Fundamental Physics in the Third Millenium

M. A. Reynolds
Embry-Riddle Aeronautical University

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Preface

Over the past 400 years, we humans have delved deeper and deeper into the fundamental nature of the universe in which we live, the matter that it contains, and how that matter interacts. These steps have taken us from the four elements of Aristotle (earth, water, air, fire) to the periodic table of Mendeleev, to the "uncuttable"[1] particles of today — quarks and leptons. In this pamphlet I will lay out the basics of the standard model of particle physics along with an overview of nuclear physics, with as little mathematics as possible. However, math is the language in which the physics is expressed, so some equations are necessary. The concepts that we have learned from the mathematical theories can be expressed in words, but a warning is required: they are so different from the concepts of everyday, macroscopic objects that it can be very difficult to be precise with the language.

From a philosophical standpoint, I will adhere to the reductionist viewpoint of Steven Weinberg, as expressed in his book, "Dreams of a Final Theory." He makes the case that at this point in history the standard model of particle physics is the most fundamental theory of the natural world. This fundamental nature means that from an understanding of particle physics, one could in principle deduce the properties of nuclei, of atoms and molecules (chemistry), of biological systems, even of life. Of course, such a prescription is not practical, nor probably even possible. As Weinberg says,

> *Right now we do not know how to use our standard model of elementary particles to calculate the detailed properties of atomic nuclei, and we are not certain that we will ever know how to do these calculations, even with unlimited computer power at our disposal. Nevertheless, we have no doubt that the properties of atomic nuclei are what they are because of the known principles of the standard model. This "because" does not have to do with our ability actually to deduce anything but reflects our view of the order of nature.*

While particle physics does not have an answer to every question — we still don't know why there are three families of quarks, for example — it is the deepest truth and contains the most autonomous set of principles that we know of at the present time. For these reasons, I feel that it is incumbent on teachers to make sure that students know at least the central tenets of the standard model. Even if it makes no practical difference in their lives, it can enhance their sense of wonder with the universe, and with mankind's amazing ability of uncover such secrets.

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[1] See page 3

An Introduction to Particle Physics

> *The opinion seems to have got abroad that in a few years all the great physical constants will have been approximately estimated, and that the only occupation which will then be left to men of science will be to carry on these measurements to another place of decimals.* — James Clerk Maxwell, 1871

Physics circa 1900

In 1895 (before the discovery of X-rays,[2] radioactivity,[3] and the electron[4]) there were two *forces*: the gravitational force and electromagnetic force; there were two *object properties*: mass and charge; and there was one *dynamical law* determining how objects respond to those forces: Newton's law of motion. (Well, Newton actually enumerated three laws, but they act as one coherent group.) These, in principle, are all that you need to predict how objects will behave dynamically. The object properties determine the strength of the forces that act on the objects, and Newton's dynamical laws predict the future response to those forces. Thus, the universe was envisioned as a great clock—once started it would continue to run forever. In fact, if one were able to measure (with infinite precision, of course) the positions and velocities of all objects in the universe at a specific time (i.e., the "state" of the universe), then the laws of dynamics along with a knowledge of the forces would allow one to predict their future positions and velocities. This is known as the "mechanistic worldview" or the "Newtonian worldview."

In addition, the thermodynamic properties of matter and its interaction with light were relatively well understood. So much so, in fact, that in 1875 the head of the physics department at the University of Munich advised **Max Planck** [Nobel Prize, Physics, 1918], the future progenitor of quantum theory, to not study physics because, as he put it, "Physics is a branch of knowledge that is just about complete. The important discoveries, all of them, have been made. It is hardly worth entering physics anymore."

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[2] Wilhelm Conrad Roentgen discovered X-rays on November 8, 1895, and was awarded the first Nobel Prize in Physics for 1901.

[3] Henri Becquerel discovered the natural radioactivity of uranium in early 1896 while investigating X-rays, and shared the Nobel Prize in Physics for 1903 with Pierre and Marie Curie.

[4] Joseph John Thomson discovered the electron in 1897 and was awarded the Nobel Prize in Physics for 1906. In reality, Thomson measured the charge-to-mass ratio of the electron in 1897, and it wasn't until 1899 that he was able to make an independent measurement of its charge (and hence its mass); the latter date, therefore, can be more definitively called the date of discovery.

However, there was little understanding of what matter was made. No theory satisfactorily explained why a particular object was endowed with its particular values of mass and charge. Many elements (such as nitrogen and oxygen) were known, and each element had a known molar mass and volume density, but no underlying reason for these properties had been successfully proposed. As you might guess, there had been hints about the microscopic structure of matter. For instance, the atomic hypothesis had been around since Democritus (c. 400 BCE), who postulated that rather than being a continuum, matter was made up of small discrete objects called "atoms". The word *atoms* comes from the Greek word $\alpha\tau o\mu o\sigma$, which means "that which cannot be cut," or "uncuttable." However, this hypothesis was nothing more than supposition until John Dalton proposed his law of multiple proportions in 1803, which states that when two elements combine to form more than one compound, the ratios of the weights are ratios of small integers.

One of the clearest sets of data was the ratio of the amounts of oxygen and nitrogen needed to make various compounds.[5] Experiment showed that

$$\frac{m_O}{m_N} = 0.57,\ 1.13,\ 1.71,\ 2.29,\ 2.86 \tag{1}$$

for the five compounds nitrous oxide (N_2O), nitric oxide (NO), nitrous anhydride (N_2O_3), nitrogen dioxide (NO_2), and nitric anhydride (N_2O_5), respectively. The five ratios are very close to the integers 1:2:3:4:5. While this suggests that matter is made of discrete clumps, it would take another hundred years before the concept was accepted by the scientific community.[6] The discrete clumps turned out not to have exactly integer mass ratios, a fact that was first conclusively shown in 1920 by William Aston, who, along with Ernest Rutherford [Nobel Prize, Chemistry, 1908] developed an accurate mass spectrograph, and whose work included the discovery of isotopes of non-radioactive elements.

Physics circa 2000

The current view of the fundamental nature of matter and the ways in which it interacts is certainly more detailed than in 1895, and it is tempting to believe that we have reached "the end." However, while there are mathematical reasons that lead us to believe we might be near the "Theory of Everything," or a "Grand Unified Theory," past experience has at least humbled physicists of the present day and they understand that what we call "fundamental" today may turn out not to be. In fact, the situation today may be compared with that of 1895. We know of more (and smaller) particles, e.g., quarks, but, for example, we still have no idea *why* the quarks have fractional electric charge or *why* they have spin $\frac{1}{2}$, nor even *why* any of the particles have the masses they do.

We now know of four *forces*: the gravitational force and electromagnetic force, but also the strong nuclear force (or "color" force) and the weak nuclear force. We also can

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[5] Friedman and Sartori, *The Classical Atom*, 1965, page 1.

[6] For a detailed look at the history of the atomic concept, see Boorse and Motz, *The World of the Atom*, which contains reprints from Lucretius to Einstein concerning the existence of atoms and subatomic particles.

	smaller → (h)	
faster ↓	**Newton**	quantum
(c)	relativity	quantum field theory

Figure 1: A schematic diagram of dynamical theories. Newton's Laws are approximately valid when velocities are small compared with the speed of light, c, and another quantity, called "action," is large compared with Planck's constant, h. Otherwise, quantum mechanics or special relativity is needed, or perhaps both. When both are needed, the combination results in a "quantum field theory," such as Quantum Electrodynamics (QED) which describes electromagnetism, and Quantum Chromodynamics (QCD) which describes the strong/color force.

enumerate many more *properties* (or attributes) of subatomic particles: mass, charge, and color, which are related to the forces, as well as others that make sense only within the quantum description of matter, properties like spin and strangeness. Finally, we have expanded Newton's description of how these particles interact, with the result that his *dynamical laws* have been modified both on a small scale (quantum mechanics) and at large velocities (special relativity), as shown in Figure 1.

The theory of *relativity* and the theory of *quanta* are the two great theoretical constructs of the early 20th century.

If you are interested in the intersection of quantum mechanics and relativity—quantum field theory—you will likely have to go to graduate school because not only are advanced mathematical tools needed, but also a thorough grounding in relativity and nonrelativistic quantum mechanics.

Relativity

Intuition is something one develops on the basis of experience.
— Alfred Schild

The most important result of Einstein's Special Theory of Relativity is the equivalence of mass and energy. The "rest energy," E_0, of an object is given by

$$E_0 = mc^2, \tag{2}$$

where m is the mass of the object and c is the speed of light

$$\boxed{c = 299\ 792\ 458\ \text{m/s}}. \tag{3}$$

This value of c is exact—it has been defined as this value—but a useful approximation (keeping three significant digits) is $c \approx 3.00 \times 10^8$ m/s. In some sense, Eq. (2) defines how much energy is locked up in the mass of an object. More importantly, this equation expanded our understanding of the rules of the natural world by replacing two "laws" that were thought to be universal (the conservation of mass as the conservation of energy) with a third law that we now believe *is* universal (the conservation of the sum of mass and energy).

In 1789, the chemist Antoine Lavoisier was the first to show that matter was conserved in chemical reactions. That is, even though the compounds may change (e.g., liquid water can be turned into gaseous hydrogen and gaseous oxygen), the quantity of matter neither increases nor decreases. He discovered this rule by carefully measuring the weights of the reagents and the products in various chemical reactions (including the gases). Today, we call this the "Law of Conservation of Mass." In the 19th century, several physicists, James Joule among them, realized that there was another conserved quantity, energy. Joule, for instance, stirred water with a paddle, and, by carefully measuring the amount of mechanical work done by the paddle and the subsequent increase in temperature of the water, was able to demonstrate that there was a "mechanical equivalent of heat." Subsequently, with the discovery of other types of energy (electrical energy, wave energy, etc.), the principle that the total energy in the universe is constant came to be accepted, and prompted Rudolf Clausius, thermodynamicist, to say in 1865

> "*The energy of the universe is constant.*"

What Einstein said with Eq. (2) is that neither of those two laws are separately true, but that the *sum* of mass and energy is a constant. In reality, he discovered a new type of energy: rest energy. In fact, c^2 can be thought of simply as a conversion factor between joules, units normally used to measure energy, and kilograms, units normally used to measure mass. In an analysis of atomic, nuclear, and particle physics, a basic understanding of the physical processes involved can be obtained by keeping track of the transformation of mass to energy, and vice versa, and not worrying about the detailed dynamics. This is similar to analyzing collisions between objects by looking only at the momentum before and after the collision, but ignoring the details of the forces that caused those changes in momentum.

Quantum Mechanics

The implications of quantum mechanics are often much stranger than those of relativity. An important new philosophical result is that there are some questions that are not "askable" in quantum mechanics, in the sense that certain quantities cannot be measured at particular times. Before we get to that, however, let's start with the reason why it is called *quantum*.

In our classical, macroscopic world, properties of objects (both intrinsic properties such as mass, and extrinsic properties such as velocity) can take on any value among a continuous range of values—there is no restriction. In the subatomic quantum world, however, some properties of particles (though not all) are restricted to a discrete set of values—they are "quantized." The best known example is probably the energy of an electron that is bound in a hydrogen atom. Unlike a planet (or asteroid, or comet) orbiting the Sun, which can have any energy, the electron is restricted to occupy certain "energy levels." Each of these levels is assigned a "quantum number," and the electron's energy can be calculated from that quantum number. In this particular case the quantum number is n, and it can take on the values $n = 1, 2, 3, \ldots, \infty$. It is simply a label for the particular quantum "state" that the electron occupies. The electron's energy when it is in state n is given by the formula

$$E_n = \frac{E_1}{n^2}, \tag{4}$$

where $E_1 \approx -13.6$ eV,[7] and E_1 is the electron energy when it occupies state $n = 1$. The average radius, r_{ave}, of the electron's orbit is another physical property that can be calculated in terms of the quantum number n

$$r_{ave} = a_0\, n^2, \tag{5}$$

where $a_0 \approx 0.0529$ nm, and is called the "Bohr radius." This structure is ubiquitous in quantum mechanics:

> A quantum number labels the state that a particular particle is in, and the physical properties of that state can be calculated from a formula that depends on that quantum number.

In the case of the hydrogen atom, n is called the "principal" quantum number.

Wave-particle duality

One of the more confusing properties is the "wave-particle" duality of light. For example, energy is a property that is usually thought to apply to particles, and frequency is a property that is usually thought to apply to waves. Light (indeed, all particles) can be thought of as having both mutually exclusive characteristics—wave and particle—and which characteristic shows itself depends on the experiment. In fact, in order to correctly interpret some experiments, *both* characteristics must be invoked. Linus Pauling has stated the situtation clearly:

> Does light *really* consist of waves, or of particles? Is the electron *really* a particle, or is it a wave?
>
> These questions cannot be answered by one of the two stated alternatives. Light is the name that we have given to a part of nature. The name refers to all of the properties that light has, to all of the phenomena that are observed in a system containing light. Some of the properties of light resemble those of waves, and can be described in terms of a wavelength. Other properties

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[7] Recall that electron volts (eV) are just another energy unit, defined as the amount of energy gained by an electron when it falls through a potential difference of one volt, and that 1 eV$= 1.602\ 176\ 53(14) \times 10^{-19}$ J, or with our usual precision $\boxed{1 \text{ eV} \approx 1.60 \times 10^{-19} \text{ J}}$. Note that the energy of the electron in Eq. (4) is its total energy (kinetic plus electric potential) which is why it is negative—the convention is that that potential energy is zero when the separation of the electron and proton is infinitely large, and therefore the potential energy is always negative.

> of light resemble those of particles, and can be described in terms of a light quantum, having a certain amount of energy, $h\nu$.... A beam of light is neither a sequence of waves nor a stream of particles; it is both.
>
> In the same way, an electron is neither a particle nor a wave, in the ordinary sense. In many ways the behavior of electrons is similar to that expected of small spinning particles, with mass m, electric charge $-e$, and certain values of angular momentum and magnetic moment. But electrons differ from ordinary particles in that they also behave as though they have a wave character, with a wavelength given by the de Broglie equation. The electron, like the photon, has to be described as having the character both of a particle and of a wave.... You might ask two other questions: Do electrons exist? What do they look like?
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> The answer to the first question is that electrons do exist: "electron" is the name that scientists have used in discussing certain phenomena, such as the beam in the electric-discharge tube studies by J. J. Thomson, the carrier of the unit electric charge on the oil drops in Millikan's apparatus, the part that is added to the neutral fluorine atom to convert it into a fluoride ion. As to the second question—what does the electron look like?—we may say that some information has been obtained by studying the scattering of very-high-velocity electrons by protons and other atomic nuclei. These experiments have given much information about the size and structure of the nuclei, and have also shown that the electron behaves as a point particle, with no structure extending over a diameter as great as 0.1 fm.[8]

As early as 1909, Einstein was beginning to understand that this was to be the crux of any physical theory of light:

> It is my opinion that the next phase of theoretical physics will bring us a theory of light that can be interpreted as a kind of fusion of the wave and the [particle] theory.

Of course, it would take until 1926 for the term "photon" to be coined, and until 1927 for Compton to receive the Nobel prize for his X-ray experiments that definitively pinned down the particle nature of light. In fact, in the conclusion of Compton's 1923 paper[9] on the subject, he holds nothing back:

> [There is] little doubt that the scattering of X-rays is a quantum phenomenon.

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[8] Pauling, *General Chemistry*, pages 80-81. The current upper limit to the "diameter" of an electron is about 10^{-7} fm.

[9] Arthur H. Compton, "A quantum theory of the scattering of X-rays by light elements," *Phys. Rev.* **21** 483-502 (1923).

e^-	**electron**	Leptons
ν_e	electron neutrino	
μ^-	muon (mu lepton)	
ν_μ	muon neutrino	
τ^-	tauon (tau lepton)	
ν_τ	tau neutrino	
u	**up quark**	Quarks
d	**down quark**	
c	charm quark	
s	strange quark	
t	top (truth) quark	
b	bottom (beauty) quark	

Table 1: The twelve elementary particles that comprise all natural and man-made matter. The three particles in boldface — electron, up quark, and down quark — comprise all known natural matter. There are six leptons (three massive leptons and three massless neutrinos) and six *flavors* of quarks.

Matter

> *If I could remember the names of all the particles, I'd be a botanist.*
> — Enrico Fermi

At its most basic level, all matter consists of combinations of 12 elementary particles, which are listed in Table 1. They can be classified into two groups, leptons and quarks: quarks interact via the strong force but leptons do not. Both types of particles interact gravitationally (i.e., they all have mass) and via the weak force. Finally, all but the neutrinos interact electromagnetically because neutrinos are electrically neutral. The original motivation for the classification of leptons in 1947 was that the electron (the only known lepton at that time) was less massive than the proton and neutron (the only known nucleons—later determined to consist of quarks), and "lepton" is from a Greek word that means small or light. (See page 15.) Of course, after the discovery of the tau lepton in 1975 and the observation that it was almost twice as massive as a proton, the original reason no longer made sense. However, with the discovery of quarks and the fact that they are the only particles to interact via the strong force, the division into leptons and quarks is appropriate, albeit for reasons that have to do with forces rather than mass.[10]

Amazingly, all natural matter that we observe in the world around us consists of only three of these particles: electrons, up quarks, and down quarks. The atoms in our bodies are comprised of electrons as well as protons and neutrons, but the proton is made up of

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[10] In addition to these 12 particles, there are the so-called "exchange particles," like the photon (denoted by the symbol γ), that mediate the four forces. These particles are also called "gauge bosons," or "intermediate vector bosons," and they are not normally considered to be matter. I will discuss them below in the *Interactions* section on page 18.

2 up quarks and 1 down quark (commonly written 'uud'), while the neutron is 2 down quarks and 1 up quark (commonly written 'udd'). In this sense, the universe is very simple. There are only three particles, which combine in a myriad of ways to make up all the wonderful objects that we see: trees, rivers, oceans, mountains, planets, stars, and galaxies.

What are the intrinsic properties of these elementary particles? Two are very familiar, mass and electric charge, and others, such as color, spin, magnetic moment, strangeness, isotopic spin, lepton number, and baryon number, are not as familiar. The nomenclature of particle physics is very complicated, but if you remember to characterize particles based on their fundamental properties, like mass, charge, etc., it doesn't matter what they are called, you will be able to understand the physics of their interactions.

You may have noticed that I didn't mention size as an intrinsic property. The reason is that all of these elementary particles are thought to be point-like and have no size. For example, the size of an electron has been experimentally measured to be less than 10^{-22} meters![11] This simply means that the electric force that an electron feels is Coulombic (i.e., $\sim 1/r^2$) down to that distance, which means that there is no reason to think that electrons have any structure at any scale. Of course, when elementary particles combine to form protons, neutrons, atoms, and molecules, the physics of their interaction occurs on a spatial scale so that the conglomerations acquire a characteristic size and shape.

There is another characteristic of these particles that has no classical counterpart: they are identical and indistinguishable. Unlike our macroscopic world, where we can paint seemingly identical objects different colors to distinguish them (billiard balls, for example), in the microscopic world there is no way to tell two electrons apart. When a cue ball, say, collides with an eight-ball and they each move off in different directions, it is clear which ball is which after the collision. However, if two electrons collide and move off, the experimenter is not able to distinguish which electron is which after the collision. It turns out that this fact has far-reaching implications on the allowable motions of these particles. The most well-known implication is the Pauli exclusion principle that is applied to electrons within atomic orbitals.

Antimatter

> *Antimatter is as much matter as matter is matter.* — Abraham Pais[12]

For every particle, there is a corresponding "antiparticle," with the same mass, but opposite electric charge, and these are listed in Table 2. The antiparticles are denoted by an overbar, or sometimes by simply changing the sign, as with the positron. Do not ascribe any mysterious properties to antimatter. As Pais implies, from an antiparticle's point of view, *we* are made of "antimatter." In fact, current cosmological theories suggest that

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[11] Hans Dehmelt, "A Single Atomic Particle Forever Floating at Rest in Free Space: New Value for Electron Radius," *Phys. Scr.* **T22** 102-110 (1988)

[12] Abraham Pais is perhaps one of the foremost chroniclers of the story of modern physics. His writings, listed in the Bibliography, are all the more valuable because he was a practitioner — he worked on the front lines in 1940s through the 1970s — and he knew and collaborated with several of the key players personally, e.g., Bohr, Einstein, Heisenberg.

e^+	positron (anti electron)	anti Leptons
$\overline{\nu}_e$	anti electron neutrino	
μ^+	anti muon (mu lepton)	
$\overline{\nu}_\mu$	anti muon neutrino	
τ^+	anti tauon (tau lepton)	
$\overline{\nu}_\tau$	anti tau neutrino	
$\overline{\mathrm{u}}$	anti up quark	anti Quarks
$\overline{\mathrm{d}}$	anti down quark	
$\overline{\mathrm{c}}$	anti charm quark	
$\overline{\mathrm{s}}$	anti strange quark	
$\overline{\mathrm{t}}$	anti top (truth) quark	
$\overline{\mathrm{b}}$	anti bottom (beauty) quark	

Table 2: The twelve elementary antiparticles.

in the early universe, a short time after the Big Bang, there was approximately as much matter as antimatter. As the universe cooled, equal amounts of matter and antimatter were annihilated, and what was left over was the small amount of matter that makes up the visible universe. The question of why there was an asymmetry between the amounts of matter and antimatter (i.e., why there wasn't exactly the same amount of both kinds) is one that still has not been answered.

Why, then, does antimatter exist? No one knows, but that appears to be the way the universe is made. However, within the rules of our current structure of theoretical physics, antiparticles are a "necessary consequence of combining special relativity with quantum mechanics."[13] **Paul Dirac** [Nobel Prize, Physics, 1933] was the first to realize this fact when he attempted to construct a relativistic wave equation for the electron in 1928 (the Schrodinger equation was not relativistic). The mathematics implied the existence of positive electrons, which later turned out to be positrons.

Mass

A particle's mass indicates how strongly it interacts via the gravitational force. The mass of the electron is

$$m_e = 9.109\ 382\ 6\ (16) \times 10^{-31}\ \text{kg},$$

or, with our typical precision, $\boxed{m_e \approx 9.11 \times 10^{-31}\ \text{kg}}$. Rather than using the SI unit of kilogram, a common practice is to quote particle masses in terms of their "rest energy." Einstein's relativistic equivalence $E_0 = mc^2$ means that the electron's rest energy is $m_e c^2 \approx 8.19 \times 10^{-16}\ \text{J} \approx \boxed{0.511\ \text{MeV}}$. (Sometimes, physicists omit the factor c^2 because it is clear

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[13] Martin and Shaw, *Particle Physics*, p. 2.

from the context that the mass is being quoted in energy units.) It is common to quote a particle's rest energy in millions of electron volts (MeV), rather than Joules. The other massive leptons, the muon and tauon, are *identical* to the electron, except for their mass. The accepted values of the lepton masses are

$$\begin{aligned} m(e^-) &= \quad 0.510\ 998\ 910(13)\ \text{MeV} \\ m(\mu^-) &= \quad 105.658\ 3668(38)\ \ \text{MeV} \\ m(\tau^-) &= \quad 1\ 776.99(29)\ \ \text{MeV} \end{aligned}$$

The first thing to notice is the progression of larger masses with the μ^- and τ^- leptons. This increasing mass is characteristic of the quarks and neutrinos as well. In fact, there are three "families" (or generations) of leptons and quarks, each composed of a lepton, its corresponding neutrino, and two quarks. The following table organizes them in this way.

Leptons	Quarks	
e^-	u	
ν_e	d	light
μ^-	c	↓
ν_μ	s	↓
τ^-	t	heavy
ν_τ	b	

The first family is the lightest, and each successive family is heavier than the previous. Similar to the leptons, the top and bottom quarks are the most massive, and the up and down quarks are the least massive. The neutrino masses also increase, with ν_e the lightest and ν_τ the heaviest. We will ignore the neutrino masses, however, because they are very small (on the order of a few eV). In fact, experiments are only able to set upper limits on their masses, and currently they are

$$\begin{aligned} m(\nu_e) &< \quad 2.2 \quad \text{eV} \\ m(\nu_\mu) &< \quad 170 \quad \text{keV} \\ m(\nu_\tau) &< \quad 15.5 \quad \text{MeV} \end{aligned}$$

These mass limits can be determined in two ways. The first comes from the energy analysis of β decay, the prototype of which is the neutron decay in Eq. (8). These "direct" measurements yield the upper limits given above. The second, "indirect," method consists of analyzing cosmological data, specifically the cosmic microwave background, and determining what neutrino mass would result in a universe different from the one we observe. This method gives an upper bound to the *sum* of all three neutrino masses of about 0.3 eV. In this book I will always assume these masses to be so small as to be ignorable in our calculations.[14]

The quark masses are more problematic because quarks have never been observed in isolation, and therefore we can only infer their masses from theoretical arguments. That is,

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[14] In 1998, the SuperKamiokande neutrino experiment determined that the different types of neutrinos can change into each other, which automatically implies that they must have mass. See Dennis W. Sciama, "Consistent neutrino masses from cosmology and solar physics," *Nature* **348**, 617-618 (13 December 1990) for an interesting discussion.

measurements of energy released in particle reactions must be used along with a theoretical structure, such as QCD (quantum chromodynamics), in order to predict the quarks' "free" mass.[15] For example, the up quark has a "free" mass of about 3 MeV/c^2, and the down quark about 6 MeV/c^2. The other quark masses are listed in the table below.

quark	mass (GeV/c^2)
u	0.003
d	0.006
c	1.5
s	0.5
t	175[16]
b	4.5

Keep in mind that the values of these masses have large error bars, and that it really only makes sense to talk about the mass of particles that can exist in isolation. Particles that can be isolated, such as protons and neutrons, have masses that *can* be experimentally measured:

$$\begin{aligned} m_p c^2 &= 938.272\,029(80) \text{ MeV} \\ m_n c^2 &= 939.565\,360(81) \text{ MeV} \end{aligned}$$

Usually, we will not need to express them so precisely, so we can use $\boxed{m_p c^2 \approx 938 \text{ MeV}}$ and $\boxed{m_n c^2 \approx 940 \text{ MeV}}$. However, we shall see that the mass *difference* between them is critical, so it's important to remember that while they are both approximately 2000 times more massive than the electron, the neutron is slightly heavier than the proton.

If you look at the free masses of the up and down quarks, it's clear that the masses of the proton and neutron are not simply the sums of the masses of their constituent particles. How can that be? The reason is because there is a significant amount of potential energy involved in assembling the proton and neutron from the quarks, and this fact highlights the need to discuss our first "modern" concept in detail, that of binding energy.

Binding Energy

The binding energy B of a compound particle of mass M is defined as the difference between the sum of the masses m_i of the individual constituent particles and the mass of the compound particle multiplied by the square of the speed of light,

$$B \equiv \left(\sum_i m_i - M \right) c^2. \tag{6}$$

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[15] A quark's free mass is the mass we would theoretically expect it to have if it could be freed from the confines of the proton or neutron. However, because the quarks can't be isolated, their free mass depends sensitively on the theoretical assumptions made about the color force. The quarks's *constituent* masses, on the other hand, can be calculated in a straightforward manner using the concept of binding energy B, introduced below, and ignoring any potential energy due to the strong force. On the other hand, the free masses of nucleons (protons and neutrons) in nuclei *can* be determined using the binding energy concept — see page 24 — because they can be isolated.

[16] This has only been recently determined accurately, from a collision of a proton and anti-proton, each with about 1 TeV of kinetic energy.

The simplest example that illustrates binding energy is the deuteron (the nucleus of deuterium, ^{2}H, also known as heavy hydrogen), which consists of one proton and one neutron.[17] The deuteron's binding energy can be calculated from the measured rest energy of the deuteron and the (isolated) masses of the proton and neutron:

$$\begin{array}{rrl} m_p c^2 & 938.272\ 029(80) & \text{MeV} \\ +m_n c^2 & 939.565\ 360(81) & \text{MeV} \\ -m_D c^2 & 1\ 875.612\ 82(16) & \text{MeV} \\ \hline = B & 2.224\ 57(20) & \text{MeV} \end{array}$$

This means that if we are able to combine a free proton and a free neutron to make a deuteron, we obtain ≈ 2.22 MeV of energy in return[18] — in the language of chemistry, it's an *exothermic* reaction. Where does the released energy go? It goes into the kinetic energy of the compound particle! In fact, combining two nucleons into a single nucleon is called *fusion*, so named because two or more particles "fuse" to form one particle. A more complicated fusion reaction occurs in the core of the sun, where four protons fuse to form one α particle (the nucleus of helium, ^{4}He).[19] That reaction, of course, is also exothermic, and is what powers the sun. These considerations lead us to the conclusion that mass is a form of potential energy:

Mass (and binding energy) is potential energy.

Another, more familiar, example is the case of the Earth and a 1-kg ball. If these two objects are infinitely far away from each other, they have well-defined masses, M_E and $m = 1$ kg, that can be measured precisely. As we bring the ball to the surface of the Earth, the Earth-ball system loses potential energy. The amount lost can be calculated from our knowledge of gravitational potential energy

$$\begin{aligned} \Delta U = \frac{G M_E m}{R_E} &= 6.26 \times 10^7 \text{ J} \\ &= 3.89 \times 10^{26} \text{ eV} \\ &= 6.93 \times 10^{-10} \text{ kg } c^2, \end{aligned} \tag{7}$$

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[17] You might think that the proton or neutron would be simpler, but they are three-particle systems, not two, and more important, the binding energy is not well defined when the constituent particles cannot be isolated.

[18] There is another unit of mass that is commonly used when binding energy calculations are made, and that is the "atomic mass unit," or "u." Here, the carbon-12 atom sets the scale so that $m(^{12}\text{C}) \equiv 12.00$ u exactly, and the conversion to kilograms is 1 u = $1.660\ 538\ 86(28) \times 10^{-27}$ kg $\approx 1.66 \times 10^{-27}$ kg. The atomic mass approximately measures the "atomic number" of the nucleus, i.e., the number of protons and neutrons. Working in atomic mass units, but keeping only six decimal places, the calculation of the deuteron's binding energy is

$$\begin{array}{rl} m_p & 1.007\ 276 \text{ u} \\ +m_n & 1.008\ 664 \text{ u} \\ -m_D & 2.013\ 553 \text{ u} \\ \hline = B/c^2 & 0.002\ 388 \text{ u} \end{array}$$

and converting to electron volts (1 u ≈ 931.494 MeV/c^2) I obtain $B \approx 2.22$ MeV.

[19] Note that Eq. (6) does not state *how* the constituent particles combine to form the compound particle; other laws of physics are needed to determine that.

where I used the constants $\boxed{G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}}$, $M_E = 5.98 \times 10^{24}$ kg, and $R_E = 6.37 \times 10^6$ m. Where did that energy (≈ 63 MJ worth) go? It went into heat, sound, etc. The Earth eventually radiated away the heat energy, and the sound energy also travels off. This leads me to make the following claim:

CLAIM: The compound object (Earth and ball) has a smaller mass than the two separate objects combined!

The mass lost is exactly 6.93×10^{-10} kg, the mass equivalent of the potential energy difference. Of course, this mass difference is extremely tiny, and cannot be measured with present day experiments, but it must exist, nonetheless. If I were to separate the Earth and ball again, it would take 63 MJ of work, and when I measured their masses, they would "recover" their original masses, because I have put energy into the system with the work that I did to separate them.

While the underlying physics of binding energy and the mass of compound objects is identical in both the classical case (Earth and ball) and the subatomic case (proton and neutron), there are some subtle differences. In the classical case, the binding energy is small compared with the rest energies of the particles involved, and we tend to think of the constituent objects retaining their identity regardless of whether they are far apart or combined. However, with subatomic particles it is often the case that the binding energy is a significant fraction of the rest energies, and the compound object is usually considered to be a different object—the constituent particles lose their identity. For example, a proton "consists" of two up quarks and a down quark: uud. However, there is another compound particle, called Δ^+, which also consists of two up quarks and a down quark. But the mass of the Δ^+ is 1232 MeV, and it is considered to be a different particle from a proton. The mass is different because the three quarks are in a different quantum state than the proton (that is, they occupy a different energy level), which means that the proton and Δ^+ have different binding energies, and hence different masses.[20] On the other hand, when the 1-kg ball is on the surface of the Earth, we still consider the Earth and the ball to be separate, distinct, objects.

A final example of an interaction involving the mass-energy relationship (and antimatter) is the decay of the neutron. A free neutron (not one that is bound in an atomic nucleus) spontaneously decays into a proton with a half life of 10.23 minutes. The reaction equation is

$$n \rightarrow p + e^- + \bar{\nu}_e, \tag{8}$$

where the electron and antineutrino must be part of the decay products in order to conserve both charge and the "lepton number," a quantity that is characteristic of the weak force. The lepton number is simply another quantum number that must always be strictly conserved, similar to electric charge. Now, the neutron is NOT comprised of a proton and electron, so there is no binding energy, but we *can* calculate the energy released in this decay by computing the difference in rest energies before and after the decay

$$Q \equiv \left(\sum_{\text{initial}} m - \sum_{\text{final}} m \right) c^2. \tag{9}$$

---

[20] This difference in binding energies can be traced to a difference in the spins of the quarks.

The symbol Q (called "reaction energy") is used rather than B to denote that this is not a compound particle, but that there is some energy that is released in the reaction. If $Q > 0$ there is energy released (exothermic) and a spontaneous decay is energetically possible. However, if $Q < 0$ then simply because of energy conservation the decay is not allowed. In the case of the neutron decay, I obtain (and you should check the math) $Q \approx 0.782$ MeV.[21] What happens to this released energy? As before, it goes to the kinetic energy of the product particles, i.e., those on the right-hand-side of the reaction equation. In some sense, you can think of a neutron as being in a higher potential energy state than a proton (because it is more massive), and since objects like to lower their potential energy, the neutron wants to turn into a proton.[22]

Classification according to mass, and particle names

There were three known particles in the 1930s: the electron, proton and neutron. With a mass of about 0.5 MeV/c^2 the electron was the lightest, and with a mass of about 1000 MeV/c^2 the nucleons (the common name for the proton and neutron) were heavy. With the discovery in 1937 of an intermediate mass particle, about 100 MeV/c^2, in cosmic rays, the particles were given "nicknames" according to their mass. Since the electron was light, it was called a "lepton," from the Greek word *leptos* ($\lambda\varepsilon\pi\tau o\zeta$) meaning "small." And, since the nucleons were massive, they were called "baryons," from the Greek *barys* ($\beta\alpha\rho\upsilon\zeta$) meaning "heavy." The cosmic ray particle was therefore called a "meson," from the Greek word *mesos* ($\mu\varepsilon\sigma o\zeta$) meaning "middle."[23] It wasn't realized until later that the intermediate mass cosmic ray particle was actually the μ^- lepton, although it was originally called the μ-meson.

Under our current naming scheme, however, a baryon has come to mean a particle that is made up of three quarks (such as a proton or neutron), a meson has come to mean a particle that is made up of a quark–anti-quark pair, and leptons are the elementary particles that do not interact via the strong force. Since any three of the six flavors of quarks can combine to form a baryon, there are 56 possible combinations, although there are more than 56 different baryons since it is possible for the same set of quarks to have different binding energies (see the comparison between the proton and Δ^+ above). For example, the sigma (Σ) baryons are combinations of one strange quark and two up or down quarks. Their quark content and masses are listed here:

Σ^+	uus	1189.4 MeV
Σ^0	uds	1192.5 MeV
Σ^-	dds	1197.4 MeV

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[21] In calculating this value, I ignored the small neutrino mass. Since the upper limit on the rest energy of the electron neutrino (and antineutrino) is about 2.2 eV, it doesn't affect the calculation at this level of precision.

[22] As far as we know, the proton is a stable particle because there is no baryon that is less massive for it to decay into, although the possibility that the half-life for proton decay is so long that we haven't noticed it yet is an active area of research. Baryon number is another quantum number that must be conserved. The electron is the lightest lepton and hence it, too, is stable against spontaneous decay.

[23] Interestingly, Hideki Yukawa, who predicted the existence of an intermediate-mass particle in 1935, initially proposed to call it a "mesotron," in keeping with the name of the electron. However, Werner Heisenberg noted that the correct Greek word was *mesos* and it had no "tr."

baryon		mass (MeV/c^2)
p	uud	938.3
n	udd	940.6
Σ^+	uus	1189.4
Σ^0, Λ^0	uds	1192.5, 1115.7
Σ^-	dds	1197.4
Ξ^-	dss	1321.7
Ξ^0	uss	1314.9

meson		mass (MeV/c^2)
π^+	$\mathrm{u\bar{d}}$	139.6
π^0	$(\mathrm{u\bar{u}}\text{-}\mathrm{d\bar{d}})/\sqrt{2}$	135.0
π^-	$\mathrm{d\bar{u}}$	139.6
K^0, $\bar{K}^0$	$\mathrm{d\bar{s}}$, $\mathrm{\bar{d}s}$	497.6
K^+, K^-	$\mathrm{u\bar{s}}$, $\mathrm{\bar{u}s}$	493.7
η	$(\mathrm{u\bar{u}}+\mathrm{d\bar{d}}\text{-}2\mathrm{s\bar{s}})/\sqrt{6}$	547.9
η'	$(\mathrm{u\bar{u}}+\mathrm{d\bar{d}}+\mathrm{s\bar{s}})/\sqrt{3}$	957.7

Table 3: Tables of the light (u, d, s quarks only), spin $\frac{1}{2}$ baryons and the light, spin 0 mesons. Note that the Σ^0 and Λ^0 have the same quark content but different masses. The heavier one, Σ^0, is an electromagnetic excited state and decays in about 7×10^{-20} s into the lighter one, Λ^0. This process is identical to that which occurs when an electron in an excited state (of higher energy) in an atom decays into a lower energy level. In both cases the decay is accompanied by the emission of a photon equal to the energy difference. Here, the energy difference is indicated by the mass difference, and a gamma ray of wavelength 2.57 fm is emitted. Also note that the π^0, η, and η' are all neutral mesons, but are just different linear combinations of the same set of three quark—anti-quark pairs.

The pi (π) mesons are composed of different combinations of up and down quarks and their anti-particles:

π^+	$\mathrm{u\bar{d}}$	139.6 MeV
π^0	$(\mathrm{u\bar{u}}\text{-}\mathrm{d\bar{d}})/\sqrt{2}$	135.0 MeV
π^-	$\mathrm{d\bar{u}}$	139.6 MeV

Note that the π^0 meson is actually a *superposition* of quark states. This means that when an experimenter "looks" at a π^0 meson, 50% of the time they will "see" the $\mathrm{u\bar{u}}$ combination, and the other 50% they will see $\mathrm{d\bar{d}}$. The factor of $\sqrt{2}$ indicates this mathematically.[24] This is just one of the strange features of quantum mechanics. Some quarks (and baryons and mesons) can be linear combinations of two (or more) independent quark states. Also note that the π^+ and π^- are antiparticles of each other, and hence have the same mass, and that the π^0 is its own antiparticle.

The Σ and π particles are just a few of the possible baryon and meson combinations that can be constructed with the six known quark flavors. A short list, along with their quark constituents, are shown in Table 3. At this time, no other combinations of quarks other than qqq and $\mathrm{q\bar{q}}$ have been observed, although there have been searches for exotic combinations such as so-called "penta-quarks," made up of four quarks and one anti-quark: $\mathrm{qqqq\bar{q}}$. In some sense this looks like a baryon and meson bound together. Either these do not exist, or their lifetimes are too short to measure.

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[24] The probability of each state occurring is equal to the square of the numerical coefficient that multiplies that state.

Electric Charge

A particle's charge indicates how strongly it interacts via the electromagnetic force. In addition, however, charge is quantized; that is, it appears in nature only as integer multiples of the fundamental unit of charge, e, which happens to be the charge of the electron,

$$q_e = -e = -1.602\ 176\ 53(14) \times 10^{-19}\ \mathrm{C},$$

or, for our purposes $\boxed{e \approx 1.60 \times 10^{-19}\ \mathrm{C}}$.[25] The other massive leptons (muon and tau) have the same negative charge as the electron, and the neutrinos are neutral. In fact the word neutrino was proposed by Wolfgang Pauli in 1930, and means "little neutral one" in Italian.

What about the quarks? What are their charges? The quarks come with *fractional* charges, that is, submultiples of e! For example, the charge on the up quark is $q_u = +\frac{2}{3}e$, and that on the down quark is $q_d = -\frac{1}{3}e$. At first sight, this appears strange. How can any particle have a fractional charge? There are two ways to reconcile this with the proposed quantization of charge. First, all this really says is that the fundamental unit of charge is not e, but is $\frac{1}{3}e$. Charge is still quantized and all particles have integer multiples of this fundamental unit. Second, because quarks are never observed in isolation (they always appear in groups of 3 — baryons — or in a quark–anti-quark pair — mesons), the charges of particles that *can* exist in isolation must be a multiple of e. So the proton and neutron have integer charges

$$\begin{aligned} q_p &= \left(+\frac{2}{3}+\frac{2}{3}-\frac{1}{3}\right)e = e \\ q_n &= \left(+\frac{2}{3}-\frac{1}{3}-\frac{1}{3}\right)e = 0. \end{aligned}$$

This second fact was helpful in convincing skeptics about the usefulness, and the ultimate reality, of quarks. The charges on the quarks are

u	c	t	$+\frac{2}{3}e$
d	s	b	$-\frac{1}{3}e$

One important fact about electric charge is that it is absolutely conserved. There is no way to transform charge into energy, as there is with mass, so the charge of compound particles is just the sum of the charges of the constituent particles. This conservation law is sometimes stated as

$$\boxed{\text{Electric charge is neither created nor destroyed.}}$$

Why? We don't know. All we know is that the violation has never been observed, so until then it remains a "law."

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[25] Keep in mind that e is a positive quantity, and that negative particles have charges that are integer multiples of $-e$.

Interactions

Gravity and electromagnetism are the two classical (non-quantum) forces. The other two "forces," the weak and strong nuclear forces are inherently quantum mechanical in nature. For this reason, you won't be able to fully understand them in detail until *after* a thorough study of quantum mechanics; however, we can discuss them now using some of the classical concepts that you already know, such as energy and momentum. This quantum nature leads to a new way of describing and understanding these forces that is completely different from our previous descriptions. Previously, you have learned about forces in two different ways. First, as "action-at-a-distance," propounded by Newton with his Universal Law of Gravitation.[26] Second, utilizing the concept of a "field," devised by Faraday (and honed by Maxwell) to explain the electric and magnetic forces. Gravity can also be described in terms of the gravitational "field," both in the Newtonian limit and in general relativity. Due to the necessity of using quantum ideas to describe the weak and strong nuclear forces, we are forced to use quantum field theory, and this third description postulates the existence of exchange particles.

For example, two electrons repel each other not because of a mysterious action-at-a-distance Coulomb force, nor even the electric field, but by exchanging photons. Just like two ice skaters who, throwing a ball back and forth, appear to repel each other, electrons exchange photons, which, due to the conservation of momentum, exert impulses on the electrons, and they appear to repel each other. The photon, therefore, is the exchange particle that "mediates" the electromagnetic force. The ice skater analogy does not work for particles that attract each other, but the concept is still valid. In his thinking that led to the proposal of the meson, the mediating particle that held the nucleons together in the nucleus, **Hideki Yukawa** [Nobel Prize, Physics, 1949] wrote

> If one visualizes the [nuclear] force field as a game of "catch" between protons and neutrons, the crux of the problem would be the nature of the "ball" or particle.

This view has three aesthetically pleasing features. First, all interactions are "local," which means that particles must be in the same location for any effect. Second, it nicely explains the $1/r^2$ nature of the electric and gravitational forces: the "density" of mediating particles must decrease as $1/r^2$ from the "source" particle, a simple geometrical effect. Finally, effects are not instantaneous, but take a finite time as the mediating particle traverses the intervening distance. A graphical method of describing interactions that automatically displays the first and third of these features is called a "Feynman diagram." A Feynman diagram of the electromagnetic interaction between two electrons is shown in Fig. 2. This is similar to a position-time graph from elementary mechanics, where the

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[26] Newton had a philosophical objection to action-at-a-distance, which he expressed in a letter in 1692: "*That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else by which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it.*"

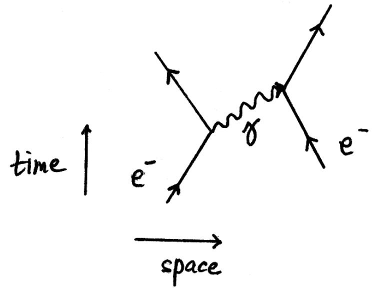

Figure 2: Feynman diagram depicting the electromagnetic interaction between two electrons. Only one spatial dimension is shown, and the time axis is conventionally drawn upward. Matter particles are depicted by straight lines, while the photon, an exchange particle, is shown as a wavy line and indicated by a γ.

trajectories of all particles are shown. Note that the photon (γ) comes into and out of existence when it interacts with an electron, and each electron undergoes a momentum change. At each "vertex" all quantities such as charge and other quantum numbers are conserved, the only exception being energy. That is, energy is created when the photon is emitted by the first electron, and then lost when the photon is absorbed by the second electron. The time interval over which the photon exists (and during which energy conservation is violated) is short enough so that Heisenberg's uncertainty principle is not violated. A photon of this type is called a "virtual" photon, so that in this third picture of interacting electrons, they do not exert a Coulomb force (at a distance), nor do they create an electric field, but they exchange virtual photons in order to exchange momentum and repel each other.

To the extent that each of the fundamental forces can be described by a quantum field theory, each force must be mediated by an exchange particle. If the photon mediates the electromagnetic force, what particles mediate the other forces? They are listed below, along with their mass, charge, spin, and color.

		mass	charge	spin	color
gravity	graviton[27]	0	0	2	no
E&M	photon	0[28]	0	1	no
color	gluons	0	0	1	yes
weak	$W^{\pm}$	80.4 GeV/c^2	$\pm e$	1	no
	Z^0	91.2 GeV/c^2	0	1	no

Our "zoo" of particles is now complete. We have 12 particles of matter, 12 of anti-matter, and 13 "gauge bosons." (There are 8 types of gluons, which are distinguished because they also carry color.)

It turns out that when the mass of the mediating particle is zero, then the interaction is long range. This makes sense for gravity and electromagnetism, in that they both are

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[27] The graviton, while postulated to exist, has not yet been observed.

[28] The current upper bound for the photon mass is 1.2×10^{-51} g. (Luo *et al.*, "New experimental limit on the photon rest mass with a rotating torsion balance," Phys. Rev. Lett. **90** 081801, 2003)

$1/r^2$ forces which means that although they become weaker with distance, they never go to zero. The weak force, on the other hand, is extremely short range because the W and Z bosons are very massive. This means that the weak force is very "weak" (hence the name) and particles must be very close to interact in this manner. The color force is also long range, but it turns out to become stronger with distance rather than weaker. The strong force, which is the force between baryons and mesons, is a short range force that is the residual, or "leftover," color force between objects that are color neutral.

Color Charge

To interact gravitationally, an object must have mass, and to interact electrically it must have electric charge. There is only one kind of mass, but there are two kinds of electric charge (positive and negative). The force between quarks is called the color force, and for a particle to feel the color force, it must have "color charge." In this case, however, there are three kinds of color charge: red, green, and blue.[29] Just like equal positive and negative electric charges "cancel" and result in a charge neutral object, so too a combination of all three colors results in a color neutral object that does not interact via the color force. The three colors, while they have nothing to do with the actual colors of light, were chosen because of the property that these three colors added together make white (something without color).

The color, therefore, is a new quantum number of the quarks. An up quark, for example, can either be in the red state, the green state, or the blue state. Anti-quarks come in "anti-colors," which can be thought of the complementary color on the color wheel; i.e., anti-red is equivalent to mixing green and blue, which gives cyan. The colors magenta and yellow are the anti-colors of green and blue, respectively. Since quarks are never seen in isolation, this means that bare color is never seen, and quarks must exist only in combinations that are color neutral. The only such combinations are three quarks (qqq), each with a different color, or a quark–anti-quark pair ($\mathrm{q\bar{q}}$), with a color and its anti-color. These, of course, are just baryons and mesons.

For example, the quarks in the π^+ meson ($\mathrm{u\bar{d}}$) must be anti colors. However, which colors does it choose? Red and cyan? Green and magenta? Blue and yellow? In fact, the mesons are composed of linear combinations (superpositions) of all three possibilities. We can write

$$\pi^+ = \frac{1}{\sqrt{3}}\left(u_r\bar{d}_r + u_g\bar{d}_g + u_b\bar{d}_b\right), \tag{10}$$

where the $\sqrt{3}$ is there just to make sure that we are counting only one quark. Essentially, the π^+ meson can be thought of consisting of $33\frac{1}{3}\%$ of each color combination.

This is exactly analogous to the logic inherent when Pauli first proposed his exclusion principle in 1924:

> In the atom there can never be two or more equivalent electrons for which ... the values of all quantum numbers coincide. If there is an electron in the

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[29] Originally, the three colors were red, white, and blue, but the concept of color neutrality is more pleasing using the well-known primary colors.

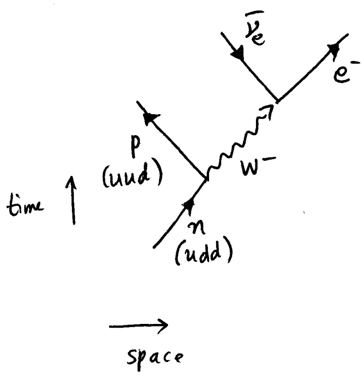

Figure 3: Feynman diagram of neutron decay. Note that only one d quark in the neutron decays into a u quark, and the other d quark and the one u quark are "just along for the ride."

> atom for which these quantum numbers have definite values then the state is "occupied."

He is essentially saying that no more than *two* electrons are allowed in each quantum state. In fact, only *one* fermion[30] is allowed per quantum state, so that meant that there must be another quantum number for the electron in an atom (beyond the three that were known) that could take on two possible values. It turned out that this quantum number was m_s, the z component of the electron's spin, which could take on the values $\pm\frac{1}{2}$. That is, the electron could be in either a spin up state, or a spin down state. With color, the logic is the same: there must be another quantum number that "allows" three identical particles to be in the "same state." In reality, since only one fermion per state is allowed, the new quantum number must take on three different values. This idea of color was proposed independently by Oscar Greenberg in 1964, and M. Y. Han and Yoichiro Nambu in 1965.

Weak force

Of the four fundamental forces, the weak force is the most difficult to describe in simple mathematical terms. It does, however, have one feature that none of the other forces have: it can change quarks and leptons from one flavor to another. Because the exchange particles ($W^\pm$ and Z^0) are so massive, the weak force acts over extremely short distances—so short, in fact that the interactions appear to be point-like, and the existence of the $W^\pm$ and Z^0 particles must be inferred from their decay products.

One example of the weak interaction is the radioactive decay of the neutron on page 14. One of the down quarks in the neutron is transformed into an up quark (making a proton), but in the process a W^- particle is created, which then decays into an electron and antineutrino. This reaction can most easily be described graphically by means of a Feynman diagram, shown in Fig. 3. The diagram depicts the decay process with the

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[30] See page 42 for a more detailed look at fermions and bosons.

spatial dimension on the horizontal axis and time running vertically. First, the neutron is transformed into a proton and a W^-: $n \rightarrow p + W^-$, and then the W^- decays: $W^- \rightarrow e^- + \bar{\nu}_e$. The net reaction is, of course, identical to Eq. (8). Note that at each vertex in the Feynman diagram electric charge is conserved. In addition, "lepton number" is also conserved at each vertex (lepton number is a quantum number assigned so that leptons have a lepton number of 1, and anti-leptons have a lepton number of -1). However, the mass is *not* conserved at each vertex: the extra mass of the W^- violates the law of the conservation of mass and energy, but does so only for a short time in accordance with the Heisenberg uncertainty principle.

One other strange thing you may notice about the Feynman diagram is that the antineutrino is depicted with an arrow directed backward in time. This is because in quantum field theories anti-particles can be thought of as particles moving backward in time. On that weird note, we now turn to more mundane matters—nuclear physics—where we ignore the sub-nuclear particles and concentrate on protons and neutrons, and on the nuclei that they comprise.

An Introduction to Nuclear Physics

For a nucleus to be stable it must have a mass which is less than the combined masses of any pair of nuclei made by subdividing it. — Hans Bethe

nucleon a proton or a neutron
nuclide a specific nucleus with Z protons and N neutrons (plural: nuclides or nuclei)
isotopes nuclides with identical Z but different N
isotones nuclides with identical N but different Z
isobars nuclides with identical A
isomer a nuclide in an excited state

The nuclei of atoms of ordinary matter consist of protons and neutrons. The atomic number Z is the number of protons in a nucleus, and N is the number of neutrons. The sum is $A = Z + N$, which is called the atomic mass number. Unlike chemical (or atomic) properties, which are determined solely by Z (because Z is also the number of electrons in the atom, and the interactions between these electron *are* chemistry), the nuclear properties depend on both the proton *and* neutron number. This is because the forces through which the nucleons interact, in addition to the electromagnetic force, are the strong and weak nuclear forces. Because they consist of quarks, both protons and neutrons interact via these nuclear forces. In fact, both protons and neutrons (nucleons) interact identically via the strong nuclear force because they have the same "strong charge."

The notation for an isotope of element X is ${}^{A}_{Z}\mathrm{X}_{N}$, which is usually shortened to ${}^{A}\mathrm{X}$. For example, the common isotope of helium, denoted ${}^{4}\mathrm{He}$, consists of 2 protons and 2 neutrons. The fact that it is helium automatically means $Z = 2$, and the number of neutrons can be determined from the values of A and Z ($N = A - Z = 2$). The less common isotope of helium is ${}^{3}\mathrm{He}$, pronounced "helium-3," which consists of 2 protons and 1 neutron. Our first task is to investigate the intrinsic properties of nuclei in the same way we looked at the elementary particles. The relevant properties are also the same: mass, electric charge and color. Color is straightforward — as we saw in our discussion of baryons above, they are all color neutral (i.e., white). Therefore, all nuclei are also color neutral. Electric charge is also simple: since each nucleus consists of Z protons, and each proton has an electric charge of $+e$, the total charge of a nucleus with Z protons and N neutrons is

$$Q_{Z,N} = +Ze. \tag{11}$$

Mass

The mass of a nuclide with Z protons and N neutrons (which I will denote by $M_{Z,N}$) is approximately given by

$$M_{Z,N} \approx Zm_p + Nm_n. \tag{12}$$

The equality is not exact because each nucleus has some binding energy (see Eq. 6). But the fact that it is a good approximation is one of the clues that led to the discovery of the periodic table—elements had atomic weights that were almost integer multiples of the atomic weight of hydrogen. For a nucleus, this binding energy is defined as

$$B(\text{nucleus}) \equiv (Zm_p + Nm_n - M_{Z,N})\, c^2. \tag{13}$$

As discussed on page 12, the constituent particles are taken to be nucleons, rather than quarks.

For example, the ^{4}He nucleus (α particle) is one of the most tightly bound nuclei, which can be seen by using the known masses of helium and hydrogen in the calculation of the binding energy:

$2 \times m_{\text{H}}$	$2\times$	1.007 825 u
$+2 \times m_n$	$2\times$	1.008 665 u
$-m_{\text{He}}$		4.002 603 u
$= B/c^2$		0.030 377 u

Converting the atomic mass units (u) to MeV results in 28.296 MeV. Since $A = 4$, this binding energy is 7.07 MeV *per nucleon*, or, as it is commonly denoted $B/A = 7.07$ MeV.[31] In this calculation, I used *atomic* masses (rather than nuclear masses) which is fine because the number of electrons on the right-hand-side of Eq. (13) cancel (hydrogen has one electron per atom and helium has two), and the electron binding energy is smaller than the uncertainty in the atomic masses.

Remember that the binding energy is a theoretical construct which says how much energy would be released *if we were able to break the compound particle apart into its constituent nucleons.* However, it is not usually possible to construct a compound particle simply by "fusing" the constituent particles. Let's take helium as an example. How is it actually created? In the core of the Sun ^{4}He is produced in a series of nuclear fusion reactions called the "proton-proton chain," and the net result of these reactions is

$$4p \rightarrow \alpha + 2e^+ + 2\nu_e + 2\gamma, \tag{14}$$

where the "α-particle" is the common name for the nucleus of ^{4}He. It is four *protons* (not two protons and two neutrons) that fuse together, but in the process (which must involve the weak interaction) two of those protons are converted to neutrons, plus the requisite positrons, neutrinos, and photons.[32] How much energy is released in this reaction? That

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[31] The binding energies of nuclei are more commonly expressed as the binding energy per nucleon, B/A, rather than just the binding energy, B, because B/A gives information on whether a given reaction (e.g., fission or fusion) is exothermic or endothermic.

[32] Note that both electric charge and lepton number are conserved, as they must be.

is, what is the Q value? Using the proton and α-particle masses (i.e., the nuclear masses), and not including the neutrino masses in Eq. (14), I get $Q/c^2 = 0.026501$ u, or $Q = 24.685$ MeV. Since Q is positive, this is an exothermic reaction.

However, if we want to analyze carefully what happens to the positrons, it's possible to use atomic masses. So, let's add four electrons to each side of the reaction in (14)

$$\begin{array}{rcl} 4p & \rightarrow & \alpha \quad + 2e^+ + 2\nu_e + 2\gamma \\ (+4e^- & \rightarrow & 2e^- + 2e^-) \end{array} \tag{15}$$

where I have grouped the electrons with their respective nuclei, but there are still two electrons left over. The positrons that were created tend to annihilate with any electrons nearby and this annihilation process results in the creation of four photons. So, in reality the net reaction is: four hydrogen *atoms* are converted into one helium *atom* plus six photons (which escape the Sun and illuminate the Earth) and two neutrinos (which head off into space and rarely interact with matter)

$$4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 6\gamma + 2\nu_e. \tag{16}$$

Of course, it's too hot and dense for neutral atoms to exist in the solar core, so again this reaction equation is a theoretical construct that allows us to properly take into account all of the energy released. Finally, therefore, we can calculate Q using atomic masses (it is simply the mass difference between one ^{4}He atom and four ^{1}H atoms), which is 26.731 MeV. This is not the same as the (theoretical) binding energy, but is the (practical) energy released. Where does this energy go? Most of it is taken away by the photons, but each neutrino carries 0.26 MeV away, on average, and this 0.52 MeV is lost forever as the neutrinos leave the sun. (Neutrinos can pass through about one light year of lead before having a significant probability of reacting.) Hence, the final energy that is available to illuminate and heat the Earth is about 26.21 MeV per net fusion reaction.

Another measure of the binding energy of a nucleus is its *mass excess*, Δ, defined as

$$\Delta \equiv M_{Z,N} - A \times (1 \text{ u}). \tag{17}$$

The dimension of Δ is mass, and therefore the dimension of Δc^2 is energy. In standard tables, such as NUBASE, the mass excess is listed in keV (rather than the actual mass or rest energy).[33] In reality, then, Δc^2 is given, and if you wish you can calculate $M_{Z,N}$ from Eq. (17).

The stability of a given nucleus can be determined using the criteria proposed by Hans Bethe in the quote on the top of page 23: for a nucleus to be unstable, it is only necessary to find *one* pair of nuclei whose combined masses are less than the nucleus in question. That is, if the value of Q is positive for the reaction where the nucleus in question splits, then the nucleus is unstable. In particular, it is found that there are no stable nuclei with $A = 5$ or $A = 8$, a fact that is extremely important in the explanation of element formation in the early universe. Let's investigate the possible isobars with $A = 5$: ^{5}H,

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[33] A third way to characterize the nuclide mass is by its "packing fraction," f, where $f \equiv \Delta/A$. This was first proposed in 1915 by Harkness and Wilson [*J. Amer. Chem. Soc.* **37** 1367 (1915)] while trying to understand why isotopes had masses that differed from integral multiples of the hydrogen mass.

^{5}He, ^{5}Li, ^{5}Be, and ^{5}B. The helium and lithium are unstable to the emission of a neutron and proton, respectively,

$$\begin{aligned} ^5\mathrm{He} &\rightarrow {}^4\mathrm{He} + n, \\ ^5\mathrm{Li} &\rightarrow {}^4\mathrm{He} + p, \end{aligned} \tag{18}$$

with reaction energies of 893.8 keV and 1.966 MeV, respectively, and half-lives of 700 ys and 370 ys,[34] respectively. ^{5}Be and ^{5}B have so many protons that the repulsive electric force overwhelms the attractive strong force. Finally, ^{5}H has too many neutrons, and decays via *double* neutron emission

$$^5\mathrm{H} \rightarrow {}^3\mathrm{H} + 2n. \tag{19}$$

Fusion

How did all the elements that we see around us come into being? Some of the stable nuclides are the end products of radioactive decay chains, which started at larger, heavier nuclei. But how were those larger, heavier nuclei formed? Why do the different elements (and isotopes) have the proportions in nature that they do? Why is the universe mostly composed of hydrogen (75%), with helium at 23% and the other elements existing as traces in the last 2%?

The answers to these questions come in three parts. According to the Big Bang model (by far the best explanatory model that we have of how the universe came to be), in the first three minutes, the originally hot "soup" of protons, neutrons, electrons, and photons, cooled and allowed some of these nucleons to fuse and form the nuclei of deuterium, helium-3 and helium-4. This process is called "Big Bang nucleosynthesis." As the universe continued to cool, the electrons were able to bind with the nuclei to form atoms. Then, stars and galaxies formed, and fusion reactions took place in the cores of stars (where it was hot enough), and the hydrogen and helium formed heavier elements such as carbon, nitrogen, and oxygen, on up to iron ($Z = 26$). This process is called "stellar nucleosynthesis." Finally, as the more massive stars ran out of nuclear fuel and gravity won out and the stars collapsed, the end result was a large explosion which was able to produce more fusion reactions and create the elements that are heavier than iron, e.g., uranium. This process is called "supernova nucleosynthesis."

Each of these three processes are complex and I can only briefly describe them here, but we can understand the basics with the knowledge we have of the electric and strong forces, and the knowledge we have of binding energy. How does a fusion reaction actually happen? A good analogy is to think of the two reacting nuclei as small magnets covered with two-sided tape. When they are far apart, the poles of the magnets are oriented in such a way that they repel each other (this repulsion is identical to the electric repulsion felt by two positively charged particles). However, if they get close enough for the tape to touch, the stickiness of the tape is strong enough to bind them together. This short-range attractive force is similar to the strong nuclear force in two ways: first, it only acts over

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[34] 1 ys = 1 yoctosecond = 10^{-24} s.

very short distances, and second, it is strong enough to overpower the repulsive electric force.

Big Bang nucleosynthesis

In 1948 Ralph Alpher, Hans Bethe, and George Gamow proposed that the process of *successive neutron capture* built up the nuclei from protons and neutrons. First, deuterons are produced

$$p + n \to d + \gamma, \tag{20}$$

and then tritium

$$d + n \to t + \gamma. \tag{21}$$

Only the nucleus is referred to in these reactions, given that the temperature was much too hot for atoms to be stable. (See Table 4 on page 28 for a description of the symbols.) These reactions occurred easily because the reactants (on the left side) do not repel each other electrically. For this reason alone, neutron capture is a very efficient process. Initially, in the hot *primordial soup*, the reverse reactions happened just as easily (for example a deuteron absorbing a photon and splitting into a proton and a neutron) so that there is an equilibrium between the number of protons, neutrons, deuterons, tritons (and, of course, photons). However, as the universe cooled, the typical photon energy was not high enough to split a deuteron, and only the forward reactions occurred. At this time, the tritons hung around long enough for some of them to decay into helions

$$t \to h + e^- + \bar{\nu}_e, \tag{22}$$

where this radioactive decay has a half life of $\tau = 12.32$ years.[35] The helions are, of course, stable, so they can capture another neutron and create the nucleus of ^{4}He

$$h + n \to \alpha + \gamma. \tag{23}$$

Does this process continue indefinitely? No. Because now there is a problem—there are no stable nuclei with $A = 5$. If ^{4}He were to capture a neutron, the resulting ^{5}He would re-emit that neutron in 700 ys, as we saw above on page 26. In the simplistic picture, this is where Big Bang nucleosynthesis ends, with an equilibrium abundance of four stable atoms: ^{1}H, ^{2}H, ^{3}He, and ^{4}He, with a relative abundance of 76% ^{1}H and 24% ^{4}He. This is called the primordial abundance.[36]

Stellar nucleosynthesis

In stars like the Sun, the main fusion reaction occurring at the core is the fusion of hydrogen into helium, as shown in Eq. (14). In addition, however, as the stars age,

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[35] A given triton doesn't have to wait 12 years to decay, because some of them decay in a much shorter time. The "half life" means that approximately half will have decayed before 12 years, and half later.

[36] In reality, there were trace amounts of ^{2}H and ^{3}He that did not capture neutrons, and there were even trace amounts of ^{6}Li, ^{7}Li, and ^{9}Be created (jumping over the mass holes at $A = 5$ and $A = 8$) but the reactions that created them were highly unlikely.)

name of atom	symbol	# of e^-	name of nucleus	symbol	nuclear mass (u)	stable?
neutron	^1n	0	neutron	n	1.008 664	no; τ = 10 min 14 sec
protium	^{1}H	1	proton	p	1.007 276	yes
deuterium	^{2}H	1	deuteron	d	2.013 553	yes
tritium	^{3}H	1	triton	t	3.015 500	no; τ = 12.3 years
helium-3	^{3}He	2	helion	h	3.014 932	yes
helium-4	^{4}He	2	alpha particle	α	4.001 506	yes

electron mass: 0.000 548 580 u neutrino mass: less than 10^{-8} u

Unit Conversions $E_0 = mc^2$

$$\begin{aligned} 1\text{ u} &= 1.660\,538 \times 10^{-27}\text{ kg} \\ &= 931.494\text{ MeV}/c^2 \\ 1\text{ J} &= 1\text{ kg m/s}^2 \\ 1\text{ eV} &= 1.602 \times 10^{-19}\text{ J} \end{aligned}$$

Decay reactions

$$\begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e \\ t &\rightarrow h + e^- + \bar{\nu}_e \end{aligned}$$

Table 4: A list of the low atomic number elements and their isotopes. While the names of the neutral atom and its nucleus are different, the symbol for the nucleus is sometimes identical with that of the atom (e.g., ^{3}H, tritium or helium-3) and sometimes it has its own symbol (e.g., t, triton). Also included are conversion factors between SI units, like joules, and those common in nuclear physics, like MeV and u, as well as the decay processes of the neutron and the triton, the two smallest unstable particles. Finally, note that "hydrogen" sometimes refers to the sum of protium, deuterium and tritium, and sometimes only to protium.

they gravitationally contract and the temperature rises in the core, allowing more highly charged nuclei to overcome the Coulomb repulsion and fuse. You might expect that the next fusion likely to occur is two α particles joining to form ^{8}Be, and you'd be correct

$$2\alpha \rightarrow {}^8\text{Be}. \tag{24}$$

However, this is an endothermic reaction and ^{8}Be is unstable to splitting right back into two α particles with a half life of 67 as (67×10^{-18} s). However, it turns out that while this time is short, it is longer than the time spent by two α particles to simply scatter past each other (i.e., collide but repel each other due to the fact that they are both positively charged). This means that some of the ^{8}Be nuclei exist long enough for a third α particle to join them and create ^{12}C

$$3\alpha \rightarrow {}^{12}\text{C} + \gamma. \tag{25}$$

This is called the "triple α process," and it is exothermic, releasing 7.4 MeV of energy. Now that there's carbon, adding another α particle results in oxygen, specifically ^{16}O. These reactions in stars are the reason that both carbon and oxygen are very common elements in the universe (and therefore here on Earth).

This process, which might be called "successive α capture," continues to create heavier and heavier elements as the star continues to contract and heat up in the core. For example, ^{20}Ne is formed when ^{16}O absorbs an α particle. The hotter, denser core is needed for the highly charged nuclei to come close enough to fuse. Remember that ^{16}O has 8 times the electric charge as one α. In addition, all of these reactions are exothermic, which means that they supply the star the energy it needs to stave off gravitational collapse. They don't supply enough energy to eliminate a slow contraction, but they do preclude a violent collapse. The exothermic nature of the fusion reactions stop, however, once the elements iron and nickel are reached. Fusing these elements to create even heavier elements actually takes more energy than the reaction releases, and at this point the star has come to the end of its life.

Supernova nucleosynthesis

The fusion of elements heavier than iron and nickel do take place in stars, but they do so at the violent end. When the exothermic fusion reactions have been exhausted, there is nothing to stop gravity from winning the battle, and the core of the star collapses quickly, creating a hot, dense region which allows more fusion reactions to occur. The most common reaction is again successive neutron capture. Iron and nickel absorb many neutrons, increasing their mass, until there are too many neutrons for the nucleus to be stable, at which point they become radioactive, undergoing β decay, which transforms some of the neutrons into protons (enough to make the nucleus stable). Then the nucleus continues to capture more neutrons, and the sequence repeats. In this way, all of the elements heavier than iron and nickel are created.

At the end of the collapse, the core of the star becomes a white dwarf, neutron star, or black hole, depending on the star's initial mass.

Artificial fusion

Because the energy released in nuclear fusion reactions is so great, it would be highly desirable to be able to construct a fusion reactor here on the Earth to mimic what the Sun does so easily in its core. The main problem with this idea is that in order for the positively charged nuclei to approach each other close enough for the strong nuclear force to dominate (and for fusion to occur), the nuclei must be moving very fast, which means that they must be hot and dense. Unfortunately, here on Earth we don't have the strong gravity of the Sun to help us contain the plasma,[37] so when we heat up a gas of hydrogen, for example, it wants to expand. There have been several methods developed to overcome this problem, two of which are called "magnetic fusion" and "inertial fusion," and without going into the details of these methods, we can look at the possible nuclear reactions that might be worthwhile exploring here on Earth.

Zeroth generation fusion reaction — the Sun

As we saw above, the net reaction that occurs in the sun is to fuse four protons into one α particle

$$4p \to \alpha + 2e^+ + 2\nu_e + 2\gamma.$$

Remember that this is not a single reaction (the probability of four protons being at the same location at the same time is vanishingly small) bur rather a sequence of five separate reactions, called the "proton-proton chain." Such a large number of reactions means that the entire sequence is extremely unlikely, even in the solar core. However, the core of the Sun is large, dense, and hot, so that even though the likelihood is low, there are enough chances that success is guaranteed. But here on Earth we do not have that luxury. It would be much simpler, and much easier, if we could determine a single exothermic reaction to use for generating fusion power. Luckily here on Earth we have a supply of two isotopes of hydrogen, deuterium (1 proton and 1 neutron) and tritium (1 proton and 2 neutrons), and these can be used advantageously.

First generation

About 0.015% of all the hydrogen here on Earth is actually deuterium, and is is simple to fuse two deuterium nuclei

$$d + d \to \begin{cases} n + h & (3.269\ \text{MeV}) \\ p + t & (4.033\ \text{MeV}) \end{cases}$$

Half the time, the two deuterium nuclei form a neutron and a helion, and the other half of the time they form a proton and a triton. It turns out that one cubic mile of seawater has enough deuterium, and hence enough stored nuclear energy, to supply the world's entire energy demands for about 25 years.[38]

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[37] "Plasma" is the term denoting an ionized gas, whose atoms are not neutral, but have the electrons removed.

[38] We can prove this with an order-of-magnitude estimate. At a density of about 1000 kg/m^3, one cubic mile of water has a mass of 4.17×10^{12} kg. Each water molecule has a mass of about 18 u (its molecular

Even though tritium is unstable with a half life of 12.3 years (see Table 4), which means that it doesn't occur naturally, the second reaction above can produce some tritium, and that tritium can be used in another "first-generation" fusion reaction

$$d + t \rightarrow \alpha + n \quad (17.59 \text{ MeV})$$

Obviously, the *d-t* reaction releases quite a bit more energy than either of the *d-d* reactions, and so is more attractive, but there are two problems. First, tritium must be produced, which costs some energy, but also this reaction releases an energetic neutron. From an engineering standpoint, it is not as easy to extract energy from neutral particles as it is from charged particles (such as the α that is also produced). In addition, the neutrons tend to make the walls of the containment vessel radioactive (remember the process of successive neutron capture?). Even with these downsides, this will be the primary reaction used in planned magnetic fusion devices, such as tokamaks.

Second generation

Are there any reactions that do not suffer from these two problems? Yes. One such reaction is between a deuteron and a helion, the nuclei of deuterium and helium-3 (^{3}He),

$$d + h \rightarrow \alpha + p \quad (18.35 \text{ MeV})$$

Because no neutrons are emitted and quite a bit of energy is released, it appears that we have found the ideal fusion reaction. However, while there seems to be enough deuterium on Earth, there is very little ^{3}He — there are only about 1.3 atoms of ^{3}He for every million atoms of ^{4}He, and helium is found primarily in natural gas deposits. In addition, the probability of fusion is significantly reduced compared with the first-generation reactions because the helion is doubly charged (2 protons and 1 neutron), which means that it and the deuteron repel each other *twice* as strongly as two deuterons. That is, they have to overcome a stronger repulsion, making it less likely that they will get close enough to "fuse." Finally, even though no neutrons are emitted in this reaction, some of the deuterons will fuse with each other producing neutrons via one of the first-generation reactions. For these reasons, an abundant source of ^{3}He is desired. It appears that the Moon could be that source.

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weight is 18.0153 g/mol), which means there are 1.39×10^{38} water molecules, each with two hydrogen atoms, or 2.78×10^{38} hydrogen (or deuterium) atoms. Since the fraction of deuterium is 1.5×10^{-4} of all hydrogen, we finally have, in our cubic mile of seawater, 4.17×10^{34} deuterium atoms, or the same number of deuterons. If we assume that we use the first generation *d-d* reaction then and average of 3.651 MeV can be released for every two deuterons. For the deuterons in our cubic mile of seawater, we can get at most (if we extract *all* the deuterium)

$$4.17 \times 10^{34} \, \frac{3.651 \text{ MeV}}{2} = 7.61 \times 10^{40} \text{ eV} \approx 10^{22} \text{ J}.$$

Estimates of remaining fossil fuels vary widely, but are typically around 10^{24} J. In 2005, the world's total energy "consumption" was 5×10^{20} J which means that our cubic mile of seawater would be able to supply about 25 years of energy for the entire world.

Third generation

If enough ^{3}He can be found, the so-called "third-generation" fusion reaction might be feasible, *h*-*h* fusion:

$$h + h \rightarrow \alpha + 2p \quad (12.86 \text{ MeV})$$

The good qualities are, again, that it is highly exothermic and there is no possibility that neutrons will be produced. However, the electric repulsion force between the two helions is twice that of the second-generation fuels, and four times that of the first-generation reactants. So, although this is by far the "cleanest" reaction in principle, the practical difficulties of low fusion probability and low abundance of ^{3}He are difficult to overcome. Recently, several people have proposed that there is adequate ^{3}He in the regolith of the lunar surface (the regolith is the dusty sand that covers most of the moon). If this is true, and if it is economically feasible to extract it from the regolith, and if we can develop cheap transportation to and from the Moon, then this third-generation fusion reaction might be the solution.

Radioactivity

> *To the chemists of the 19th century the atom and the element represented each in its sphere the uttermost limit of chemical subdivision or disintegration, and at the same time the point beyond which it was impossible for experimental investigation to proceed. If it were queried what there was beyond, nothing but more or less vague and fruitless speculations were forthcoming. This line of demarcation, for so long regarded as insurmountable, has now been swept away, at all events in principle. Nowadays the inner structure of atoms and the laws regulating that structure belong to the problems that can be made the subject of discussion in a thoroughly practical and at the same time fully scientific manner, thanks to the exactness of the measurements which have been taken. The results already arrived at are not only of the utmost importance in themselves, but derive perhaps a still greater significance from the numerous possibilities, wholly unsuspected ten or twelve years ago, which have been thrown open for the continuance of the work of investigation in this department of science.* — Presentation of 1908 Nobel Prize in Chemistry to Ernest Rutherford

Historical Background

On March 1, 1896, Antoine Henri Becquerel discovered radioactivity. His motivation was to look for X-rays (recently discovered in November 1895 by Roentgen) from phosphorescent materials, and he was familiar with the phosphorescent properties of uranic salts,[39] which, of course, contain uranium. He wrapped a photographic plate (a piece of glass covered with a photographic emulsion) in black paper, and placed on the paper a piece of a phosphorescent substance. He exposed the combination to the sun for several hours, in the expectation that the sunlight would cause the uranium to phosphoresce, and that

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[39] Specifically, Becquerel used uranyl disulfate, $K_2UO_2(SO_4)_2 2H_2O$.

phosphorescent light from the uranium would penetrate the black paper and leave an image on the emulsion. It worked as expected, but then there came a week of cloudy weather and the sun did not shine. Becquerel put his plates and uranium in the cupboard for a week (without being exposed to sunlight), and for some reason he decided to develop those plates, even though he expected nothing. However, his intuition was correct, and he discovered that the plates showed an image, just as if it had been in the sun!

This was the first step in the discovery and understanding of radioactivity, and the "rays" that must have been emanating from the uranium were called "Becquerel rays."

One week later, on March 9, Becquerel discovered that the rays could discharge an electroscope, which meant that the rays were charged. At that time there were two types of rays known, cathode rays (which had been shown by Thomson to be electrons) and light rays (which had been shown by Maxwell and Hertz to be electromagnetic waves). Of course, the "X-rays" of Roentgen would turn out to be high-frequency electromagnetic waves, and the Becquerel rays were nothing but electrons, but that was not clear for quite a while. In fact, the uranium sample emitted both electrons and α-particles, but the α particles were easily stopped by the paper and so did not contribute to the darkening of the emulsion.

Becquerel's family was quite prodigious. Along with his grandfather, Antoine César, his father, Alexandre Edmond, and his son, Jean, the four of them continuously held the chair of physics at the Museum of Natural History in Paris from 1838-1948, a span of 110 years! The four of them studied many aspects of physics, including thermoelectric phenomena, luminescence, infrared spectroscopy, magnetic polarization by crystals, and magneto-optics. In fact, after his discoveries, Antoine Henri said, "These discoveries are only the lineal descendants of those of my father and grandfather on phosphorescence, and without them my own discoveries would have been impossible."

The second step in the understanding of radioactivity came in 1898 when Marie and Pierre Curie found that the element thorium ($Z = 90$) was also radioactive. In addition, they discovered two new elements due to their radioactivity, which they named polonium and radium. These latter two they found by chemically isolating them from their sample of pitchblende. Pitchblende is a black mineral, mainly $\mathrm{UO_2}$, but it also has some impurities, and these are what the Curies found. The Curies won the 1903 Nobel Prize in Physics, jointly with Becquerel, for their investigation into radioactivity. In addition, Marie won the 1911 Nobel Prize in Chemistry for the discovery of polonium and radium. At this point, even though radioactivity was not at all understood yet, two questions had become common: Where did the energy associated with the activity come from? and: Were *all* elements radioactive (but perhaps with very long lifetimes)?

The third event in our story occurred in 1899 when Rutherford deduced that there were two different types of Becquerel rays: α rays and β rays. They were distinguished by their ability to penetrate matter: α rays were easily absorbed in a few centimeters of air (Becquerel's black paper absorbed them); β rays were more penetrating—it took several cm of air before they were absorbed. Later it was determined that α rays were actually the nuclei of ^{4}He, and β rays were electrons. In 1900, Paul Villard in Paris observed a third type of ray emitted by radium that was even more penetrating than β rays (but it was not charged), and he called them γ rays. These, of course, were photons, but that was not determined until 1914.

The final piece of the puzzle, the fourth step, was put in place in 1902 when Rutherford and **Frederick Soddy** [Nobel Prize, Physics, 1921] developed their "transformation theory." This theory was an explanation of what was occurring during radioactive decay: In modern terminology, a "parent" nucleus was transformed into a "daughter" nucleus when an α or β ray was emitted. Soddy had originally suggested the term "transmutation theory," but Rutherford objected, believing that people would think they were proposing medieval alchemy. In fact, though, that was *exactly* what they were doing: radioactivity was changing one element into another! Another part of the transformation theory was the observation that the process of transformation decayed exponentially with time. They discovered this while investigating a gas called "thorium emanation," which we now know was an isotope of radon, ^{220}Rn. Most of the daughter elements were solids at room temperature, so that they remained locked in the original rock. Radon, however, is a gas, and so when it is created as a part of a series of radioactive decays it can be easily isolated. Rutherford and Soddy found that no matter when they started observing, the activity of ^{220}Rn was reduced by half in one minute, and this allowed them to describe radioactivity mathematically as an exponential decay.

Natural and artificial radioactivity

When radioactivity was initially being investigated, and it was realized that the "rays" carried enormous amounts of energy, the answers to two questions were being sought by most scientists. First, where did the energy come from? Initially, the energy was thought to be contained in the atom, but in 1903 Pierre Curie and Albert Laborde showed that 1 g of radium could heat 1.3 g of water from melting to boiling in 1 hour. This was quite a bit of energy, and it caused some to consider abandoning the principle of the conservation of energy. Second, were all elements radioactive? It was possible that elements only appeared to be stable, but in reality had very long half lives.

Natural radioactivity

If we were to wait a long enough time, then all radioactive elements would decay, and only stable elements would be left. The age of the Earth is finite, however, and any radioactive elements that were present at the time of the Earth's formation must have a sufficiently long half life in order to still be around in sufficient quantities to be observed. There are three nuclides that have half lives that are comparable to the 4.5 Gy age of the Earth. Those three are listed in the following table (along with ^{237}Np).

element	τ	series	stable end
^{232}Th	14 Gy	$4n$	^{208}Pb
^{237}Np	2.3 My	$4n+1$	^{209}Bi
^{238}U	4.5 Gy	$4n+2$	^{206}Pb
^{235}U	0.71 Gy	$4n+3$	^{207}Pb

Each of these elements is at the start of a "radioactive decay series," in which successive α and β decays occur until a stable element is reached. Since each α decay changes A by

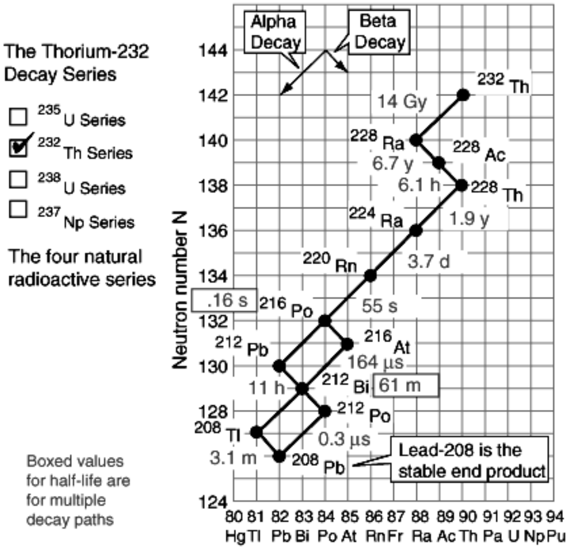

Figure 4: A nuclide chart for the $4n$ decay series, which starts with ^{232}Th and ends with the stable element ^{208}Pb. In between it creates several other nuclei that were observed by Rutherford and Soddy, for example, ^{224}Ra and ^{220}Rn. Note that some nuclei, for example ^{216}Po and ^{212}Bi, can decay in two different ways. From HyperPhysics, `http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/radser.html`

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4 units, and β decays do not change A, this means that each step in a given series will consist of isotopes of atomic mass number A that differ by 4. Hence, all the elements in the ^{232}Th series, for example, will have mass numbers that are multiples of 4, or given by $4n$. Those of the ^{237}Np series will have mass numbers given by $4n + 1$, etc.

The isotopes in the ^{237}Np series are not naturally occurring on Earth because of the short half life of ^{237}Np compared to the age of the Earth. All other elements *are* observed, with varying abundances. The reason is because it is thought that a supernova (or supernovas) provided the material that eventually condensed to form the solar system. The physics of supernovas is fairly well understood, including the heavy elements that are produced in nuclear reactions during the violent explosion. Calculations show that approximately equal numbers of ^{238}U and ^{235}U are produced. However, the half life of ^{235}U is much shorter than that of ^{238}U, so that today in the Earth, there is significantly more ^{238}U than ^{235}U.

There are no isotopes of any element above lead ($Z = 82$) in the periodic table that are stable. This is why lead is the common end product of each series. Bismuth ($Z = 83$) has one "quasi-stable" isotope, ^{209}Bi, whose half life is 19×10^{18} years. It was originally thought that ^{209}Bi was stable, since no radiation had been detected. However, the mass excess of ^{209}Bi predicts that it should α-decay into an isotope of thallium, ^{205}Tl, which it does.

Rutherford and Soddy, for example, started with thorium (^{232}Th), which decays after several steps into thorium X (^{224}Ra), and this then α decays into ^{220}Rn, "thorium emanation." (See Fig. 4.) This thorium emanation, a gas, is what led them to their discovery of the exponential decay law. They observed that the activity of this gas decreased rapidly, with a half life of about 1 minute (today it is measured at $\tau = 55.6$ s). Determining this sequence of events was not simple. At first, they thought that thorium itself transformed into the emanation. However, they soon discovered a previously unknown component of thorium compounds, which they called thorium X and which could be chemically separated from thorium. After separation, they found that it was the *thorium X* that produced the emanation. This led them to believe that thorium itself was inactive. A second discovery showed that the separated thorium continued to produce thorium X, and the activity of the separated thorium X decreased with time. A glance at Fig. 4 reveals that the situation is more complex that this, and it's a wonder that Rutherford and Soddy were able to deduce what they did.

There are two other long-lived radioactive isotopes that act as "clocks" and allow us to determine the ages of rocks in the Earth. These are rubidium (^{87}Rb) and potassium (^{40}K). Finally, ^{14}C is continually produced in the atmosphere from the bombardment of cosmic rays, and this is the basis for "carbon dating," which can determine the ages of objects that have been alive in the past, such as trees.

Artificial radioactivity

In 1934, Irène Joliot-Curie and Frédéric Joliot produced the first "artificial" radioactive substance, phosphorus-30. They bombarded aluminum[40] with α-particles from the decay of polonium

$$^{27}\mathrm{Al} + {}^{4}\mathrm{He} \rightarrow {}^{30}\mathrm{P} + n.$$

^{31}P is the only stable isotope of phosphorus, and ^{30}P undergoes β^+ decay into the stable isotope ^{30}Si

$$^{30}\mathrm{P} \rightarrow {}^{30}\mathrm{Si} + e^+ + \nu_e,$$

with a half life of about 2.5 minutes. Thus, they were able to "activate" normal matter, i.e., take stable aluminum and create radioactive phosphorus, and for this they received the Nobel Prize in Chemistry for 1935. They had found, in effect, the "philosopher's stone," that age old quest to turn one element into another. Although they didn't create gold, their work had profound implications for the human race.

Subsequently, Enrico Fermi and his laboratory in Rome bombarded stable elements with neutrons, and were able to create many new radioactive isotopes. For this he won the Nobel Prize in Physics for 1938. It turns out that many particles will work as a tool to transmutate a nucleus: protons, deuterons, α-particles, neutrons; but neutrons, due to their neutral electric charge, tend to have the easiest time penetrating the nucleus.

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[40] Their sample was 100% of the isotope ^{27}Al because it is the only stable isotope of aluminum.

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Particle Discovery Timeline

1874 electron coined by Stoney

1897 electron discovered by Thomson

1905 light quantum proposed by Einstein

1911 nucleus discovered by Rutherford

1913 isotope coined by Soddy

1920 proton named

1925 Rhenium [Re] discovered by Walter Noddack and Ida Tacke (later Noddack), by concentrating it from gadolinite. They claimed they discovered technetium [Tc] (they named it masurium) but this is controversial.

1926 photon coined by Gilbert Lewis

1928 positron predicted by Dirac (discovered 1933)

1930 neutrino proposed by Pauli (observed 1956)

1931 deuterium discovered by Harold Urey (^{1}H called 'protium')

1932 neutron discovered by Chadwick

1933 positron discovered by Anderson, and Blackett & Ochialini using a cloud chamber.

1935 meson predicted by Yukawa

1937 muon discovered by J. C. Street and E. C. Stevenson in a cloud chamber.

1937 Technetium [Tc] discovered by Emilio Segré [Nobel Prize, Physics, 1959] and Carlo Perrier.

1940 ^{14}C discovered by Martin Kamen and Sam Ruben at the University of California, Berkeley, Radiation Laboratory.

1941 nucleon invented by Christian Møller

1946 lepton invented by Pais and Møller

1947 pion discovered

1947 Λ^0 (uds) in cosmic rays; first "strange" baryon; long lifetime (10^{-8} s), hence weak force.

1949 Δ resonances by Fermi and H.L. Anderson

1950 π^0 discovered

1950s Technetium (^{43}Tc) identified in stellar spectra by Paul Merrill at Mt Wilson.

1954 baryon coined

1955 anti-proton discovered by Owen Chamberlain and Emilio Segré, for which they earned the Nobel Prize in Physics, 1959.

1956 anti-neutron discovered

1956 electron neutrino observed by Cowan and Reines

1961 ρ, ω, η, and K^* discovered

1962 muon neutrino discovered by Lederman, Schwartz, and Steinberger at Columbia University

1962 hadron coined

1964 quarks discovered by Friedman, Kendall and Taylor (1990 Nobel)

1964 Ω^- discovered at Brookhaven

1964 charm quark proposed

1968 quarks experimentally confirmed by Friedman, Kendall, and Taylor; a SLAC-MIT collaboration

1974 charm quark discovered by Richter and Ting at SLAC and Brookhaven (the $J/\psi = c\bar{c}$ was discovered simultaneously in November: the "November revolution")

1975 tau lepton discovered by Martin Perl at Stanford

1977 bottom quark discoverd by Lederman at Fermilab ($\Upsilon = b\bar{b}$)

1983 $W^{\pm}$ and Z^0 by Carlo Rubbia and Simon van der Meer at CERN

1995 top quark discovered at Fermilab, CDF and D0 collaborations

2000 tau neutrino discovered at Fermilab

Some Advanced Concepts

Cosmic Rays and Muons

> *Coming out of space and incident on the high atmosphere, there is a thin rain of charged particles known as the primary cosmic radiation.* — Cecil Powell [Nobel Prize, Physics, 1950]

The charged particles that make up the "primary" cosmic rays are protons, α particles, heavier nuclei, and electrons, and they impact the Earth from all directions and with various energies. Most of these are protons (about 80%), second in abundance are α particles (about 14%), while electrons make up less than 1%. When they impact nuclei in the atmosphere — mostly oxygen and nitrogen nuclei — their energies are such that they create "showers" of hadrons, mostly pions, along with some kaons, and anti-protons, and anti-neutrons. These then decay into photons, electrons, positrons, neutrinos, and muons (which themselves decay into electrons and neutrinos). These are all called "secondary" cosmic rays.

Where do the primary cosmic rays come from? Some come from the sun (mostly due to solar flares), most come from galactic supernovae, and a few with the highest energy are suspected to originate from outside the Milky Way. You might suspect the solar wind—a neutral plasma that consists of low energy protons, electrons, and helium nuclei—as a source of cosmic rays. Due to their low energies, however, these particles are stopped from reaching the atmosphere by the Earth's magnetic field, except in the polar regions. While they have enough energy to cause aurora, they do not cause showers of secondary subatomic particles.

How many are there? About 1 charged particle per second per cm^2 impacts the Earth.[41] This is a far cry from the 6×10^{10} neutrinos s^{-1} cm^{-2} that come from the Sun.

What are their energies? The typical kinetic energy of these particles is about 10 MeV to 100 MeV, although there are some at higher energies. Figure 5 shows the distribution of the measured energy per particle. In fact, the cosmic ray with the highest energy has been measured at 48 J! These ultra-high energy cosmic rays are suspected to be extra-galactic, as there is no plausible mechanism of acceleration to these energies by a supernova, for example. Again, compare these energies to those of solar neutrinos that have only 0.26 MeV.

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[41] Henley and Garcia, *Subatomic Physics*, page 597.

What happens to the secondary cosmic rays? The pions decay via the following modes

$$\pi^0 \rightarrow 2\gamma \tag{26}$$
$$\pi^\pm \rightarrow \mu^\pm + \nu, \tag{27}$$

where the neutral pions decay electromagnetically with an average lifetime of 8.4×10^{-17} s, and the photons subsequently create electron-positron pairs. Most of the energy of the original cosmic ray follows this path. Some of the energy goes into charged pions, which decay into muons with an average lifetime of 2.6×10^{-8} s. This long lifetime indicates that the decay is due to the weak interaction, and is therefore relatively unlikely. The muons then decay into electrons (or positrons) and neutrinos

$$\mu^\pm \rightarrow e^\pm + 2\nu, \tag{28}$$

and their average lifetime is 2.2 μs, also a weak interaction.[42]

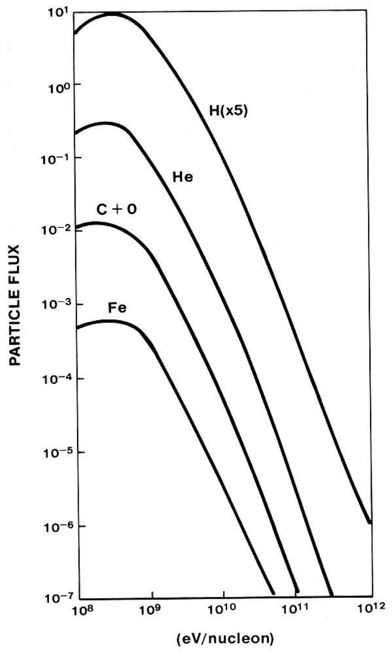

Figure 5: The energy spectrum of the different nuclei that make up cosmic rays. Carbon and oxygen are lumped together. From Friedlander, *Cosmic Rays*, Figure 6.4.

What happens to these secondary cosmic rays as they pass through the atmosphere? First of all, in addition to possible decay, the charged particles cause ionization of the atmospheric molecules and therefore lose energy. For example, a typical muon loses about 2 GeV of kinetic energy before it hits the ground (if it hasn't decayed yet), and by the time they do reach the ground, the average muon energy is about 4 GeV. Secondly, the showers spread out laterally from the direction of the primary cosmic ray. The main hadronic core (pions, etc.) covers a few meters by the time it hits the ground, and the electromagnetic particles (electrons, positrons, photons) have spread further, about 100 m. Finally, the muons have spread the furthest, almost 1 km.

The Pauli Exclusion Principle and classification according to spin

The spin quantum number s is a measure of the particle's intrinsic angular momentum, i.e., it's rotation, or spin.[43] Of course, particles can also have orbital angular momentum (like the electrons orbiting a nucleus in an atom, or like the Earth orbiting the Sun), but their intrinsic angular momentum is a property of the particle, like mass or electric charge, that does not change. Angular momentum is one of those properties that the laws of quantum mechanics state must be quantized, or discrete. The quantum number can

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[42] Recall that the weak force is responsible for changing one family of quarks or leptons into another.

[43] You can visualize this as if the particle were actually spinning on an axis, like the Earth. However, the elementary particles are thought to be point objects, and therefore spinning does not make any sense from the classical point of view. You must be content to accept the fact that spin is a fundamental property of particles, and does not have to associated with any rotation.

take on either integer or half integer values, but no others. Just like placing subatomic particles in three categories depending on their mass (lepton, meson, or baryon), we can place all particles into one of two categories depending on their spin.

fermions	—	half-integer spin, $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...$
		⇒ MUST obey the Pauli Exclusion Principle
bosons	—	integer spin, $s = 0, 1, 2, ...$
		⇒ no exclusion principle

Fermions are named after Enrico Fermi and Paul Dirac, who developed “Fermi-Dirac statistics” to describe this type of particle, and bosons are named after Satyendra Bose and Albert Einstein, who developed “Bose-Einstein statistics.” All of the twelve elementary particles have $s = \frac{1}{2}$, or in the jargon of particle physics, they are “spin one half” particles.

What is Pauli’s exclusion principle? In 1924 he stated it in the following manner

> In the atom there can never be two or more equivalent electrons for which ... the values of all quantum numbers coincide. If there is an electron in the atom for which these quantum numbers have definite values then the state is “occupied.”

Prior to this, from observations of atomic spectra when the atoms are placed in magnetic fields, it had been determined that each electron had three quantum numbers, n, ℓ, and m_ℓ. The first, n, is a label for the shell, ℓ labels the subshell, and m_ℓ is the so-called “magnetic quantum number” because it would split the spectral lines only when the atom was placed in a magnetic field. It was realized that *two* electrons could be placed in each of these quantum states, and so a fourth quantum number for the electron, m_R, was proposed by Samuel Goudsmit which could take on the two values $m_R = \pm\frac{1}{2}$. This now doubled the number of allowed states, and Pauli’s principle works. As you might guess, m_R is nothing but m_s, the z component of the electron’s spin. That is, two electrons can occupy a single state, but with the caveat that one must be spin up and the other spin down. This implies, therefore, that they are really occupying *different* quantum states, since the external configuration (i.e., position) as well as the internal configuration (i.e., spin) must be included in the definition of “quantum state.”

The strong nuclear force

If nucleons are color neutral, what holds them together in the nucleus of an atom? The answer is the *residual* color force, also called the strong force, that exists because the color force between two color-neutral nucleons does not exactly cancel. The situation is similar to the force that electrically neutral atoms exert on each other. This residual electric force exists because the electric charge in the atoms (the positive charge in the nucleus and the negative charge in the electron cloud) are not in exactly the same location. This means that they act like electric dipoles, and two electric dipoles exert a force on each other that has a shorter range than the Coulomb force between bare charges.

The Coulomb force falls off as $1/r^2$, and you can show that the force between dipoles falls off as $1/r^4$. It becomes weaker more quickly as the dipoles move apart, and therefore they must be close together to feel a significant force. This weak residual force is also known as the "van der Waals" force, postulated by **Johannes van der Waals** (Nobel Prize, Physics, 1910) to obtain an equation of state for a non-ideal gas that included a liquid phase.

The exact same partial cancelation occurs with the color charge of the quarks in a nucleon. The fact that the three quarks in a nucleon are not in exactly the same location means that there will be a nonzero residual color force, which is usually called the strong nuclear force. If this is truly a "force," then using our new description of forces it must be mediated by an exchange particle. This exchange particle is a pion, or π meson, and is what Yukawa envisioned as holding the nucleus together. He knew that the force must be short range, because atomic nuclei do not compress as more nucleons are added—they have a relatively constant density. As Yukawa correctly deduced, this implies that nucleons only interact with their "nearest neighbors," and do not feel any attraction to distant nucleons on the other side of the nucleus. As I have stated on page 19, short-range forces must be mediated by massive exchange particles, and a range of 1 fm corresponds to a mass of about 100 MeV/c^2, which is very close to the mass of the pion.

One final note on terminology. Quarks are the only elementary particles that have color and interact via the color force. Baryons and mesons are the only particles that are composed of quarks. Therefore, baryons and mesons are the only particles that interact via the strong nuclear force. Collectively, baryons and mesons are called "hadrons," meaning a particle that exerts and feels the strong force.

Shell model

Certain nuclei are especially tightly bound, which means that they have a large binding energy per nucleon, a large B/A. One of these is ^{4}He, as well as the other "even-even" nuclei (those with an even number of protons and an even number of neutrons), e.g., ^{12}C, ^{16}O, and ^{20}Ne. In an attempt to understand this structure and regularity, quantum mechanics has been used to create a "shell model" of the nucleus, where the nucleons arrange themselves in shells similar to the electron shells in an atom. This model is more complicated than the atomic model because in the atomic model the electrons all orbit in the strong electric field of the nucleus and the inter-electron interaction is weak. In the nuclear shell model, however, there is no central object in a nucleus, however, so each nucleon moves in a "field" due to all the other nucleons combined. This makes the nucleus a "many-body" problem at its most fundamental level.

Electrons in atoms are the most tightly bound in the inert gases, listed in the right-most column of the periodic table. This is because in these atoms the outermost electron shell is filled. Their atomic numbers are

2	10	18	36	54	86	118
He	Ne	Ar	Kr	Xe	Rn	Uuo

where I've listed the element symbol below these special values of the atomic number.

There are also special values of the number of nucleons, and these are called MAGIC NUMBERS. For nuclei, these magic numbers are

2	8	20	28	50	82	126

If a nucleus has either N or Z equal to one of these "magic numbers," then that nucleus is especially tightly bound. If both N *and* Z are magic, then that nucleus is called "doubly magic." For example, ^{4}He and ^{16}O are both doubly magic. This means that an extra nucleon added to one of these nuclei is especially loosely bound. ^{4}He takes this to an extreme, since it requires about 20 MeV to remove a proton or neutron,[44] but an additional proton or neutron is not bound at all.[45]

The higher magic numbers are less striking, but they exhibit observable effects, nonetheless. Tin, for example ($Z = 50$), is the element with the largest number of stable isotopes, ten. Also, in α decay, when the emission of an α particle removes the 125th and 126th neutrons from a nucleus (which should be strongly bound), the resulting energy of the α particle is much lower than when the 127th and 128th neutrons are removed (which are weakly bound).

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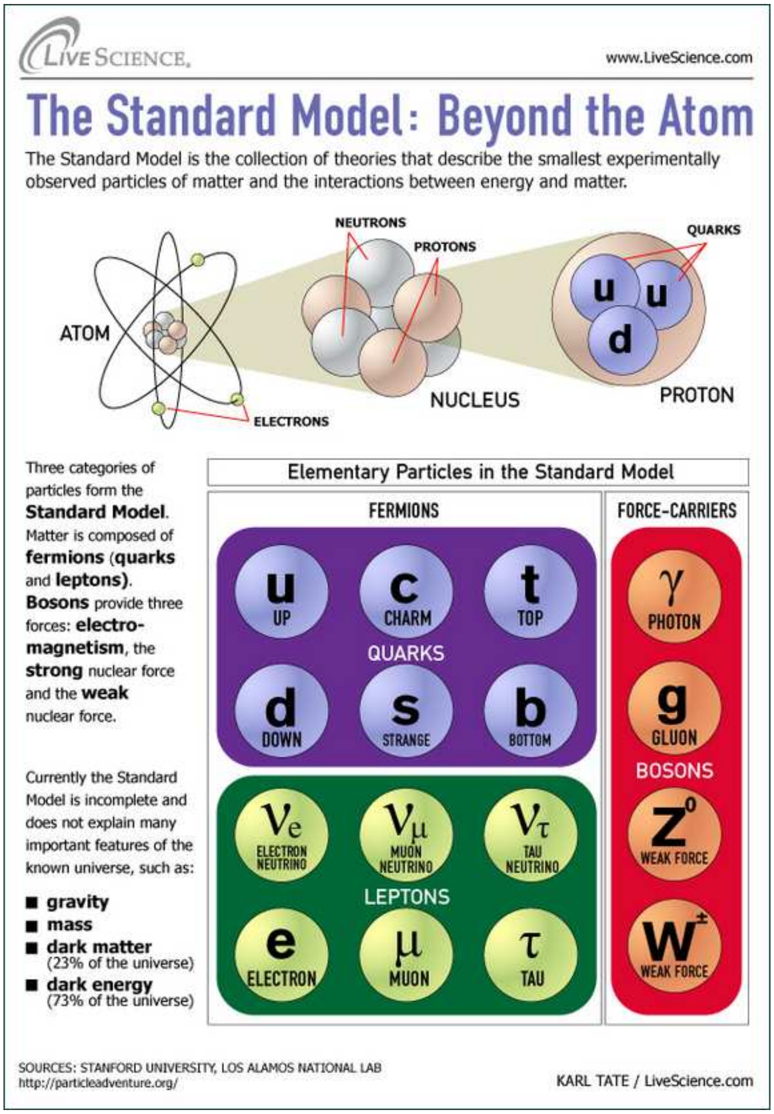
[44] You can calculate that it takes 20.58 MeV to remove a neutron and 19.81 MeV to remove a proton from an α-particle.

[45] Recall that there are no stable nuclei with $A = 5$.

Science without epistemology is — in so far as it is thinkable at all — primitive and muddled. — Albert Einstein



The Standard Model: Beyond the Atom

The Standard Model is the collection of theories that describe the smallest experimentally observed particles of matter and the interactions between energy and matter.

Three categories of particles form the **Standard Model**. Matter is composed of **fermions** (**quarks** and **leptons**). **Bosons** provide three forces: **electro-magnetism**, the **strong** nuclear force and the **weak** nuclear force.

Currently the Standard Model is incomplete and does not explain many important features of the known universe, such as:

- **gravity**
- **mass**
- **dark matter** (23% of the universe)
- **dark energy** (73% of the universe)

Elementary Particles in the Standard Model

FERMIONS			FORCE-CARRIERS
u UP	c CHARM	t TOP	γ PHOTON
	QUARKS		
d DOWN	s STRANGE	b BOTTOM	g GLUON
			BOSONS
ν_e ELECTRON NEUTRINO	ν_μ MUON NEUTRINO	ν_τ TAU NEUTRINO	Z^0 WEAK FORCE
	LEPTONS		
e ELECTRON	μ MUON	τ TAU	$W^\pm$ WEAK FORCE

SOURCES: STANFORD UNIVERSITY, LOS ALAMOS NATIONAL LAB
http://particleadventure.org/

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