Black holes, quantum information, and the foundations of physics

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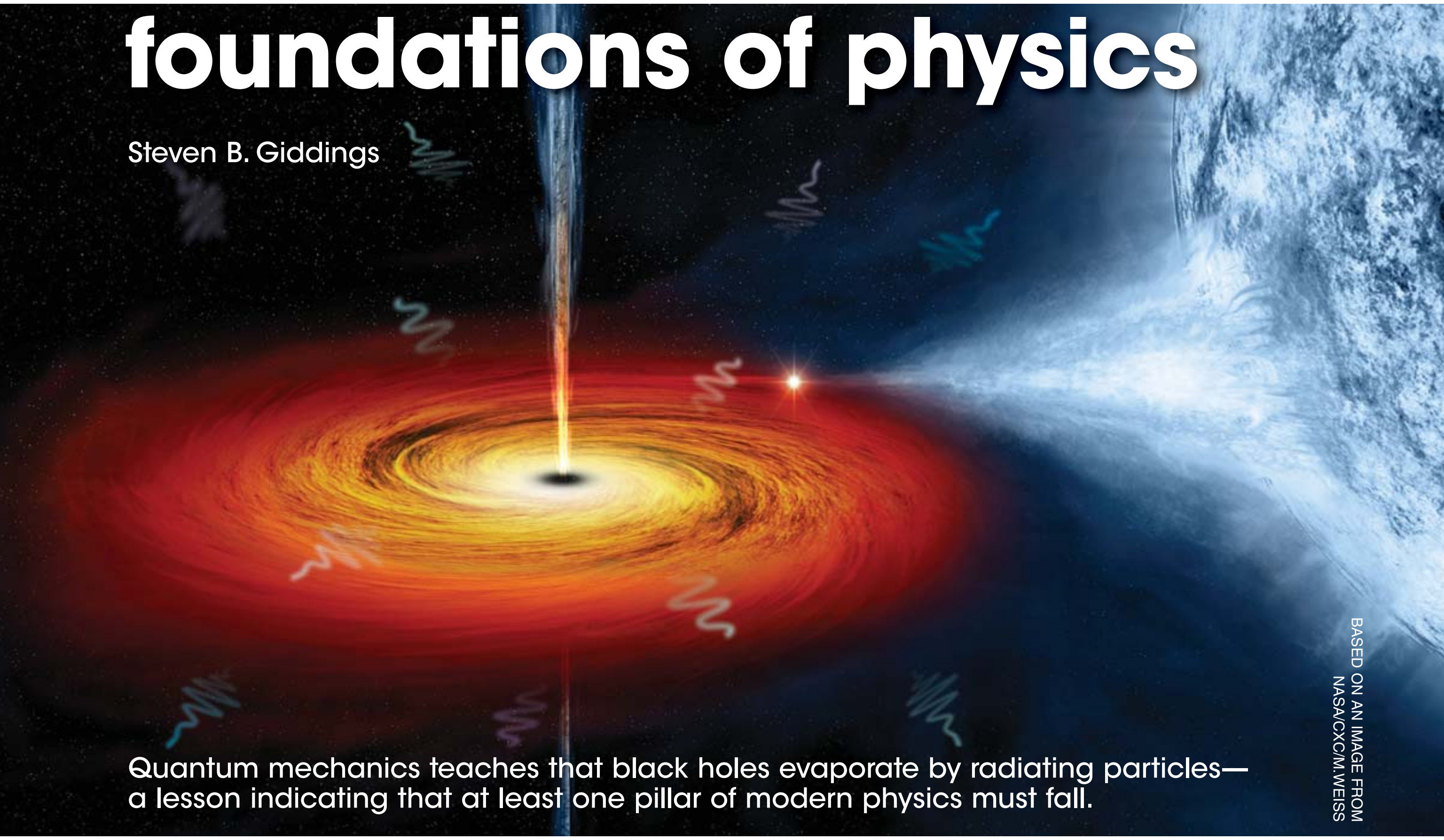

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Quantum mechanics teaches that black holes evaporate by radiating particles—a lesson indicating that at least one pillar of modern physics must fall.

Black holes are perhaps the most profoundly mysterious objects in the universe. We have excellent evidence that they exist—even that they are ubiquitous throughout the cosmos. But their very existence threatens to overthrow the current foundations of physics, specifically locality and the fundamental role of spacetime. This radical conclusion follows from a question a child could ask (and many readers probably did ask as children): What happens to stuff thrown into a black hole?

Our best gravity theories—general relativity and modifications of it—predict black holes, and evidence for them has grown steadily. The effects of black holes are particularly prominent on galactic scales. They appear to be central engines in many galaxies, creating spectacular phenomena such as active galactic nuclei, quasars, and massive jets that span hundreds of thousands of light-years and contain more than a million solar masses of material. (See the article by Jon Miller and Chris Reynolds in PHYSICS TODAY, August 2007, page 42.) Evidence has also accumulated for their role in galaxy formation, and astrophysicists estimate that most galaxies harbor a central black hole. The best evidence comes from our own galaxy, which hosts a central object of 4 million solar masses. Beautiful work imaging stellar orbits has constrained the object size to no more than 1000 times the expected radius of a black hole (see PHYSICS TODAY, February 2003, page 19); the near future should see direct imaging reach down to the black hole radius. We have no plausible description of such objects as anything other than black holes.

Observations may speak strongly in favor of black holes, but the theoretical framework of quantum field theory offers no consistent explanation for those remarkable objects. Our understanding of them may guide a new approach to the foundations of physics.

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A penetrating thought experiment

The basic notion of a black hole is simple enough to fascinate schoolchildren: It is an object whose escape velocity exceeds the speed of light c. The idea goes back to natural scientist and Anglican rector John Michell, who in 1783 observed that an object with the density of the Sun but 500 times its radius would be a black hole. The nonrelativistic Newtonian equations he used give the correct relativistic formula for the black hole radius. His argument can be summarized beginning with the formula for the energy E of a particle of mass m moving in the gravitational potential of a spherical mass M:

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r},$$

where v is velocity, r is radial distance, and G is Newton's constant. A particle just barely escapes to infinity if $E = 0$; if it has an initial velocity $v = c$, the condition of marginal escape determines an initial radius $R(M) = 2GM/c^2$, now called the Schwarzschild radius for mass M. For an Earth-sized mass, R is approximately 1 cm; for a solar mass, it is about 3 km.

Our current classical understanding of gravity is via general relativity and Einstein's equations. In 1916, mere weeks after their final formulation, Karl Schwarzschild presented a solution giving the gravitational field of a spherically symmetric mass. That Schwarzschild solution is the simplest possible black hole; figure 1 illustrates its basic features.

In Einstein's theory, gravity is described through the curvature of spacetime, and in the center of the black hole, the curvature goes to infinity. That infamous singularity indicates a breakdown of physics, but one far removed from scrutiny. In particular, since in classical physics nothing escapes the region within the so-called event horizon located at R, the singularity has no effect outside the black hole. The need for physics to smooth out the singularity is nonetheless one of the motivators for pursuing a quantum theory of gravity. Many feel that a correct quantum description will resolve such singularities.

As theorists well know, quantum gravity presents challenges beyond singularity resolution. At the outset it seems straightforward to quantize general relativity, whose dynamical variable is the spacetime metric. Quantum mechanics instructs us to consider wavefunctions of dynamical variables; those states evolve via a unitary evolution operator that typically is given by the exponential of the Hamiltonian multiplied by the time and divided by the imaginary unit i. In gravity, we thus consider wavefunctions of the metric. Then straightforward inference yields the analogue of the Schrödinger equation, called the Wheeler–DeWitt equation.

But the devil is in the details. When one tries to make the resulting expressions precise and, in particular, studies them via perturbation theory, an infinite number of infinities arises throughout the quantum amplitudes of the theory. That proliferation of infinities is the problem of nonrenormalizability.

Singularities and nonrenormalizability have been primary foci in decades of quantum-gravity research. However, a further problem that has not received the same focus seems to me to be even more profound, and its resolution likely more central to a quantum description of gravity. That problem arises from a thought experiment.

Thought experiments have a long tradition in elucidating difficult physical principles. Imagining light rays seen from moving trains or falling elevators was instrumental to understanding special and general relativity. Mentally probing quantum particles with the light waves of Heisenberg's microscope drove at limitations to measurement and at the essence of quantum mechanics and the uncertainty principle.

An important thought experiment for quantum gravity—perhaps the key thought experiment—is to imagine two quantum particles colliding at sufficiently high energy and small impact parameter to make a quantum version of a black hole. How does one then predict the outcome?

Density matrices and missing information

Pure and mixed states, their density matrices, and entropy are easily illustrated in two-state systems such as that of a spin that can be up or down. Here, the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \tag{1}$$

is an example of a pure quantum state, with corresponding density matrix

$$\rho_1 = |\psi\rangle\langle\psi| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|). \tag{2}$$

In contrast, the density matrix

$$\rho_2 = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) \tag{3}$$

is that of a mixed state, meaning that it cannot be expressed in the form $|\phi\rangle\langle\phi|$ for any state $|\phi\rangle$. The von Neumann entropy, defined as $S = -\mathrm{Tr}(\rho \log \rho)$, is a measure of purity ("Tr" stands for the matrix trace); the equality $\rho_1 = |\psi\rangle\langle\psi|$ implies $S(\rho_1) = 0$, and one easily finds $S(\rho_2) = \log 2$. Evolution from a density matrix like ρ_1 to one like ρ_2 violates unitarity and loses quantum information.

Tracing out degrees of freedom in a pure state typically yields a mixed state. For example, as a toy model for the degrees of freedom inside and outside a black hole, consider a two-spin system with correlated spins and a pure state given by

$$|\psi'\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\downarrow\rangle_B). \tag{4}$$

Tracing $\rho' = |\psi'\rangle\langle\psi'|$ over system B gives

$$\rho_A = \sum_{i=\uparrow,\downarrow} {}_B\langle i|\rho'|i\rangle_B, \tag{5}$$

a density matrix of the form displayed in equation 3.

Black hole production and evaporation

Classical physics gives a first step to predicting the results of the thought experiment. The basic intuition is that if two ultrarelativistic particles collide at distances that are small compared with the Schwarzschild radius of their center of mass energy E—that is, $R(E/c^2) = 2GE/c^4$, they concentrate enough energy to form a black hole.

There are important checks of that intuition. Analytic study of the gravitational field of colliding high-energy particles reveals the presence of a black hole; see figure 2 for an elaboration. And more recent powerful numerical work in general relativity

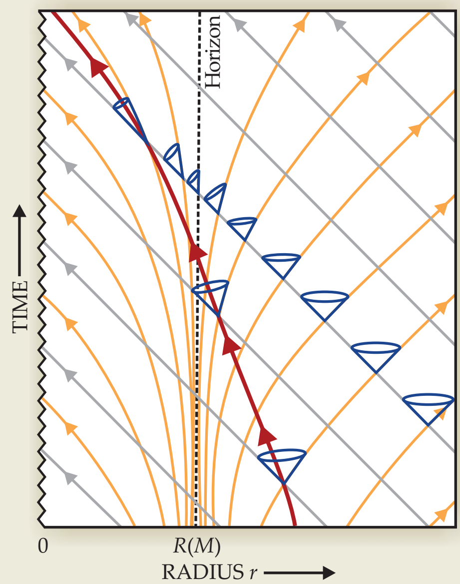

Figure 1. The spacetime geometry of the Schwarzschild black hole solution can be depicted in different ways. In this representation, ingoing light rays always travel along ingoing lines heading toward the top and left at 45°; outgoing light rays asymptotically approach 45° lines at large radius r. Massive particles, with their slower speeds, must travel within the light cones (blue) between outgoing and ingoing light rays, as illustrated by the red path. No light ray can escape to infinity from inside the vertical dotted line, the horizon located at the mass-dependent Schwarzschild radius $R(M)$. Instead, any trajectory beginning inside the horizon is pulled to a central point, the singularity at $r = 0$, where spacetime curvature becomes infinite.

has confirmed the formation of black holes in high-energy collisions.

Quantum physics modifies the classical picture. In particular, the quantum spreading of a particle can, in principle, prevent horizon formation. The scale of the spreading is comparable to the particle's de Broglie wavelength, which for large E is roughly $\hbar c/E$ ($\hbar$ is Planck's constant). If that wavelength is smaller than the would-be horizon size, $R(E/c^2)$, a black hole can form despite quantum spreading. The formation criterion is satisfied if E is larger than the Planck energy $E_P = \sqrt{\hbar c^5/G}$. At energies much higher than E_P, this quantum delocalization is relatively unimportant, and since the spacetime curvature in the vicinity of the horizon is small, the quantum picture appears well approximated by the classical picture. Interestingly, the above considerations can be thought of as significantly altering the conclusions drawn from Heisenberg's-microscope thought experiments that probe short distances at high energies; evidently, gravity yields a new fundamental limit on resolution, of size $R(E/c^2)$.

Quantum physics leads to another effect not present in the classical theory: black hole evaporation.[1] By analyzing quantum fields in the semiclassical geometry of a large black hole, Stephen Hawking showed that particles are radiated in a process similar to quantum tunneling. As a result, the black hole shrinks until it ultimately reaches a mass comparable to the Planck mass $M_P = E_P/c^2$. At that juncture, Hawking's calculation must fail due to large curvatures and quantum fluctuations.

The question is, What happens then, at the finale of our thought experiment? Our current physical framework offers no sensible answer. Even for a thought experiment, that deficiency is deeply troubling. More disturbingly, theorists have devised TeV-scale theories of gravity in which black holes might be produced in high-energy colliders such as the Large Hadron Collider at CERN (see the article by Nima Arkani-Hamed, Savas Dimopoulos, and Georgi Dvali, PHYSICS TODAY, February 2002, page 35). If such theories describe the world, the question of the finale becomes a true experimental issue.

The fate of quantum information

Unitary evolution is a key property of quantum mechanics. It is closely related to reversibility: Unitary evolution takes a given initial state to a definite final quantum state and preserves the state's normalization and thus probability. It can therefore be run backwards to recover the initial state, a fundamental property that corresponds to quantum mechanics conserving quantum information.

To see unitarity's conflict with black hole evaporation, we need to better understand the latter. Hawking radiation is nearly thermal, with a temperature inversely proportional to the black hole radius. A thermal state is one with a large amount of missing microscopic information, typically characterized by its entropy. For black holes, Jacob Bekenstein, now at the Hebrew University of Jerusalem, argued that the entropy is proportional to the area A of the black hole horizon, $S_{BH} = Ak_Bc^3/4G\hbar$, where k_B is Boltzmann's constant.

To understand the origin of the missing information in Hawking's calculation, first note that observers falling into a classical black hole would see nothing unusual happening at the horizon. So one would expect to find near the horizon a state that behaves essentially like the vacuum. However, quantum mechanics produces vacuum fluctuations. As illustrated in figure 3, the strong gravitational field of the black hole separates those fluctuations into the outgoing Hawking particles that make up the Hawking radiation and partner excitations that fall into the center of the black hole. Outside observers see only the radiation, so in giving a quantum description of it, they should construct a quantum density matrix formed by summing over the degrees of freedom inside the black hole. But the Hawking particles and missing partners are correlated, so the density matrix describes a mixed quantum state rather than a pure one. The so-called von Neumann entropy of that mixed state characterizes the missing information carried by the partners inside the black hole. The box on page 31 provides a simple example to illustrate the above concepts.

That black holes contain missing information follows from the locality of quantum field theory (QFT). I have described the missing information in terms of Hawking partners, but one could also drop PHYSICS TODAY articles or other information-rich objects into a black hole. Once inside, the information cannot escape the black hole without propagating faster than the speed of light. And in QFT, locality is conventionally formulated as the proscription against faster-than-light relay of information.

Hawking initially advocated that after a black hole reaches the Planck mass, it disappears in a final burst, along with all its missing information.[2] Since the black hole could have formed from a pure quantum state but the post-evaporation state is mixed, with radiation carrying lots of entropy, the evolution is not quantum mechanical and instead is of some more general form that violates unitarity.

When particles collide at ultrahigh energies,[3] as in the key thought experiment introduced earlier, a two-particle pure state scatters and produces a multiparticle state that is, in Hawking's proposal, not pure. Evolution from pure states to pure states would be described by a scattering matrix (the S-matrix); Hawking's proposal[2] generalizes that matrix to a linear mapping, from density matrices to density matrices, that is not unitary. In particular, the generalized mapping discards information.

The problem with that nonunitary evolution is Murray Gell-Mann's dictum, borrowed from author T. H. White, that in quantum physics, anything not forbidden is compulsory. If black holes destroy quantum information and can also contribute to virtual processes—for example, loops in Feynman diagrams—the breakdown of unitarity will spread to pollute all of physics. The result, as first argued by Thomas Banks, Michael Peskin, and Leonard Susskind,[4] is that the whole world would experience nonunitary, non-energy-conserving evolution and would appear thermal with an enormous temperature of order E_P/k_B. Although a small minority dispute that conclusion,[5] it is widely regarded as correct.

To save quantum mechanics, one might consider the alternative in which black holes don't completely disappear but leave behind remnants that store the missing information. Those would be particle-like, Planck-mass objects, and they could be long-lived or stable. But to contain missing information from an arbitrarily large initial black hole, they would need to have an arbitrarily large number of internal states that describe the possible configurations of the Hawking partners or all the things that could have been thrown into the black hole.

The remnants should also obey quantum mechanics, and in particular, nothing seems to forbid remnant pair production generalizing the formation of Hawking particles and partners. But basic quantum rules imply that if an object can be pair produced in a way that is independent of a degeneracy of internal states, then the total production rate is proportional to that degeneracy. For the putative remnants, the degeneracy is unbounded. And that means unlimited remnant production during any physical process in which there is enough energy available to create a pair—no matter how small the amplitude may be to create a pair with a specific internal remnant state. The world would explode into remnants.

In short, quantum information can't escape, because of locality; can't be destroyed without violating quantum mechanics and energy conservation; and can't be left behind without sacrificing stability. This is the essence of the apparent paradox of black hole information.

Where did Hawking go wrong?

Hawking's approach was semiclassical, and it's worth asking whether it yields a true paradox by providing a sharp calculation of the missing information. One might, for example, try to calculate the quantum state of the black hole, trace over the inside degrees of freedom to find the outside density matrix, and determine the von Neumann entropy characterizing the black hole's missing information. In calculating quantum fluctuations about the classical geometry, Hawking approximated that state. One can improve on his work by using the freedom in relativity to choose a convenient time slicing—that is, to define surfaces of constant time—and specifically by choosing spatial slices that span the interior and exterior of the black hole but avoid the singularity. Such "nice" slices can be constructed fairly explicitly. Then one can try to calculate the quantum state on those slices, in a systematic perturbative expansion in Newton's constant G.

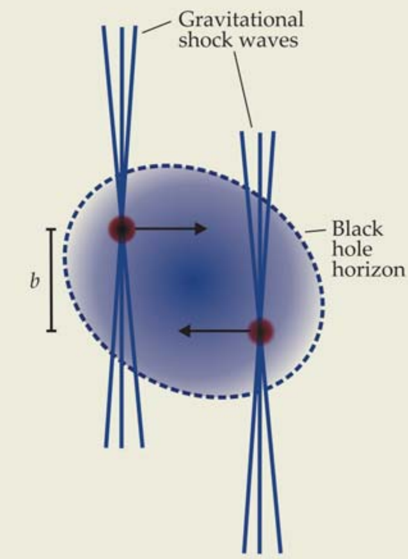

Figure 2. As with electric and magnetic fields, gravitational field lines of ultrarelativistic particles pancake into the transverse plane and form gravitational shock waves. Outside the shock waves, spacetime is essentially flat. Theorems of general relativity say that a black hole horizon will result for impact parameters b significantly smaller than the Schwarzschild radius given by the center-of-mass energy of the colliding particles. Black hole formation thus limits the ability to resolve shorter distances using high-energy collisions.

Such a procedure meets obstacles. The form of the quantum state can be determined over a short time range, but sharply calculating the quantum state and entropy for the Hawking radiation over long spans of time has proved challenging. In particular, the nice slices eventually become an extreme and not necessarily valid description of the dynamics. That issue, and related difficulties with the perturbative expansion, apparently becomes problematic before an evaporation time of order $R(M)S_{BH}(M)/(k_Bc) \propto (M/M_P)^2R(M)/c$. Interestingly, arguments based on quantum information theory indicate that information needs to start escaping from the black hole on such a time scale.[6] Also, perturbative calculations of the quantum state in an analogous system, an expanding inflationary universe, appear to suffer a similar breakdown of perturbation theory over an analogous time scale.

Thus varied evidence suggests that the question "Where did Hawking go wrong?" is answered by pointing to the lack of a sharp perturbative calculation of the lost information over relevant time scales, and indicates that Hawking's story therefore does not generate a true paradox.[7] And many physicists, including Hawking himself, now expect that the missing information does escape the black hole as it shrinks. But the question remains as to what physics is responsible for the leakage.

Indeed, the information problem represents a basic conflict among foundational physical principles. Those include quantum mechanics, specifically its unitary evolution; Lorentz invariance and its generalization, local frame independence; and locality. Those principles imply that nature is described by QFT; the generalization with local frame independence underlies QFT's extension to general relativity and curved spacetime.

Such a basic conflict among principles signals that one or more must be modified. Among them, locality seems the least robust in a quantum description of gravity. Attempts to modify the others have typically foundered on the shoals of inconsistency and conflict with experiment. Although locality in QFT is closely linked with causality and thus consistency, locality is difficult to formulate precisely in a theory of gravity. Whichever principles require modification, if information does indeed escape a black hole, the laws of physics operate in an unfamiliar and novel way.

An answer from string theory?

One approach to the information problem has been to seek appropriate modifications in an existing framework. A leading contender is string theory, which has had success in addressing the problem of nonrenormalizability and limited success in resolving singularities. String theory modifies locality in two ways. First, strings are extended, not pointlike. The second modification is via holography.[8]

Various theorists have suggested that strings' extended nature could facilitate information escape, for example, because high-energy collisions would excite extended strings.[9] But closer examination has found that strings behave a lot like particles when forming black holes and has not supported a resolution via extendedness.

Holography is the idea that a gravity theory in a bulk region has an equivalent description in terms of a QFT confined to the region's boundary (see the article by Igor Klebanov and Juan Maldacena, PHYSICS TODAY, January 2009, page 28). In principle, the unitary boundary theory should provide a unitary bulk description of black hole formation and evaporation. But here, too, the devil is in the details. One needs a sufficiently detailed dictionary connecting bulk and boundary theories that, for example, could provide the *S*-matrix on scales that are small compared with the bulk curvature radius. Theorists have not yet been able to find such a detailed dictionary and have encountered obstacles to deriving a fine-grained description of the bulk gravity theory.

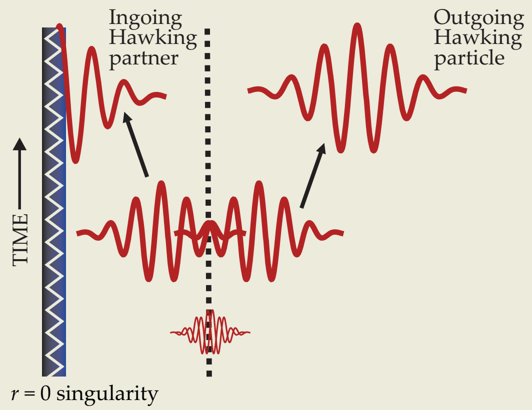

Figure 3. In the geometry of a black hole, Stephen Hawking discovered, vacuum fluctuations near the horizon (vertical dashed line) get pulled apart and turn into outgoing particles that escape from the black hole and partner excitations that fall into the singularity at radius $r = 0$. Outside and inside excitations have quantum correlations, and the outside state is missing quantum information corresponding to the inside state.

Holography is commonly associated with the idea of complementarity,[8] which proposes that observables inside and outside a black hole are complementary in analogy to Bohr's complementarity of variables like position and momentum in quantum mechanics. The result would be that inside and outside observations can't be simultaneously discussed in a common physical description. If complementarity were correct, it would likewise represent a radical departure from local QFT.

Modifying locality

In 1992, even before holography and complementarity, I proposed the possibility of a resolution in which some new, nonlocal physics relays information from inside a black hole to outside the horizon. I considered a scenario, involving what were called massive remnants, in which an initial black hole transitions to a new kind of object with information-carrying states and an interface outside the would-be horizon; figure 4 illustrates the concept. The massive remnant can interact with the outside world or decay, and by either process return any missing information. The massive-remnant scenario does not respect locality, at least with respect to the semiclassical spacetime geometry of the evaporating black hole: The object's surface must expand from near the center of the black hole to outside the horizon and thus move faster than light.

Over the past couple of decades, theorists have devised various specific realizations of the basic massive-remnant scenario. One is the fuzzball,[10] in which string-theory excitations describe states of

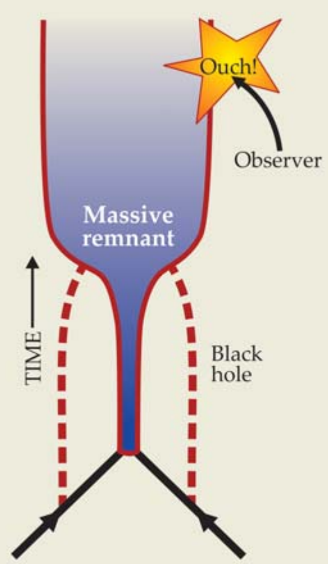

Figure 4. Massive-remnant scenarios are nonlocal. In these models, a black hole transitions to a massive object whose surface lies outside, or possibly at, the location of what would be the horizon (dashed lines on either side of the origin). In this illustration, the black hole is formed from the collision of two particles (black lines). To reach the horizon, the surface must propagate faster than the speed of light, which violates the locality of quantum field theory. An infalling observer encounters the remnant surface at a high velocity—compare falling into a neutron star—and, barring a miracle, experiences strong disruption. Variants of this general scenario include so-called fuzzballs and firewalls.

the massive remnant that replaces the black hole. A more recent variant is the firewall,[11] which describes a degenerate limit in which the surface of the remnant is right at the horizon.

Massive remnants are significantly different from conventional black holes. Unlike in the black hole case, for which an infalling observer would not notice anything dramatic at the horizon, an observer falling into a remnant would experience a painful impact on its surface, somewhat analogous to falling onto the surface of a neutron star. Moreover, one generically expects the reflection of energy from a massive remnant's surface. Some physicists have speculated that fuzzballs avoid painful interactions and reflection through a form of complementarity,[10] but skepticism rightfully remains.

Apparently, physics needs some nonlocality to accommodate unitary evolution and the escape of information from a black hole—at least with respect to the approximate semiclassical geometry. One might investigate what minimal deviations from the conventional physical description are required. Does unitarity require a massive remnant, such as a fuzzball or firewall, or can there be more mild forms of nonlocal information transfer?

One proposal is that a black hole slowly leaks information through small effects that would have a minimal impact on an infalling observer. I have been seeking to describe such evolution in a series of papers that parameterize quantum information transfer between subsystems corresponding to a black hole and its environment and to give specific models[12] that go beyond the important work of reference 6 and of Patrick Hayden and John Preskill.[13] A possibly key observation is that the necessary flow of information can be transmitted by as little as one quantum with wavelength of order $R(M)$ being emitted per amount of time $R(M)/c$. The relay of information to those quanta and their subsequent escape from the black hole, schematically illustrated in figure 5, need not do violence to an observer that encounters them.

Departure from locality is radical—it overthrows a basic pillar of QFT—but it seems to be the least radical approach to the conundrums of gravity. Nonlocal evolution has been repeatedly explored but is usually easily discarded because the modifications of QFT it implies lead to acausality and paradoxes. Any sensible nonlocal departure from local QFT would need to fit into a tightly constrained framework and in particular reproduce the predictions of QFT in everyday circumstances not involving black holes. The difficulties of suitably modifying QFT call into question our fundamental description in terms of spacetime. One alternative possibility is that Hilbert space is more fundamental and that spacetime and locality emerge as approximations from a network-like structure of Hilbert spaces.[12,14]

In my view, the need for a consistent, unitary quantum description of black holes (and also of cosmology) should be taken as an important guide to the new physical principles involved. For example, dynamics must furnish a gravitational analog of the unitary S-matrix consistent with important physical and mathematical constraints.[3]

The present unitarity crisis has similarities with the stability crisis of the classical atom. In that earlier chapter, classical physics gave consistent evolution—though not in agreement with experiment—until the electron reached the charge center. However, the correct description of the atom required introducing the fundamentally new principles of quantum mechanics to describe physics inside the Bohr radius. Those principles evolved out of initially ad hoc attempts to account for the physics of the atom, even at the price of sacrificing classical mechanics. Possibly, once again, the rigidity of structure surrounding correct physics will provide crucial guidance.

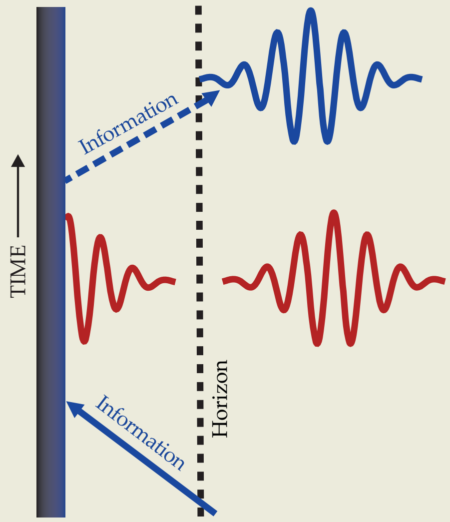

Figure 5. Information can leak slowly from a black hole in a proposed scenario that postulates a departure from the locality of the conventional quantum field theory picture.[12] Information from matter or Hawking excitations resides in the black hole for a finite time, but then emerges as quanta (blue waveform) in the near environment of the black hole that then escape to infinity and alter the usual pattern of Hawking radiation (represented by red quanta). The corresponding particles need not harm an infalling observer if they are created with sufficiently long wavelengths.

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