How To Solve Problems

M. A. Reynolds

1 Advice

Practice, practice, practice; and have fun!

Problem solving, like many other endeavors, requires constant practice. If you take a break for a significant period of time, you will lose your skills. It's very similar to learning to play a musical instrument, or learning to speak a language: you must practice often (every day is desirable) for a short length of time. Thirty minutes a day is much preferable to three and a half hours once a week, even though the total time expenditure is the same. Frequent practice hones your skills and keeps them sharp.

In addition to practice, if you don't enjoy what you are doing, you will not stick with it, and you will not be motivated to succeed. Don't look on it as work, but as a game—race with your friends, reward yourself for a good session of problem solving.

There are many books that examine problem solving from different perspectives, but what they all have in common are lots of problems for you to solve. Since they try to be applicable to a wide variety of areas, their problems are in some sense generic, and you should find one of the books that tickles your fancy, has the types of problems you are interested in, and makes it fun. Several are listed in the bibliography

2 Before You Begin

Think!

In order to retain the fun-like properties and expunge the work-like properties, I find it helpful to not consider (at first) what the problem is asking you to do. If, from the very beginning, you are worried about calculating this quantity or that quantity, or drawing a certain picture, it will seem too much like work. Therefore, the first step should be just to understand the setup.

Ask yourself the following questions: "What is going on?", "Have I seen any part of this before?", "What's the easiest to understand?".

If you want to calculate something, calculate something that interests *you*. Chances are, if it is not a required answer, it will help you get to the required answer.

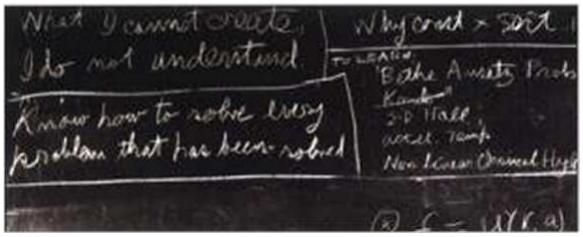

Figure 1: Feynman's blackboard

3 How to Become an Expert Problem Solver

Know how to solve every problem that has been solved. — Richard Feynman

I believe that one of the biggest issues that separates expert problem solvers from novices is confidence. When first glancing at a problem, an expert may have no more idea how to solve the problem than the novice, but he has confidence that he will be able to do so. He tries a method, and if that doesn't work, he tries another and another, until it's solved. Of course, he has many methods in his bag of tricks.

How does one gain this confidence? By successfully solving lots and lots of problems. How does one successfully solve lots of problems? By failing over and over again. The only way is to try as many different methods as it takes. Each failure, however, imparts knowledge, and you learn to recognize patterns – that is, in *this* kind of situation, *that* method works most of the time.

4 The Appearance of Success Without Working

No one is too good for work. — John D MacDonald

Many successful students appear to not have to work at all and still they earn good grades. However, I would bet that behind most (if not all) of these "brains," lies hours of hard work, unseen. Professional physicists have this type among them, too. One such example was Paul Ehrenfest. He had file cabinets full of problems that he had solved, but he hadn't published. When a junior colleague would come to his office to ask what he thought of a particular problem that the junior colleague had solved, many times Ehrenfest would open his file cabinet, pull out a file with his solution, look at it and tell the colleague that they got it right (or wrong, as the case may be).

The famous Nobel laureate Richard Feynman also was a monster calculator. He would sometimes declare an opinion on some controversial subject at a conference that seemed 'off the cuff.' But if you looked at his opinion, it was one that could not have been formed or concluded without many pages of calculation.

The moral of these stories is to solve, solve, solve. In fact, the quote at the beginning of Section 3 was one that Feynman had written on his blackboard in his office, and remained there after he died. He had also written, "*What I cannot create I do not understand,*" which

some have misquoted as "*That which I cannot derive I do not understand.*" For physics applications, I like the second version better.

5 How to Succeed Working Hard

In addition to being a great calculator, however, Feynman also had wonderful intuition. He often seemed to guess the answer to a problem, and only then figure out a mathematical way to show that his solution was correct. Freeman Dyson once said about him

> The reason Feynman's physics was so hard for ordinary people to grasp was that he did not use equations. The usual way theoretical physics was done since the time of Newton was to begin by writing down some equations and then to work hard calculating solutions of the equations. This was the way Bethe and Oppenheimer and Schwinger did physics. Feynman just wrote down the solutions out of his head without ever writing down the equations. He had a physical picture of the way things happen, and the picture gave him the solutions directly with a minimum of calculation. It was no wonder that people who had spent their lives solving equations were baffled by him. Their minds were analytical; his was pictorial.

Of course, this view was somewhat controversial, but I think that the truth lies somewhere in between: Feynman was both a great calculator *and* very intuitive. In any case, one of Feynman's colleagues facetiously suggested the "Richard Feynman Problem Solving Algorithm"

1. Write down the problem.
2. Think very hard.
3. Write down the answer.

This is similar to my advice above: before solving the problem, think about it, understand it deeply — and this understanding may require some calculation, but not necessarily a calculation of the required quantities. Of course, I would probably split step 3 into two parts: 3a. Calculate the answer, 3b. Write down the answer. Most of us are not able to do that calculation in our heads.

6 400-year-old advice

> *I think therefore I am. — René Descartes*

René Descartes was one of the first to discuss the so-called "scientific method." Such a method works as well for solving problems as it does for investigating nature — this is because they are the *same activity*! Descartes said in *Discourse on the Method*:

> A multitude of laws often hampers justice, so that a state is best governed when it has only a few laws which are strictly administered; similarly, instead of the large number of laws which make up logic, I was of the opinion that the four following laws were perfectly sufficient for me, provided I took the firm and unwavering resolution to stick to them clearly at all times.
>
> The first was never to accept anything as true if I did not clearly know it to be so; that is, carefully to avoid precipitate conclusions and preconceptions, and to include nothing more in my judgement than was presented clearly and distinctly to my mind, so that I had no reason to doubt it.
>
> The second, to divide each of the difficulties I examined into as many parts as possible, and as might be necessary for a proper solution.
>
> The third, to conduct my thoughts in an orderly fashion, by starting with the simplest and most easily known objects, so that I could ascend, little by little, and step by step, to more complex knowledge; and by giving some order even to those objects which appeared to have none.
>
> And the last, always to make enumerations so complete, and review so comprehensive, that I could be sure of leaving nothing out.

The second and third parts seem to be the most helpful. Break down each problem into small, easily understood pieces. Solve each piece; then put them together to solve the entire problem. Some problems appear to be unsolvable at the beginning, but that's because the solver tries to do it all at once. Forget about the final answer, but try to obtain information about a small part of the problem. Once you've succeeded there, attack another small part, then another, etc. Finally, the entire problem will be done.

A Bibliography

1. Ants, Bikes, and Clocks: Problem Solving for Undergraduates, William Briggs
   QA 63 .B75 2005

2. How to Solve It, G Polya, Princeton University Press, 1973 (2nd ed).
   QA 11 .P6 1971

3. Patterns of Problem Solving, Moshe F Rubinstein, Prentice-Hall, 1975.
   QA 63 .R82

4. The Whole Brain Solution, Tricia Armstrong, Pembroke, 2003.
   http://www.stenhouse.com/productcart/pc/viewPrd.asp?idProduct=8156