whose center is the origin and whose radius is 1. The field is therefore in a vacuum state whenever the magnitude of the vector is 1, regardless of its direction. The complete energy surface has a curious structure. At large distances from the origin it looks much like a paraboloid, although it is steeper. Near the origin the surface reaches a minimum at all points on the circle where the magnitude of $\vec{\varphi}$ is equal to 1, then rises again inside this circle. The surface looks like a quartic paraboloid with a dent poked into its apex [*see illustration on page 102*].

What does this energy surface imply about the possible configurations of a two-dimensional vector field? One consequence of particular importance to this discussion is that the field can have zero intrinsic energy if the magnitude of the field vector is everywhere equal to 1, regardless of the vector's direction. Consider a field whose magnitude and orientation have been measured at all points on a plane outside some arbitrary domain *D*. The magnitude of the vector has been found to be equal to 1 everywhere, and its direction is such that the vector always points away from the origin, which is inside the domain *D*. Hence outside *D* the plane is filled with vectors that all have unit length and that seem to radiate from *D*. Everywhere outside *D* the field is in a vacuum state, since the unit vector corresponds to zero intrinsic energy. The question is: Can the vacuum state also extend inside *D*, so that the field has zero intrinsic energy everywhere? It cannot. The proof that it cannot is topological in nature and it shows that *D* must envelop a region of finite energy: a two-dimensional soliton.

Imagine taking a walk around the domain *D*, carrying an arrow that always points in a direction parallel to that of the field. On returning to the starting place the arrow will be pointing in the same direction it was when the walk began, but it will have described a full circle, a rotation through 360 degrees. Now imagine examining some microscopic region inside *D*. It might seem most plausible for the pattern of radiating vectors to be reproduced at smaller scale in this region, but a simple argument shows that such a field configuration is impossible. If the field were to make a 360-degree rotation within a microscopic region, the rate of spatial variation and hence the potential energy would be enormous. As the area of the sample approached zero the potential energy would be unbounded. It follows that if a small enough sample area can be chosen, the field within it must be uniform: the vectors at all points must have not only the same magnitude but also the same direction. Thus a microscopic sample of *D* would reveal a field in which all the vectors are parallel. An arrow carried around this sample would never deviate from its original orientation; it would not rotate at all.

The procedure of carrying an arrow around a closed loop in a field illustrates a theorem in topology. The theorem states that if the field has no discontinuities and does not vanish at any point, then the arrow must rotate an integer number of times during any circuit. The arrow can fail to rotate at all, in which case the integer is zero, or it can turn once, twice and so on, but it cannot make half a turn. No proof of this theorem will be presented here, but it is supported by common sense, which indicates that the arrow must return to its original position at the end of any complete loop.

The theorem is satisfied by the loop



GAUGE FIELDS alter the description of a soliton in two or three dimensions. For a soliton to exist, a field radiating from a domain, *D*, must be in a vacuum state outside *D*. Setting the magnitude of the field equal to 1 makes the intrinsic energy zero, but the potential energy must also be taken into account. Because a radiating pattern of vectors changes direction from point to point, the potential energy does not seem to be zero. The quandary is resolved by introducing a new field, the gauge field, which brings with it a prescription for transporting a frame of reference from point to point. Outside *D* the frame rotates by exactly the amount needed to keep the direction of the field vector constant at all points (when the direction is measured with respect to the transported frame). In a small region inside *D* neither the vector nor the frame rotates. With the introduction of the gauge field the potential energy of the original field is eliminated but the topological confinement of the soliton is retained. It is no longer the original field vector but the gauge field whose net rotation changes from 360 to zero degrees.

outside *D*, where the vector rotates through 360 degrees, and by the microscopic loop inside, where the rotation is zero. What happens, however, during some loop at an intermediate scale? At some point, as the size of the region examined is reduced, the rotation of the field must make a transition from 360 to zero degrees. The topological theorem forbids a loop with a fractional rotation, but it does include two escape clauses. One is the possibility that the field has a discontinuity, where its value changes abruptly from one point to the next. That solution is excluded, however, by the requirement that all physical fields exhibit only smooth and continuous changes. The only remaining possibility is that somewhere inside *D* the value of the field falls to zero. At that one point the net rotation can change from 360 to zero degrees because where the field vanishes the direction of the vector is undefined.

This elaborate argument has been presented for one purpose: to prove that inside the domain *D* there must be a point where the magnitude of the field is not equal to 1; instead it must be equal to zero. It then follows that at this point the intrinsic energy is not zero. In fact, the requirement that a field vary only by smooth increments creates an entire region inside *D* where the energy is greater than zero. This lump of energy cannot spread out over the plane; it is confined by the "twist" where the net rotation of the field changes by one turn.

A three-dimensional soliton is similar in structure, and its confinement can be explained by analogous arguments. The points of minimum intrinsic energy now make up the surface of a sphere, and the intrinsic energy rises both inside this sphere and outside it. A field configuration corresponding to these vacuum states can again be postulated for all points outside some domain, but now the domain encloses a spherical volume rather than a circular area. The same topological argument shows that the vacuum state cannot be extended to all regions inside the domain, and so there must be a region where the field has a value that gives an intrinsic energy greater than zero. This ball of energy is a three-dimensional soliton, and it closely resembles an elementary particle in ordinary space.

In the foregoing analysis of two- and three-dimensional solitons an important element has been neglected. The configuration of the field outside the domain *D* has been defined as the vacuum state because it has zero intrinsic energy, but no account has been taken of its potential energy. Because the field vector changes direction at every point, the potential energy is not zero. Indeed, it can be proved that the potential energy is infinite, so that the field configuration not only fails to describe a localized wave but also seems to be unattainable.

There is a way out of this impasse, but it is not a simple one. The infinite potential energy outside the domain *D* can be reduced to zero by postulating a new kind of field, called a gauge field. In the past several years gauge fields have become much-studied structures, not only in physics but also in mathematics, where they are known by a different name: connections. I shall not explore the workings of a gauge field in detail but shall concentrate on those properties that are important to the theory of solitons.

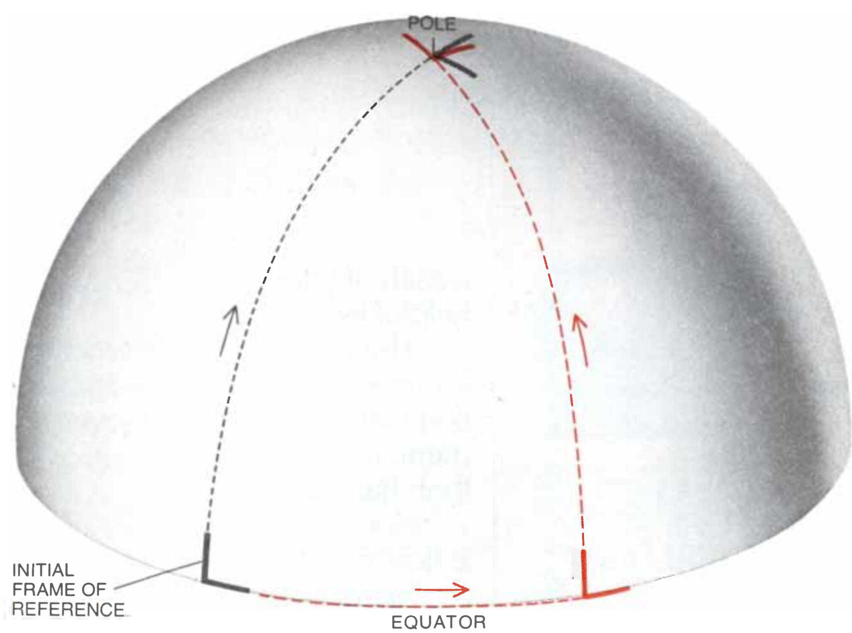
In order to eliminate the potential energy of the field without also destroying the soliton, the change in orientation of the vector from point to point must be abolished, while the overall rotation of the field by 360 degrees must be preserved. Although these two goals may seem to be irreconcilable, they are both accomplished by a gauge field.

In describing the two-dimensional field it was assumed implicitly that the direction of the vector would be measured at each point with reference to some fixed set of coordinate axes in the abstract plane where all possible values of $\vec{\varphi}$ are plotted. The frame of reference could be arbitrary, but it remained the same throughout space: it was a global frame. The introduction of a gauge field allows the frame of reference to rotate as one moves between neighboring points. The potential energy is then evaluated by measuring the variation of $\vec{\varphi}$ not with respect to a global frame of reference but rather with respect to local frames that can change from point to point.

Suppose that somewhere on the perimeter of *D* the field vector makes an angle of 30 degrees with a chosen frame of reference. If the frame is a global one, then after traveling 90 degrees around a circle the vector will have assumed a new orientation of 120 degrees. A suitably arranged gauge field, however, allows the frame of reference to rotate with the vector; no matter where a measurement is made, the orientation of the vector field is effectively constant and so the potential energy vanishes. On the other hand, the notion of a twist in the field configuration is preserved: it has been transferred from the field $\vec{\varphi}$ to the gauge field. Now it is the local frame of reference that undergoes a complete turn in a closed path around *D*.

It is important to emphasize that the gauge field does not simply specify the orientation of many independent frames of reference. Instead it prescribes how the orientation of a single frame changes when the frame is displaced. If it were necessary only to measure the changing orientations of a frame in a two-dimensional space, then it would suffice to specify one number at each point: the angle of rotation. Actually a two-dimensional gauge field is defined by two numbers at each point. One number tells how much the frame rotates during a displacement along the *x* axis and the other tells how much it rotates during a displacement along the *y* axis.

Because the gauge field is a prescription for transporting a frame between



TRANSPORT of a frame of reference according to the prescription of a gauge field is analogous to transport over the surface of the earth. If the frame is transported from its initial position on the Equator directly to the pole, it assumes one final orientation (*black*), but moving it first along the Equator and then to the pole yields a different final orientation (*color*). Hence the rotation during a displacement depends on the path followed. The rotation results from the curvature of the earth. In abstract space a gauge field has an effect equivalent to curvature.

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neighboring points rather than a prescription for setting up a different frame at each point, the net rotation of the frame during a displacement depends on the path taken. In going from Rome to London by way of Berlin the frame might rotate by some definite angle. If the frame is carried from Rome to London by way of Paris, the angle of rotation will in general be quite different. Moreover, on a round trip—say from Rome to Berlin to London to Paris and back to Rome—the frame will generally not return to its original orientation.

The notion of transporting the frame on the earth's surface is not entirely metaphorical. Imagine setting up a frame of reference, consisting of two orthogonal axes, at some point on the Equator. Then transport the frame to the North Pole by sliding it parallel to itself, first by heading due north along a meridian, then by following the Equator for a distance before turning north along a different meridian. When the two frames meet again at the pole, their orientations will be different. The discrepancy results from the curvature of the earth.

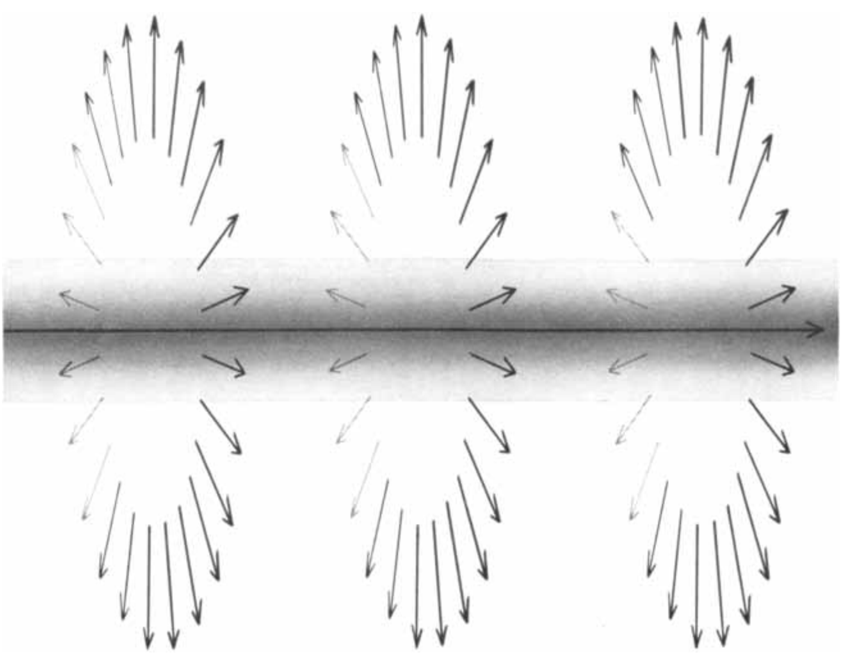
There are special configurations of a gauge field where a frame transported around a closed loop does return to its initial orientation. In those configurations the rotation of the frame during a displacement is independent of the path taken. The gauge field is then said to be in a pure gauge form. It carries no energy and is in its vacuum state. When the final orientation of the frame does depend on the path, the gauge field carries energy, which becomes larger as the dependence on the path becomes more pronounced.

It is now possible to reexamine the energy of a two-dimensional field that includes a soliton. The introduction of a gauge field eliminates the potential energy of the field $\vec{\varphi}$ outside the domain *D*. Moreover, the gauge field itself is in a pure gauge form there and carries no energy because the frame of reference returns to its original orientation after any closed loop. The gauge field cannot continue in a pure gauge form throughout the interior of *D*, however. The frame rotates by 360 degrees when it is carried around a closed loop outside *D*, but there must be a small loop inside *D* where it does not rotate at all. By extension there must be loops of intermediate size where the frame rotates by some angle between 360 and zero degrees. In that case the frame does not return to its original orientation after the circuit, and so the gauge field must carry energy.

The twist in the field configuration outside *D* is now embodied in the gauge field. Both fields, the $\vec{\varphi}$ field and the gauge field, are in their vacuum state outside *D*, but the continuity of the field configuration demands that the fields carry energy inside the domain. The soliton that emerges is again made stable against dispersion by the topology of the field configuration.

The prototypical gauge field is the one that appears in Maxwell's theory of electromagnetism. It describes the relative orientation of frames of reference in an abstract space with two dimensions. The components of a vector field $\vec{\varphi}$ lie in this space. This field is not the electromagnetic field itself but describes the charged matter to which the electromagnetic field is coupled. In realistic situations $\vec{\varphi}$ and the gauge field are both defined at all points in three-dimensional space, although the abstract space where $\vec{\varphi}$ lies is still two-dimensional, the two dimensions corresponding to the two possible signs of charge.

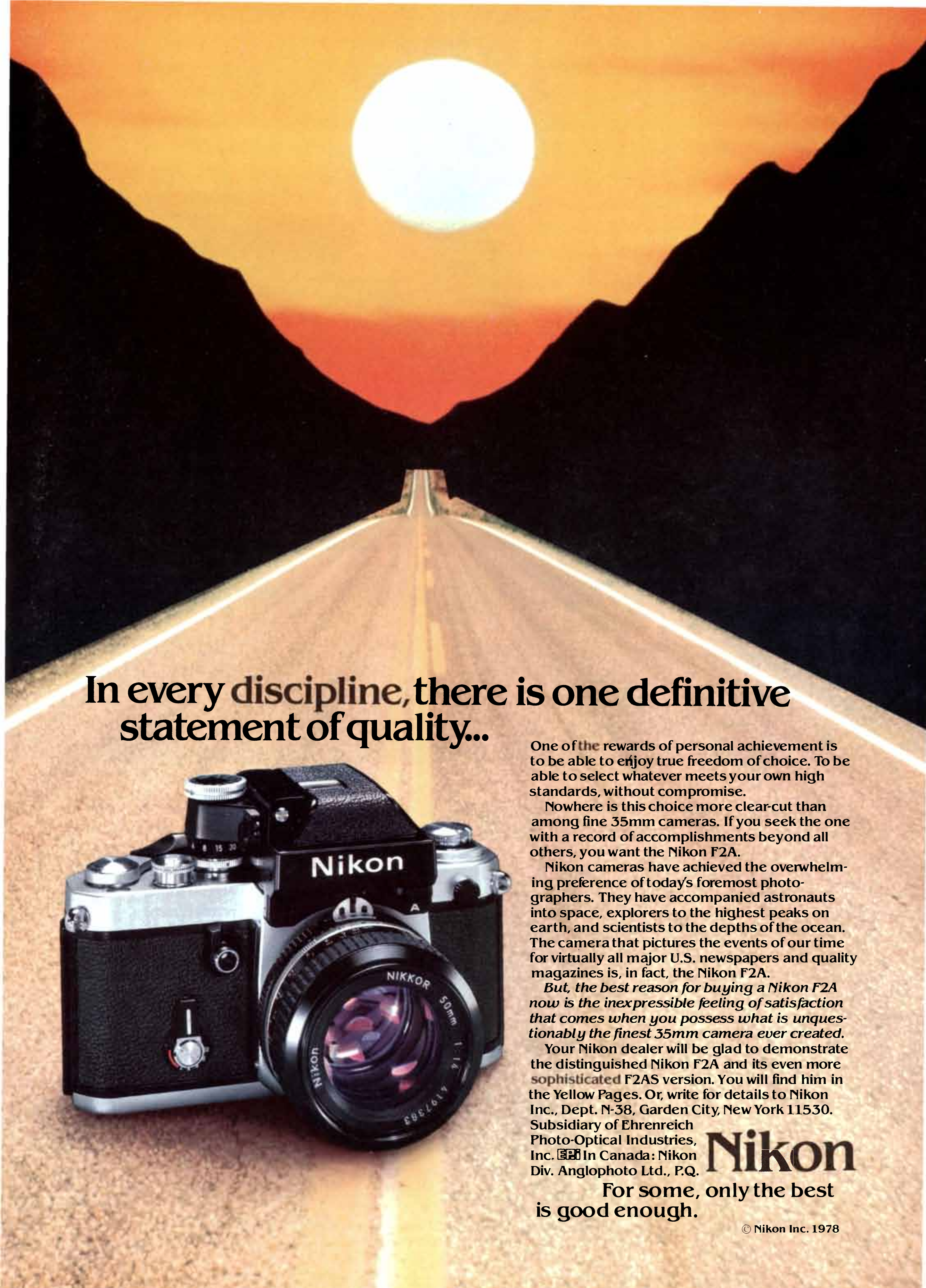
In most configurations of the electromagnetic field solitons are not possible because the minimum of the intrinsic energy corresponds to a zero value of the matter field $\vec{\varphi}$. In superconductors, however, the minimum is found at a field of nonzero magnitude. The field configuration then allows the existence of two-dimensional solitons, which do appear in superconducting materials. They manifest themselves as tubes of magnetic flux, where the fields are arranged in a vortexlike structure and the energy is confined to a narrow tube at the center of the vortex. When the vortexes are viewed in cross section, they resemble the two-dimensional soliton described above.



TUBE OF MAGNETIC FLUX in a superconductor is confined to a small cross section by a field that incorporates many two-dimensional solitons stacked with cylindrical symmetry. The superconducting electron pairs are described by a vector field with multiple vacuum states and with an intrinsic energy that is at a minimum when the magnitude of the field is greater than zero. The magnetic field is described by a gauge field. The twist in the topological configuration of the two fields bounds the magnetic flux and prevents it from spreading out in space.

A gauge field defined in an abstract space with more than two dimensions is substantially more complicated than the electromagnetic field. The additional complexity arises because there are many axes around which the frame of reference can rotate, and in general the result of rotations performed around different axes depends on the order in which they are made. Gauge fields of this kind are called Yang-Mills fields, after C. N. Yang of the State University of New York at Stony Brook and Robert L. Mills of Ohio State University, who first discussed them in 1954. It was some time before the mathematical techniques needed to deal with the fields were mastered, but Yang-Mills fields have now been given a crucial role in the physicist's description of nature. A Yang-Mills field is the essential element in a theory that seems to unify two of the four fundamental forces of nature, the weak force and the electromagnetic one. Another Yang-Mills theory, although it is less well developed, might eventually explain a third fundamental force, the strong or nuclear force.

The discovery that solitons might be generated in a theory that describes a matter field coupled to a Yang-Mills field was made independently by Gerhard 't Hooft of the University of Utrecht and by Alexander M. Polyakov of the Landau Institute for Theoretical Physics in Moscow. By recognizing that the theory can have multiple vacuum states with a nontrivial topology, they discovered the three-dimensional soliton. It is a soliton of this kind that might exist as a real elementary particle. Such a soliton particle would be very massive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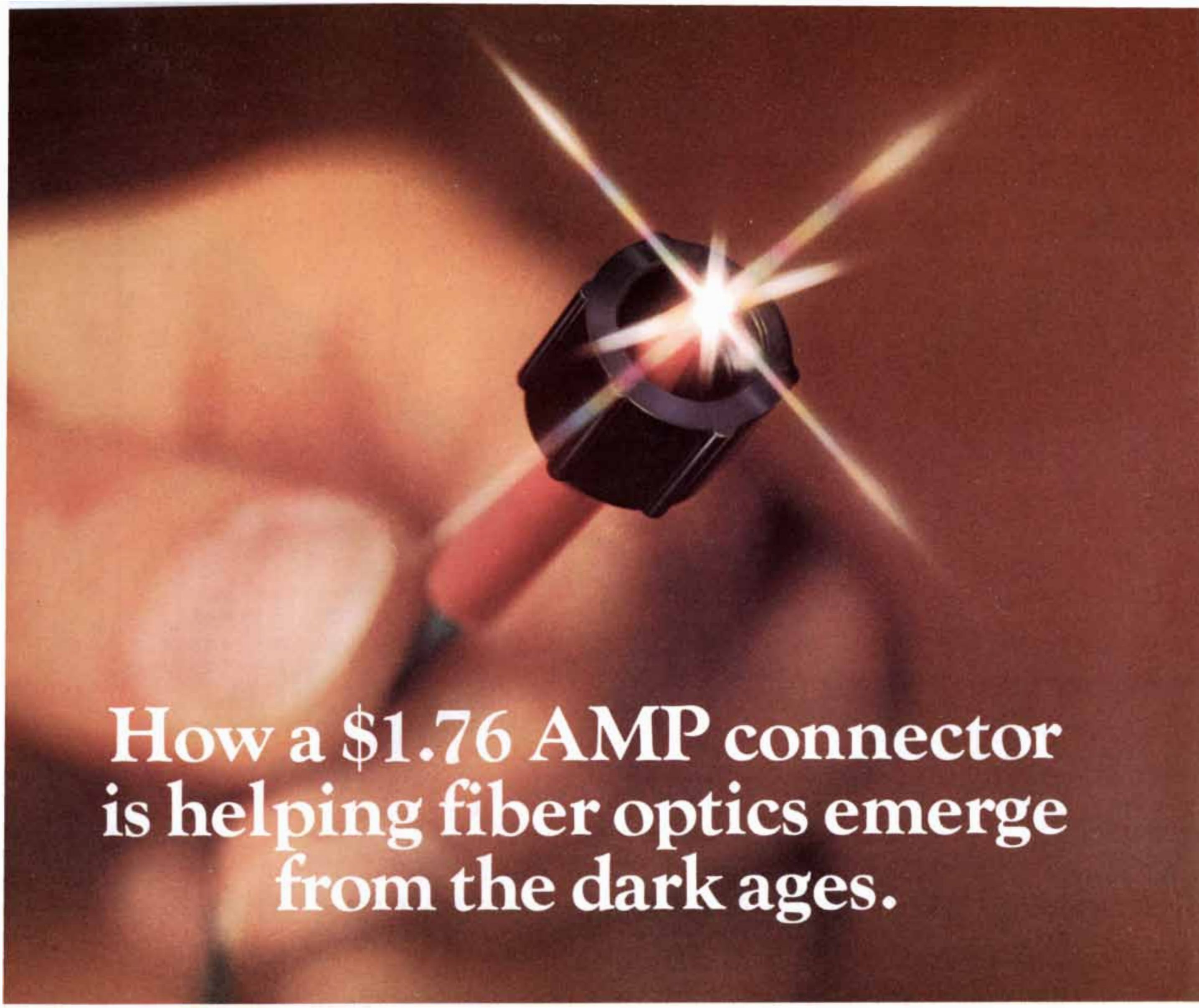

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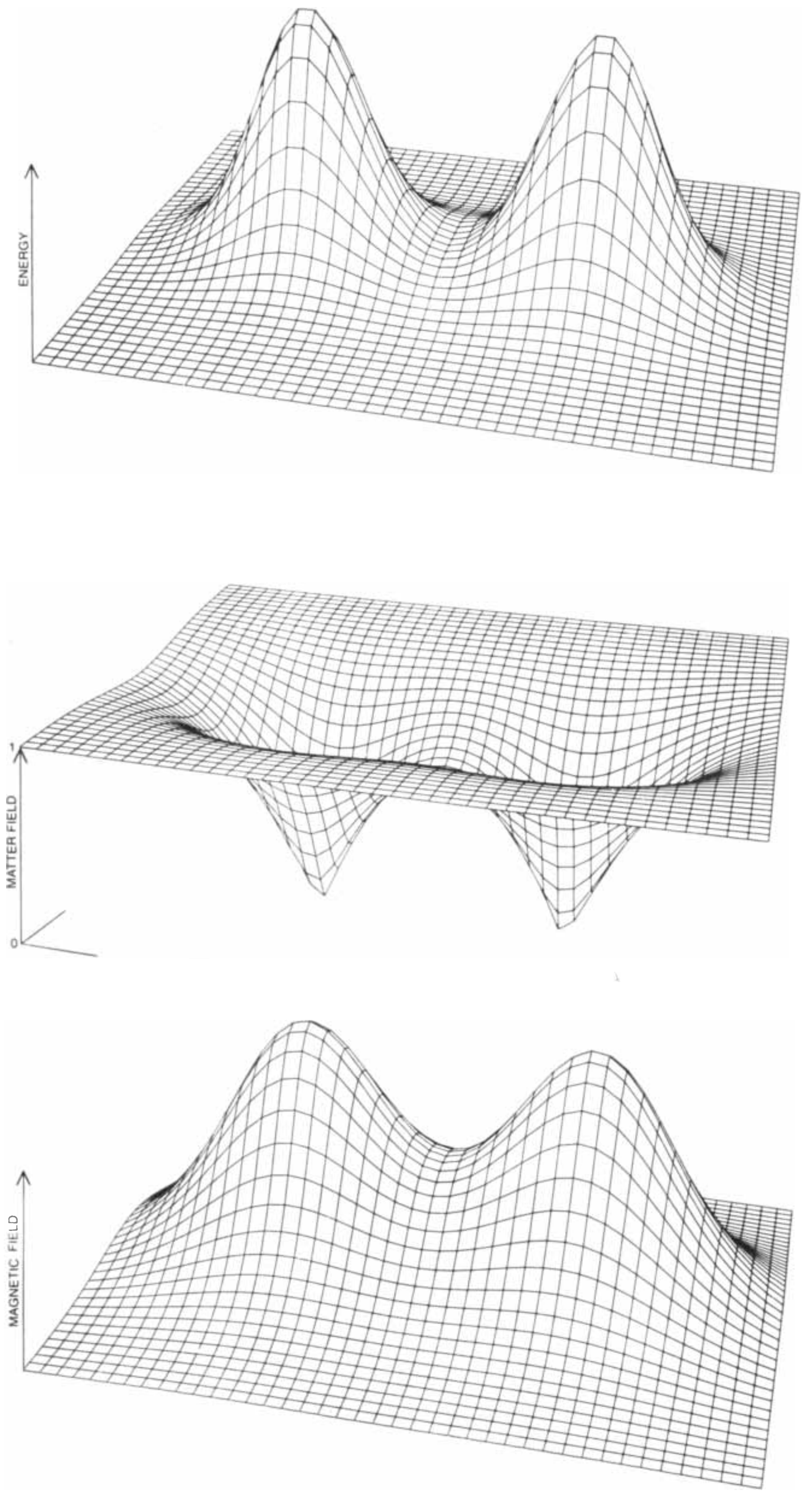
and would be a magnetic monopole, that is, it would carry an isolated magnetic charge. This property, which none of the known particles share, also derives from the topology of the field; in the same way that the twisted arrangement of vacuums confines energy it also encloses a unit of magnetic charge.

The theoretical exploration of the properties of these particles has only just begun, but a few interesting findings have already emerged. One result concerns relations between two fundamental categories of particles, the fermions and the bosons. These categories are distinguished by the intrinsic angular momentum, or spin, of the particles, and by their statistics, or behavior in groups. The spin of a fermion is a half integer, the spin of a boson an integer. The quantum-mechanical statistics of the particles specify that no two fermions can occupy the same state, whereas bosons can be brought together in unlimited numbers. Two fermions can combine to form a composite particle with the properties of a boson, just as two half integers add up to an integer. It would appear, however, that there is no way to combine bosons to yield a fermion.

When the field has only a global vacuum, the prohibition on making fermions out of bosons is indeed absolute, but it is not so when solitons are present. A mechanism for the conversion has recently been found by 't Hooft and Peter Hasenfratz, who is now at the University of Budapest, and independently by Roman W. Jackiw of the Massachusetts Institute of Technology and me. In the presence of a soliton a system with half-integer spin can emerge from a field whose only components are bosons. Alfred S. Goldhaber of Stony Brook has shown that the system would have not only the spin characteristic of a fermion but also the statistics.

Another novel result, obtained by Jackiw and me, shows that a fermion might be split in half under the influence of a soliton. We found a mode of interaction between solitons and fermions where the structure of a soliton is altered by the field of a fermion. The soliton exists in two states of identical energy, one state having the character of half a fermion and the other state that of half an antifermion.

The prospects for making and detecting soliton particles in the laboratory are quite uncertain. They depend in large measure on what theory is ultimately found to describe most accurately the interactions of elementary particles. If the theory is one that admits soliton solutions, then it is generally acknowledged that the solitons will appear in nature. There are many candidate theories, however; some of them incorporate field equations with a topology suitable for solitons but others do not. It is no surprise that soliton particles have not yet been observed in experiments



MERGER OF TWO FLUX TUBES in a superconductor illustrates the coalescence of two solitons. The matter field, whose magnitude is related to the density of superconducting electron pairs, is shown in the middle graph; this is the field in which the solitons appear. The matter field has two depressions, where the density of electron pairs falls to zero; everywhere else it rises toward a uniform value of 1. The energy of the field, shown in the upper graph, is zero at the periphery and rises to a maximum at each point where the density of electron pairs is zero. Thus the two regions where the matter field declines represent confined quantities of energy. The magnetic field passing through the solitons is shown in the lower graph; it too is confined to the regions of diminished electron-pair density. The graphs were constructed with the aid of a computer by the author and Laurence Jacobs of Brookhaven National Laboratory.

with particle accelerators. The mass of such a particle, measured in energy units, is thought to be some trillions of electron volts. One trillion electron volts is roughly 1,000 times the mass of a proton and more than four times the mass of a uranium atom. It will be at least several years before accelerators can create such heavy particles.

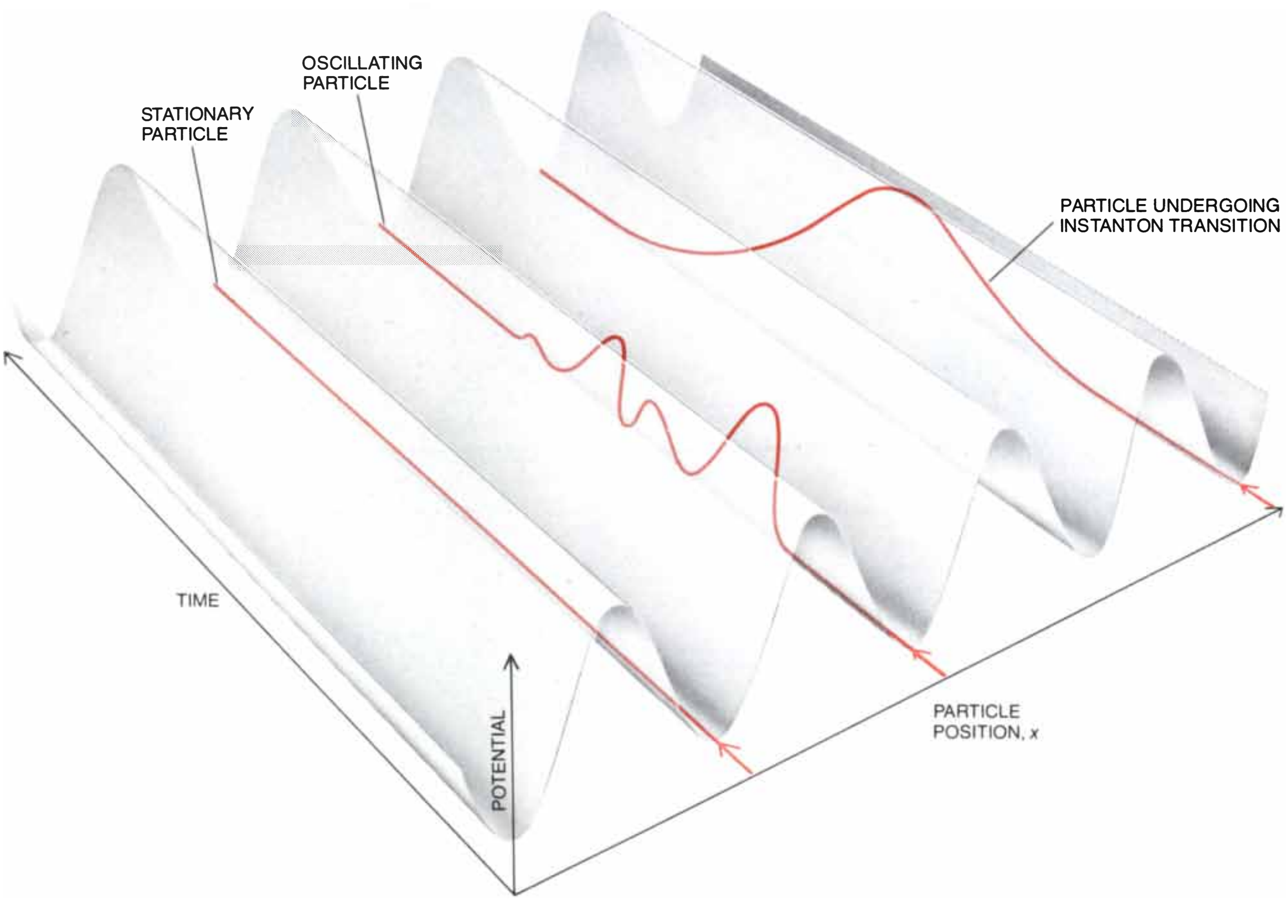
All the solitons described above are structures that are localized in space (whether the space has one, two or three dimensions). They are of interest because they are confined permanently to a definite region of space. In the past few years another kind of soliton has been discovered, one that is confined to a small region both in space and in time. It is a phenomenon that exists only at a particular place and at a particular moment. This new kind of soliton, which has been given the name instanton, is interpreted not as an object but as an event, not as a particle but as a quantum-mechanical transition between various states of other particles.

The nature of an instanton can be elucidated by examining the motion of a particle under the influence of a potential, which specifies the value of a force acting on the particle. For the sake of simplicity the potential can be made one-dimensional, and for instantons to appear it must be made periodic, that is, there must be several equivalent but distinct points of minimum energy. There are many examples of such potentials in nature; a familiar one is a roller coaster, where the potential is the gravitational field of the earth and a point of minimum potential energy is encountered at the bottom of each drop. It should be pointed out that the potential energy in this system is not that of a field but that of a particle in a field. In the example of the roller coaster potential energy is proportional to height above the lowest point on the track.

A graph of a simple periodic potential shows a straight line, which represents the space where the particle moves, and an undulating line (perhaps a sine curve), which gives the value of the potential for each point in the one-dimensional space. In order to trace the evolution of the system another axis, representing time, must be added to the graph. The undulating line is then converted into an undulating surface. Moving across the undulations is equivalent to a change in position; moving parallel to them is not a motion in space but instead reveals a steady state of the system at successive moments.

A particle in a state of minimum energy lies at the bottom of one well in the potential curve, and it can be expected to remain at rest there forever. The trajectory of the particle in time is therefore a straight line that follows the bottom of the valley. A particle might also oscillate about the equilibrium point at the bottom of a well, a motion that is represented along the time axis by a sinuous line wandering from one side of the valley to the other.

The event of greatest interest is the movement of a particle from a stable position at the bottom of a well over a potential barrier and into an adjacent well. On the time axis this evolution corresponds to a line that begins in one valley of the potential surface, climbs over



INSTANTON is a soliton confined not only to a region in space but also to a moment in time. It is interpreted as a quantum-mechanical transition between two states of motion of a particle. The particle moves in a one-dimensional potential, or force field, and the evolution of its motion is recorded on a potential surface that extends along the time axis. The history of a particle that remains stationary at one of the points of minimum potential is represented by a straight line that follows a valley in the surface. A particle oscillating about an equilibrium point traces an undulating curve in one of the valleys. The trajectory of an instanton climbs from the bottom of one valley, crosses a ridge and returns to stable equilibrium in an adjacent valley. The path describes the motion of a particle that disappears at one point of equilibrium and reappears at another. The transition is equivalent to the quantum-mechanical phenomenon of tunneling.

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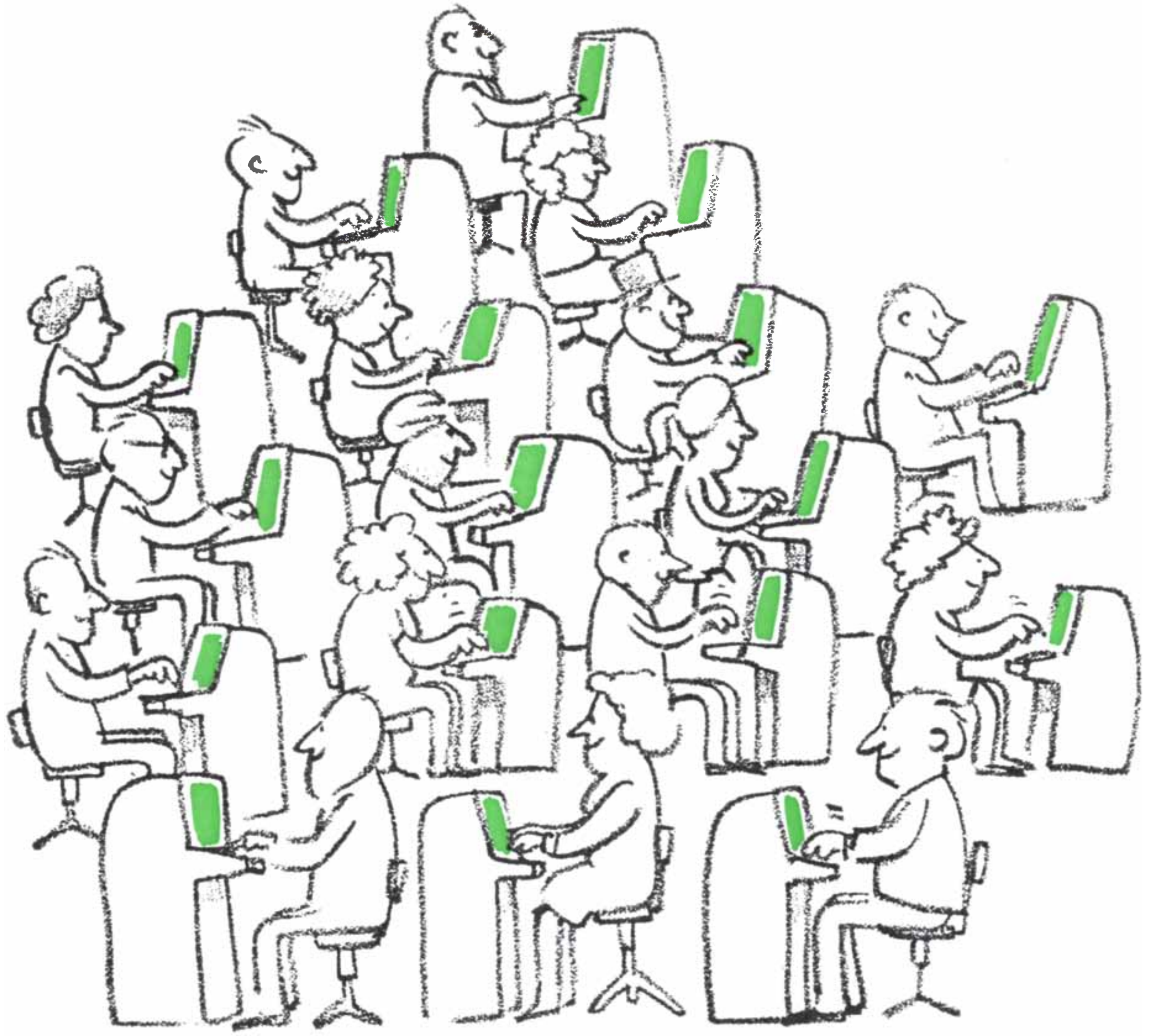

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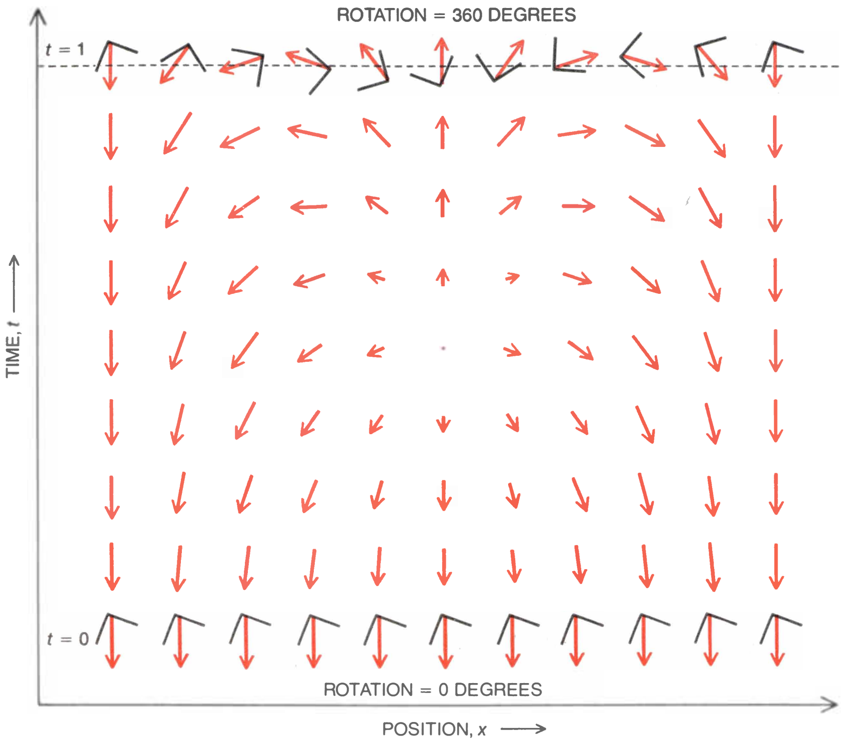

EVOLUTION OF A FIELD during an instanton transition exhibits a topological twist much like that of a purely spatial soliton. Initially ($t = 0$) the field is uniform, being made up of vectors that are all parallel and all of unit magnitude. This configuration is assumed to describe a vacuum state. At a later moment ($t = 1$) the vectors again have unit magnitude, but they are found to rotate through one turn. If the frame of reference is defined by an appropriate gauge field, the later configuration can also be regarded as a vacuum state. For the field to evolve from the first state to the second one, however, its net rotation must change from zero to 360 degrees, a transition that is topologically possible only if the field vector somewhere declines to zero. At that point the intrinsic energy of the field is greater than zero, and so the instanton describes an evolution from one vacuum state to another through a state with finite energy.

a ridge and descends into a neighboring valley. At one moment the particle is stationary at a point of minimum energy; the next moment it is found still at rest but at another minimum-energy position. The transition between these states is an instanton.

In the classical physics that preceded the development of quantum mechanics such a transition was impossible. If a train of roller-coaster cars is at rest at the bottom of a hill, one can predict with confidence that it will not spontaneously climb over the hill and come to a stop in the next valley. Energy would be conserved in this imaginary process—at least for an ideal, frictionless roller coaster—because all the energy that would have to be expended to raise the train to the top of the hill could be recovered on the downhill run. The transition is nonetheless forbidden because in classical physics energy must be conserved at all moments, not merely in a final accounting.

Quantum mechanics provides a kind of deficit financing that makes the instanton transition possible. A seeming violation of the conservation law is allowed provided that the violation does not last too long and that the books balance in the end. Through this mechanism, which is called tunneling, a particle can cross a potential barrier even though it has too little energy to surmount the barrier. The instanton is a structure in a classical field theory that describes this fundamentally quantum-mechanical process.

The graph of an instanton is geometrically identical with the graph of a soliton in the sine-Gordon field; only the labels on the axes have been changed. Indeed, instantons were first found as ordinary solitons in a field theory of four spatial dimensions. The discovery was made by A. A. Belavin, Polyakov, A. S. Schwartz and Yu. S. Tyupkin of the Landau Institute, who correctly interpreted the soliton not as a four-dimensional object but as an evolution of fields in three spatial dimensions and one time dimension. They also showed that instantons must appear in a large class of field theories, including the theories most commonly applied to the interactions of elementary particles.

Soon afterward the significance of instantons for the structure of the quantum-mechanical vacuum was analyzed by 't Hooft, by Curtis G. Callan, Jr., and David J. Gross of Princeton University and Roger F. Dashen of the Institute for Advanced Study and by Jackiw and me. (It was 't Hooft, incidentally, who suggested the name instanton.) The existence of instantons implies that the vacuum state in quantum mechanics is not unique but has a periodic structure, rather like the potential valleys of the electromagnetic field inside the lattice of atoms in a crystal. Of course, the periodic field in a crystal results from an orderly arrangement of atomic nuclei, whereas the pattern generated by the instanton solutions is an intrinsic structure of space-time. The discovery of the structure had been quite unexpected.

Although instantons are a recent innovation in field theory, they have already led to the resolution of a disturbing puzzle in the physics of subnuclear particles. The puzzle concerns the masses of the particles called mesons, which are thought to be composite objects made up of the more fundamental entities called quarks. Each meson consists of a quark and an antiquark bound together by a gauge field. Two of the quark types (labeled u and d) and their corresponding antiquarks ($\bar{u}$ and $\bar{d}$) are thought to be comparatively light. Since there are four ways to make a quark-antiquark pair from these objects ($u\bar{u}$, $u\bar{d}$, $d\bar{u}$ and $d\bar{d}$), it would seem there should be four mesons of comparatively low mass. Three such mesons have been known for many years: they are the negative, positive and neutral pi mesons, or pions, which have masses equivalent to an energy of about 140 million electron volts. The fourth light meson, which seems to be an inescapable prediction of the theory, has never been found.

There is another particle, however, that could fill the role. It is the eta meson, and it has all the appropriate properties except one: its mass is about 550 million electron volts. The introduction of instantons has now explained the anomaly of the eta meson's mass. The instantons appear as excitations, localized in space and time, in the gauge field that binds the quarks together. They alter the distribution of mass among the mesons because they have different effects on the various quark combinations. Roughly speaking, an instanton is transparent to a pion, but it acts as an obstacle to the propagation of an eta meson and thereby increases its effective inertial mass.

Solitons and instantons are the offspring of field theories that can be forbiddingly complex, yet they have a rich and elegant mathematical structure. Indeed, physicists investigating solitons have discovered that mathematicians have been studying equivalent objects for many years entirely for their geometrical interest. Mathematical analysis and physical intuition have forged powerful tools to expose the nature and properties of the soliton.

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